Universidade do Estado do Rio de Janeiro
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Different viewpoints for the Gribov problem in Euclidean Yang-Mills Theories

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Tese apresentada, como requisito parcial para obtenção do título de Doutor, ao Programa de Pós-Graduação em Física, da Universidade do Estado do Rio de Janeiro.

Orientador: Prof. Dr Marcio André Lopes Capri

Coorientador: Prof. Dr Matthieu Tissier

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DEDICATION

This thesis is dedicated to my mom Cintia and to my brother Daniel.

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ABSTRACT

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The infrared regime of Yang-Mills theories is still an unsolvable problem in theoretical physics. The attempt to describe in a clear and suitable way the phenomenon of confinement of gluons and quarks is far from being finished. In this manuscript, we study two different viewpoints for the quantization of Yang-Mills theories by taking into account the effects of Gribov copies. The first one is the well-known Gribov-Zwanziger framework reinvented by including bosonic and fermionic gauge-invariant local composite fields through a detailed construction of a nonperturbative BRST symmetry. This gives us the possibility to extend this model to another class of gauges in a correct manner, the so-called linear covariant gauges. Then, we prove its renormalizability to all orders in a loop expansion by using the algebraic renormalization method. The other one is the Serreau-Tissier approach, in this framework we establish a good explanation for the generation of the gauge field (gluon) mass added in the particular Curci-Ferrari phenomenological model proposed by M. Tissier and N. Wschebor by using the symmetry restoration phenomenon. To accomplish that, we also discuss the similarities between the nonlinear sigma models in two space-time dimensions and quantum chromodynamics.

Keywords: Quantum Field Theory. Supersymmetry. Yang-Mills theories. Gauge fields (Physics).

RESUMO

TERIN, R. C. Terin *Different viewpoints for the Gribov problem in Euclidean Yang-Mills Theories.* 2020. 157 f. Tese (Doutorado em Física) – Instituto de Física Armando Dias Tavares, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, 2020.

A descrição dos efeitos da região infravermelha das teorias de Yang-Mills ainda é um problema não resolvido na física teórica. Uma fundamentação que descreva de maneira clara e consistente o fenômeno do confinamento de quárks e glúons está longe de ser obtida. Desta forma, nesta tese estudar-se-á duas diferentes abordagens à quantização das teorias de Yang-Mills que consideram os efeitos das cópias de Gribov. O primeiro cenário é o bem estabelecido modelo de Gribov-Zwanziger reescrito de maneira BRST-invariante, com isso, adicionar-se-á na correspondente ação, campos compostos locais invariantes de calibre tanto bosônicos quanto fermiônicos. Portanto, estender-se-á o modelo para outros calibres que não o de Landau, por exemplo, os calibres lineares covariantes. Assim, realizar-se-á o estudo da prova da renormalizabilidade deste sistema a todas as ordens em teoria de perturbação através do método da renormalização algébrica. O segundo cenário a ser desenvolvido nesta tese foi apresentado na última década, o chamado modelo de Serreau-Tisier, usando este ponto de vista, obter-se-á finalmente uma boa explicação, de primeiros princípios, para a geração da massa do campo de calibre (glúon) presente no modelo particular de Curci-Ferrari através do fenômeno da restauração de simetria. Apresentar-se-á também uma discussão sobre as semelhanças entre o modelo sigma não linear em duas dimensões e a Cromodinâmica Quântica.

Palavras-chave: Teoria quântica de campos. Supersimetria. Yang-Mills, Teorias de. Campos de calibre (Física).

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INTRODUCTION

Quantum Field Theory (QFT) is the mathematical tool for our current understanding of particle physics. The quantum field theoretical interpretation of elementary particles has been successful since it was first established in quantum electrodynamics (QED) in the late 1920ties. The first applications to elementary processes were the spontaneous decays of excited atoms, the Compton effect and the electron-electron scattering. The next development was achieved in the late 1940ties, for example, the extension of the covariant perturbation expansion by Tomonaga, Schwinger and Feynman in addition with the idea of renormalization allowed to compute higher order corrections to the elementary processes of electrons and photons. This led to a prediction for the anomalous magnetic moment which was found experimentally very close to its observed value for the free electron from the Dirac theory of relativistic quantum mechanics (RQM). Therefore, this is an example of how QFT can describe or even predict some phenomena in nature.

