CONCLUSION

In the present work we have developed a finite-difference time splitting scheme to solve an anisotropic damped Kuramoto-Sivashinsky equation, which comes from the continuum theory and is an option to describe a surface eroded by ion bombardment. We dealt with realistic coefficients in a dimensionless analysis.

The MMS was employed for code verification, and a second-order convergence was detected for coarser meshes comparison, while results between first and second-order convergence came up for more refined meshes, suggesting a possible issue with the manufactured solution stability. Regarding the scheme's stability, the tests revealed that for $\Delta\tau \leq 2.0$, the numerical scheme was sufficiently stable with a grid spacing $\Delta X = 1.0$.

Spatiotemporal chaotic structures appeared for the undamped case, whose dynamics fell continuously for the long time. A chaotic oscillatory pattern rose from the simulation with $\alpha = 0.05$, reaching a better ordered structure than the one for the undamped result, while maintaining a highly constant kinematic after the emergence of the hexagonal modes. Defectless hexagonal periodic structures were obtained for higher values of the damping coefficient, with an angle of incidence $\theta = 30°$.

We performed a linear stability analysis, and used its results to guide our investigations on the evolution of the anisotropic DKS equation on preexisting patterns. There was a thorough discussion concerning the regimes before and after the emergence of hexagonal modes. Nanohole patterns were obtained in the long time for the majority of the evaluated initial patterns, using $\theta = 30°$. Small variations in the wavenumber of the monomodal starting morphology were able to influence the retention of defects in the final structure.

Based on the previous work of Rost and Krug, we investigated a case when the nonlinearities compensate each other. This was achieved using $\theta = 66.17°$, and an irregular oscillatory ripple structure with a clear orientation in the $\vec{1}_X$ direction was obtained.

In summary, it is possible to affirm that the obtained results were physically consistent with the sputtering phenomenon, reproducing ripple morphologies and nanohole pattern formation experimentally attained by other authors. Further works could make advances in the investigation of the anisotropic DKS equation coefficients, seeking the possible acquisition of new kinds of patterns with the simulations. A reproduction of nanodot structures would be particularly interesting.

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APPENDIX A - PUBLICATIONS

- ⋆ VITRAL, E., WALGRAEF, D., PONTES, J., ANJOS, G.R., MAGIAVACCHI, N. *Nano-patterning of surfaces by ion sputtering: numerical study of the Kuramoto-Sivashinsky equation by implicit time splitting*, Proceedings of the ABM Week 70th Annual Congress, 2015.

- ⋆ VITRAL, E., WALGRAEF, D., PONTES, J., ANJOS, G.R., MAGIAVACCHI, N. *Nano-patterning of surfaces by ion sputtering: numerical study of the damping effect on the anisotropic Kuramoto-Sivashinsky equation*, Proceedings of the 23rd ABCM International Congress of Mechanical Engineering, 2015.