The idea of symmetry being a fundamental actor for the development of fundamental theories in physics comes from the last century. The main argument is that all the physical laws have their origin in some symmetry, *i.e.*, all interactions in nature are determined when objects (physical observables) are invariant under the action of certain operators on them. This is the well-known "principle of symmetry". The Standard Model (SM) was constructed five decades ago and it is at the heart of our understanding of three of the four fundamental interactions: electromagnetic, weak and strong ones. The gravitational interaction is missing in the SM. Up to date the SM describes remarkably well the physics than we expected. For example, the detection of the Higgs boson in the Large Hadron Collider (LHC) (1) was responsible for giving such intellectual credit to the model.

The strong interaction, as mentioned before, belongs to the SM of particle physics, this interaction is responsible for keeping the cohesion of atomic nuclei in the presence of the repulsive electromagnetic interaction between charged protons. The particles that experience the strong interaction are known as hadrons. They can be classified in two families, the baryons (*e.g* protons, neutrons) and the mesons (*e.g.* pions). Theoretical and experimental physicists have been investigating the dynamics of hadrons for decades. An important contribution to this field is the quark model, proposed by Gell-Mann and Zweig in 1964 (2, 3). Looking at the abundance of different hadrons observed in particle detectors it is clear that baryons which are particles composed by an odd number of valence quarks and mesons composed by one quark and one antiquark are not elementary particles in the naive sense of the word “elementary”. The observed symmetry patterns between the hadrons, described by quantum numbers known collectively as "flavor"(which is like a multidimensional extension of isospin) strongly suggested a composite structure, since it would take only a small number of sub-nuclear particles, with the correct fundamental symmetry characteristics, to permit the construction of the large variety of observed particles. Also, experimental hadron physics determined very precisely the partonic substructure of the nucleon leaving unquestionable that the parton structure at low energies is formed by spin $\frac{1}{2}$ particles called quarks and spin 1 particles known as gluons, these fundamental particles are one of the building blocks of our universe.

Indirect experimental evidence of the existence of quarks was obtained and it was found that there exists six different types, namely: (u) up, (d) down, (s) strange, (c) charm, (b) bottom and (t) top. It was realized that the different flavors did not provide a sufficient set of quarks to explain what was indirect observed. One of the problems was the attempt to find a force that would attract the quarks together so strongly that they could never (or only rarely) be free particles and yet would not show up as a comparably strong force among the observed hadrons, seen as bound configurations of quarks. The second issue was with the Pauli exclusion principle which assumes that the two-particle wave function must be antisymmetric. Then, let us make a brief remark at this moment. Let us consider the Bohr orbits of two electrons in the helium atom. If we forget about the spin degree of freedom and if we consider the interactions between electrons to be small, the Pauli exclusion principle would impose to insert one electron in the ground state and other one in the first excited state, at odds with experimental evidences. This principle turns out to be save when one has both electrons in an antisymmetric spin state, which corresponds to spin zero. In order to solve a problem similar to that of helium, it was proposed in the period of 1964 and 1965 by Greenberg, Han and Nambu (4, 5, 6) a solution of these issues by introducing extra quantum numbers to describe quarks, as a consequence each flavor of quark come with three degrees of freedom. During 1970ties the gauge theories had become more popular, thus the idea of binding quarks with a gauge field was developed. In that decade, Gell-Mann, Fritzch, Leutwyler and Bardeen (7, 8) introduced the term "color"to describe these additional degrees of freedom and proposed a gauge theory based on the color $SU(3)$ symmetry group. Therefore, it was explained nicely the Pauli exclusion principle and the color charge which is the basis for a binding force for the quarks [1].

At very high energies, i.e, through small distances, QCD is asymptotically free. This property, which was discovered by Politzer, Gross and Wilzeck states that quarks and gluons behave in this regime like free particles (9, 10). These authors in their seminal works discovered that the theory at the quantum level produces a fundamental, dimensionfull scale Λ_{QCD} which controls the modification of the coupling constant g with energy

[1] There was a proposal that quarks could not be bosons nor fermions, it was analyzed specially by O. W. Greenberg (4), however has not proved successful.

scale. The coupling decreases with increasing energy, this event was confirmed in the kinematic variation of cross-section measurements from deep-inelastic scattering data. The decrease is fast enough that QCD keeps its self-consistency in all extreme energy regimes: high center-of-mass scattering energies, high temperatures, large baryon chemical potentials, etc. Therefore, one consequence of asymptotic freedom is that the QCD Lagrangian is scale invariant in the regime of energy much higher compared to the masses of the quarks, and the interactions of these fundamental particles are determined by the dimensionless parameter g.

It is well-established that quarks and gluons have not been detected outside hadrons and in spite of the fact that the confinement hypothesis was formulated several decades ago our understanding of the confinement mechanism(s) still lacks a better understanding. Thus, in opposite to other nonperturbative phenomena of interest in QCD like the dynamical breaking of chiral symmetry, $U_A(1)$ anomaly and formation of relativistic bound states, the phenomenon of confinement has conflicts with nowadays widely accepted foundations of quantum field theory (QFT). It is believed that most of the features of QCD, and in particular the mechanism of confinement can be understood by studying the YM theories (11) that consist in the pure gauge sector of QCD. The quantization problem for nonabelian gauge theories within the framework of perturbation theory was first addressed by Feynman (12), DeWitt (13, 14) and Faddeev-Popov (15). In the low energy sector, nonperturbative techniques are required because the standard perturbation theory is invalid, this occurs as a consequence of increasing value of the gauge coupling. Therefore, if some framework in the continuum could give us an easy and a rigorous way to treat the problem of the infrared sector, it would be possible to have a fundamental explanation about the phenomenon of the confinement of quarks and gluons. Being more specific, the difficulty for solving the confinement issue lies in the fact that the standard techniques which proved to be efficient in QED, are not applicable in the low energy regime of QCD. In QED the gauge coupling is small enough, so one can apply the standard perturbation theory, which means that one can write down a power series in the gauge coupling.

Even if one omits quarks in QCD is still possible to consider the remaining theory as confining one. Despite the fact that there is no real experimental evidence, lattice simulations have shown that gluons form bound states which we call glueballs [2]. Therefore, it is already interesting to investigate pure QCD without quarks, and try to find out what happens. The nonperturbative techniques, based on the studies of the Dyson-Schwinger

[2] However it is still possible to have unconfined gluons and bound states. Generally, we must impose that the gluons do not belong to the physical spectrum by construction of a physical subspace using the well-known BRST symmetry (Becchi-Rouet-Stora-Tyutin) and its algebra (16, 17, 18).

equations, functional renormalization group, Kugo-Ojima criterion, Gribov-Zwanziger approach and its refined version have provided a fruitful ground for a better understanding of the behavior of the two-point Landau gauge gluon correlation function in the infrared region, see (19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43). One could say that confinement is hidden in the behavior of the gluons. The output of these investigations is in quite good agreement with the lattice data on the gluon propagator, which exhibits a violation of the reflection positivity (44, 45, 46, 47, 48). This peculiar behavior of the gluon propagator is commonly interpreted as a signal of gluon confinement, due to the impossibility of attributing a physical meaning to the gluon as an excitation of the spectrum of the theory.

In virtually all analytic approaches to nonabelian gauge theories, it is necessary to fix the gauge. This is done through the Faddeev-Popov quantization which offers consistent results within the perturbative treatment of gauge theories. As was shown by Gribov in (35), the Faddeev-Popov method is however based on some hypothesis which are not well established outside the perturbative regime. The problem arises from the fact that a local gauge-fixing condition is not enough to fix completely the gauge freedom, allowing for gauge equivalent configurations, the so-called Gribov copies, even after imposing the gauge-fixing condition. It was soon realized that this is not a particular problem of some specific gauge-fixing, but an intrinsic problem related to the nontrivial geometrical structure of nonabelian gauge theories (49).

The Gribov-Zwanziger (GZ) approach and its refined version (RGZ), which is one of the possibilities to study nonperturbative effects in QCD and it was originally proposed to by-pass the Gribov issue, has been under strong analysis in the recent years, with the main interest on setting up its BRST invariance, we refer to (50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64) for previous efforts. In (65), the existence of an exact BRST invariance of the Gribov-Zwanziger action and its refined version has been reached through the use of a nonlocal, transverse and gauge-invariant field $A^{h,a}_{\mu}$, introduced in (66, 67), which enables a manifest BRST-invariant framework.

One of the subjects which will be developed in my thesis consists in propose an extension of the investigation done in (65, 68, 69, 70, 71, 72, 63, 73, 74, 75) about the renormalizability properties of the local and BRST invariant RGZ action including, at this time, the nonlocal fermionic gauge-invariant composite fields in the Dirac fundamental representation and it's own horizon-like function using the linear covariant gauges (LCG) as a gauge-fixing condition. For instance, as the theory is nonlocal, we must first localize it by adding auxiliary fields and then prove using the algebraic renormalization method (76) to all orders in a loop expansion the renormalizability of the RGZ framework in LCG including the fermionic gauge-invariant composite fields.

In the last decade an alternative formulation for the improvement of the Faddeev-Popov to deal with Gribov's problem was proposed by J. Serreau and M. Tissier in Landau and nonlinear covariant gauges (77, 78, 79). This construction is based on averaging first over Gribov copies with a pseudo nonuniform weight in a way that their degeneracy is lifted. Then, a second average over the gauge field configurations must be realized when the Yang-Mills action is performed. Thereby, this gauge-fixing condition can be cast in a local field theory form by means of the replica trick method (80, 81), which is very useful for disordered systems in statistical physics.

From lattice simulations it was observed that the gauge field (gluon) has a massive behavior. Keeping this in mind, several attempts to generate the gluon mass with the gauge-fixing described in (77) were proposed but none of them were completely satisfactory. In the original work done by J. Serreau and M. Tissier, a gluon mass was generated under the questionable assumption that two limits can be inverted. In (79), the gluon mass was generated due to the presence of collective effect in an extension of the gauge fixing to nonlinear Curci-Ferrari-Delbourgo-Jarvis gauge. Unfortunately, in the limit where this gauge coincides with the Landau gauge, the mass tends to zero. In this manuscript, we finally give a satisfying solution to the generation of the mass in this context.

This thesis is organized as follows: In chapter (1) we make a review about the Yang-Mills theories and its perturbative quantization given by Faddeev-Popov method emphasizing the importance of the BRST symmetry for a local formulation for those theories. Moreover we present the criteria about confinement developed by Kugo and Ojima, the Neuberger 0/0 problem and the necessity of improvement of the Faddeev-Popov gauge-fixing procedure in infrared regime due to the presence of the Gribov problem which brings the theory the well-known Gribov copies. In chapter (2) we present two different viewpoints to deal with Gribov ambiguities in the continuum. The first one, the so-called Gribov-Zwanziger and its refined version, is more famous and it has been studied for almost 43 years. Therefore, we review the proposal developed by V.N. Gribov whom implemented the no-pole condition. Some years later an improvement was done in terms of the horizon function by D. Zwanziger. Furthermore, the soft breaking of BRST symmetry and the formation of dynamical masses are also discussed. The second one to be presented in chapter (2) is the Serreau-Tissier framework. We will review some aspects of this approach, associated with the gauge-fixing proposal and its functional integral formalism in Landau gauge. Furthermore, this approach can be formulated by the extremization procedure to give the possibility of lattice simulations. Then, we reconsider the way that J. Serreau and M. Tissier proposed a continuum formulation by a local action which is free of Gribov ambiguities and prevents the Neuberger problem of the Faddeev-Popov procedure. In chapter (3) the particular gauge-fixing developed by J. Serreau and M.Tissier will be reformulated by addressing the question of whether the phenomenon of symmetry restoration is realized or not in this particular model. Therefore, this chapter finally gives a satisfying solution to the generation of the gauge field mass in this context. In chapter (4) we present the construction of the Gribov-Zwanziger framework in a BRST invariant way with a bosonic gauge-invariant composite field when nonperturbative effects are considered on the functional integral and on the own operator responsible for the BRST transformations in Landau gauge and then its extension for linear covariant gauges in a local and correct manner. In chapter (5) we identify a local refined Gribov-Zwanziger action in linear covariant gauges including the fermionic gauge-invariant composite fields and present the Ward identities, which are of great importance to the study of the renormalizability of this model by the algebraic renormalization method. In chapter (6) we show that our proposal for the RGZ action including gauge-invariant local fermionic fields is renormalizable to all orders in a loop expansion bringing a fundamental scenario where perturbative computations are well-defined. Finally we present our conclusions and future perspectives.

This manuscript is only a little part of the works developed during these four years of P.h.D (63, 64, 82, 83, 84, 85, 75). Being more specific, in (63), we worked out the first attempt to prove the renormalizability of the Gribov-Zwanziger action quantized in linear covariant gauges for pure Yang-Mills theories. An all order proof was established. However in this work the model was developed in the approximation $A^h_\mu = A^T_\mu$, i.e. the horizon function was written only in terms of the transverse component of the gauge field. In the work (64) the gauge-invariant operator $A^2_{\min}$, and corresponding gauge-invariant transverse field configuration A^{ah}_μ were investigated in a general class of gauge-fixings, which shared similarities with 't Hooft's R_ζ-gauge used in the analysis of Yang-Mills theory with spontaneous symmetry breaking. The construction showed to be a perfectly well-behaved model in the ultraviolet which turns out to be renormalizable to all orders in a loop expansion. In particular, the pivotal role of the transversality constraint $\partial_\mu A^{ah}_\mu = 0$ was underlined throughout the paper. It is precisely the direct implementation of this constraint in the local action which made a substantial difference with respect to the conventional Stückelberg theory. In fact the component of the Stückelberg propagator which gave rise to non-renormalizable ultraviolet divergences was removed. In particular, one can observe that, similar to what happens in the case of 't Hooft's R_ζ-gauge, the use of the general class of gauge-fixings provided a mass μ^2 for the dimensionless Stückelberg field ξ^a. This a welcome feature which can be effectively employed as a fully BRST invariant infrared regularization for ξ^a in explicit higher loop calculations.

In (82), the $\mathcal{N} = 1$ supersymmetric Yang-Mills theory in the presence of the composite local operator A^2, $A_\mu \gamma_\mu \lambda$ and $\bar{\lambda}\lambda$ was analysed. The Wess-Zumino gauge and the Landau gauge-fixing condition were adopted, all the above operators have been included in the starting action by means of the construction of a generalized BRST operator encoding both gauge and supersymmetry transformations. Further, using the algebraic renormalization procedure, an all order proof of the renormalizability of the resulting action was achieved. This work can be considered as a first step towards a possible understanding of the formation of the dimension two condensate $\langle A^2 \rangle$ in $\mathcal{N} = 1$ Super Yang-Mills and of its eventual relationship with the well established condensate $\langle\bar{\lambda}\lambda\rangle$. In the work (83) we took a first step towards the understanding of Stückelberg-like models in supersymmetric nonabelian gauge theories. The gauge-invariant transverse field configuration V^H was investigated in the supersymmetric Yang-Mills theory with Landau gauge. An auxiliary chiral superfield Ξ was introduced to compensate the gauge variation of the vector superfield V, thus preserving gauge invariance of the composite field V^H. This gauge-invariant composite field has allowed the construction of a local BRST-invariant massive model. Both V and Ξ are dimensionless, which led to ambiguities in defining both the mass term and the gauge-fixing term. However, working with a generalized gauge-fixing term, we found that the model is renormalizable to all orders of perturbation theory.

In (84), we discussed the $\mathcal{N} = 2$ supersymmetric mechanics with one (real) central charge for the multiplet $(1, 2, 1)$. A prescription to obtain deformed $\mathcal{N} = 2$ models by central charge was developed. To establish this in a superfield approach, we introduced deformed covariant derivatives, which took into account the new terms related to the central charge. As an application, we obtained a deformation of one-dimensional nonlinear sigma model. Also, we have recast the particular nonlinear sigma model of the (86) and shown an equivalence between the two prescriptions for some specific transformations. However, we noted that an introduction of deformed derivatives allowed us to implement this extended supersymmetry in a more simple way, once we maintained the superfields and it was not necessary to decompose the Lagrangian in components and add counterterms to recover the supersymmetry. Also, we considered an implementation of the superalgebra in two-dimensional field theory. We interpreted the central charge as a momentum operator of the spatial-dimension v. In this assumption, the supersymmetric transformations (with central charge) are fully fixed. As an application, we discussed a supersymmetric model which exhibited topological configurations in the bosonic sector and nontrivial fermionic solution.

In (85), we concluded the algebraic proof of the renormalizability of a $\mathcal{N} = 1$ super-Yang-Mills theory for SU(N) group in a supersymmetric version of the maximal Abelian gauge (MAG). The proof presented was analogous to that one presented in (83) in the case of Landau gauge. The main difference was that the gauge symmetry group was explicitly splited into its diagonal and off-diagonal parts. This split was made obvious from the diagonal rigid symmetry and the consequent generalized Jacobi identities enjoyed by some invariant tensors presented in the model. It was observed that there was a mass degeneracy among the $(N^2 - 1)$ directions of the group. Therefore, once we had at our disposal this SUSY version of MAG, it was possible to partially break the mass degeneracy and define two different mass parameters, one for the $(N - 1)$ diagonal components and the other one for the $N(N - 1)$ off-diagonal components.

In the work (75), we pursued the previous investigation started in (74, 87) by introducing, in addition of the gauge-invariant composite fields A^h_μ and $A^h_\mu A^h_\mu$, their spinor gauge-invariant counterparts $(\psi^h, \bar{\psi}^h)$. The main result obtained is that the starting action in presence of the gauge-invariant composite operators $(\psi^h, \bar{\psi}^h)$ is renormalizable to all orders in perturbation theory. This work was the first step to deal with the operators $(\psi^h, \bar{\psi}^h)$. Such operator will be employed in this manuscript in order to construct an effective kind of matter horizon function, in analogy with the so-called Gribov-Zwanziger horizon function (38) enabling to restrict the functional integral to the Gribov region Ω, to get rid of the Gribov copies in linear covariant gauges.

1 THE INFRARED REGIME OF YANG-MILLS THEORIES

1.1 Nonabelian gauge symmetry

When C.N. Yang and R.L. Mills (YM) (11) developed a gauge theory for nonabelian fields it was considered rather a curiosity. However, nowadays nonabelian gauge theories are known as the fundamental field theories due to their rich underlying structure. The YM theories are invariant under local gauge transformations of the internal group symmetry $SU(N)$ (88). Additionally, the $SU(N)$ gauge transformation is done by the unitary operator

$$U = e^{-ig\omega^a T^a}, \tag{1}$$

where g is the gauge coupling and ω^a denotes a local (x-dependent) transformation parameter with the group being defined by the generators T^a in the fundamental representation, which are Hermitian and traceless. Moreover, a runs from $a = 1, ..., N^2 - 1$ and is the internal group symmetry index known as the color index. The generators obey commutation relations of the form

$$[T^a, T^b] = if^{abc}T^c. \tag{2}$$

The real numerical factors f^{abc} are the well-known structure coefficients of the group and are antisymmetric in all the indices. It is also defined that the group generators are normalized such that

$$Tr(T^a T^b) = \frac{1}{2}\delta^{ab}. \tag{3}$$

For SU(2), we can choose $T^a = \frac{1}{2}\sigma^a$, where σ^a is a Pauli matrix, and $f^{abc} = \epsilon^{abc}$, where ϵ^{abc} is the completely antisymmetric Levi-Civita symbol. For the construction of a gauge theory using the SU(N) symmetry group, it is required that the gauge fields have to belong to some group representation and, with this, it proves useful to rewrite the theory not in terms of $(N^2 - 1)$ A^a_μ fields, although in terms of a matrix field A_μ, that we define as (38, 88),

$$A_\mu = T^a A^a_\mu, \tag{4}$$

with T^a, as previously mentioned, the generators in the fundamental representation. The gauge transformation then read

$$A_\mu^U = U(x)A_\mu(x)U^\dagger(x) + \frac{i}{g}U(x)\partial_\mu U^\dagger(x), \tag{5}$$

where A_μ are $N \times N$ matrices traceless and Hermitian; $U(x)$ is a special unitary matrix. For the infinitesimal gauge transformation case with the related parameter $\omega(x)$, the equation (1.1) will be

$$A_\mu^U(x) = A_\mu - \partial_\mu\omega(x) + ig[A_\mu(x), \omega(x)] = A_\mu(x) - D_\mu\omega(x), \tag{6}$$

where the covariant derivative reads:

$$D_\mu\omega(x) = \partial_\mu\omega(x) - ig[A_\mu(x), \omega(x)], \tag{7}$$

one still needs a kinetic term for $A_\mu(x)$. Let us define the field strength as

$$\begin{aligned} F_{\mu\nu} &\equiv \frac{i}{g}[D_\mu, D_\nu] && (8)\\ &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu(x), A_\nu(x)]. && (9) \end{aligned}$$

Thereby, it can be checked that the field strength transforms covariantly under gauge transformations as

$$F'_{\mu\nu} = U(x)F_{\mu\nu}(x)U^\dagger(x). \tag{10}$$

With this result, the pure YM action in the four dimensional Euclidean space using SU(N) as the gauge symmetry group [3] is written in the following way:

$$S_{YM} = Tr\int d^4x \left(\frac{1}{2}F_{\mu\nu}F_{\mu\nu}\right), \tag{11}$$

where S_{YM} is a gauge-invariant action, and can serve as a kinetic term for the SU(N) gauge field [4]. Moreover, from (9) one also sees that S_{YM} includes interactions among the gauge fields. A theory of this type, with nonzero f^{abc} , is called nonabelian gauge theory or Yang–Mills theory.

[3] The Wick rotation is only done through the Minkowiski to Euclidean space in the infrared sector, *i.e.*, the Faddeev-Popov operator is only defined in the Euclidean space.

[4] Note, however, that the field strength itself is not gauge-invariant, in contrast to the situation in U(1) gauge theory.

1.2 Gauge-fixing procedure

It is well established that one needs to fix the gauge first in order to implement a field-theoretical approach to correlation functions be directly computed. Indeed, the 2-point vertex function is transverse and therefore not invertible. This implies that the gauge field propagator, one of the building blocks of field theory is not defined. Also, the conjugate momentum of A_μ, $\Pi_\mu = F_{0\mu}$ is null for the temporal mode $\mu = 0$ making impractical the use of canonical quantization. These are standard characteristics of gauge theories and are related to the presence of equivalent field configurations, i.e, when a gauge transformation is done, two field configurations A_μ and A^U_μ are physically identical. In the functional integral formalism, the expectation value of a gauge-invariant observable $\mathcal{O}_{inv}$ is defined as

$$\langle \mathcal{O}_{inv}[A] \rangle = \frac{\int \mathcal{D}A\, \mathcal{O}_{inv}[A] \exp(-S_{YM}[A])}{\int \mathcal{D}A \exp(-S_{YM}[A])} \,. \tag{12}$$

The gauge invariance implies that two field configurations A_μ and A^U_μ that are associated by a gauge transformation have the same contribution to both numerator and denominator of the r.h.s of eq.(12). Moreover, we define the gauge orbit as the set of field configurations that are all related to one another by gauge transformations. Therefore, the idea of gauge-fixing consists in restraining the path integral in (12) to just one field configuration (one representant) per gauge orbit. The usual implementation of this idea was proposed by DeWitt, Faddeev and Popov in 1967 (14, 15). We shall characterize it in the sequence by starting with the following generating functional [5]

$$Z = \int \mathcal{D}A^U_\mu \exp\bigg(- S_{YM}[A^U_\mu] \bigg) \,, \tag{13}$$

which is ill-defined for the gauge-invariant YM action (11) due to the gauge freedom of the fields. One tries to select only one representative per gauge orbit by imposing a gauge condition (an additional constraint), for instance the Landau gauge condition

$$\partial_\mu A_\mu = 0. \tag{14}$$

As this manuscript is based on Landau and linear covariant gauges, we are going to fix the gauge as

$$\partial_\mu A^a_\mu = f^a(x), \tag{15}$$

[5] We neglect the Schwinger sources for the first two chapters of this manuscript.

which makes possible to analyze the quantization of Yang-Mills action for both gauge conditions.

1.2.1 The Faddeev-Popov method

We incorporate the gauge condition (15) via the following nontrivial trick. This attempt is to choose in the path-integral only one representative per gauge orbit obeying the gauge condition referred previously. With this in mind and following (89) in details, Faddeev and Popov (15) proposed the following integral

$$\mathcal{Q} = \int \mathcal{D}A_\mu \, \mathcal{T}[A_\mu] B\Big[f[A_\mu]\Big] \det \mathcal{F}[A_\mu], \tag{16}$$

where $\mathcal{D}A_\mu$ represents the volume element composed by the gauge fields A_μ, $\mathcal{T}[A_\mu]$ is considered as a functional of the gauge fields A_μ, furthermore $\mathcal{T}[A_\mu]$ obeys the following gauge-invariance requirement

$$\mathcal{T}[A^\zeta_\mu]\mathcal{D}A^\zeta_\mu = \mathcal{T}[A_\mu]\mathcal{D}A_\mu \,, \tag{17}$$

with A^ζ_μ being the gauge-transform of A_μ with a gauge parameter ζ^a. Moreover, $B\Big[f[A_\mu]\Big]$ is a weight function defined for general functions $f^a(x)$ which imposes the gauge condition. For the last, $\det \mathcal{F}[A_\mu]$, is established by

$$\det \mathcal{F}[A_\mu] = \left(\frac{\delta f^a[A^\zeta_\mu, x]}{\delta \zeta^b(y)}\right)\Big|_{\zeta=0}, \tag{18}$$

we must take into account the same integral $\mathcal{Q}$ now using as the integration variable the transformation of A_μ with a new parameter u, i.e.,

$$\mathcal{Q} = \int \mathcal{D}A^u_\mu \, \mathcal{T}[A^u_\mu] B\Big[f[A^u_\mu]\Big] \det \mathcal{F}[A^u_\mu]. \tag{19}$$

We have to make some remarks about this expression before go on, *e.g.*, $u^a(x)$ is any arbitrary however fixed set of gauge transformations parameters and (19) can be compared as a simple changing like the integral $\int_{-\infty}^{+\infty} p(x)dx$ turning out to be $\int_{-\infty}^{+\infty} p(y)dy$. Now, let us enjoy the gauge invariance of $\mathcal{D}A_\mu$ and $\mathcal{T}[A^u_\mu]$ to redefine the eq. (19) as

$$\mathcal{Q} = \int \mathcal{D}A_\mu \, \mathcal{T}[A_\mu] B\Big[f[A^u_\mu]\Big] \det \mathcal{F}[A^u_\mu]. \tag{20}$$

As $u^a(x)$ was defined as an arbitrary set of gauge transformations, the *l.h.s* of (20) is independent on it. Therefore, one can at this time integrate (20) with respect to u with some auxiliary weight $\rho[u]$, which will be defined later, thus one can infer that

$$\mathcal{Q}\int \mathcal{D}u\rho[u] = \int \mathcal{D}A_\mu\, \mathcal{T}[A_\mu]C[A_\mu]\,, \tag{21}$$

where $C[A_\mu]$ is given by,

$$C[A_\mu] = \int \mathcal{D}u\rho[u]B\left[f[A^u_\mu]\right]\det\mathcal{F}[A^u_\mu]\,. \tag{22}$$

At this moment, let us define $\tilde{u}$ a new parameter related to the product between the gauge transformations of the parameters ζ and u, *i.e.*,

$$(A^u_\mu)_\zeta = (A^{\tilde{u}}_\mu)_{(u,\zeta)}\,. \tag{23}$$

If one applies the chain rule of partial functional differentiation, one has

$$\mathcal{F}^{ab}_{x,y}[A^u_\mu] = \frac{\delta f^a[(A^u_\mu)^\zeta, x]}{\delta\zeta^b(y)}\bigg|_{\zeta=0} = \int d^4w\, \frac{\delta f^a[(A^{\tilde{u}}_\mu), x]}{\delta\tilde{u}^c(w)}\bigg|_{\tilde{u}=u} \frac{\delta\,\tilde{u}^c(w)}{\delta\zeta^b(y)}\bigg|_{\zeta=0}\,, \tag{24}$$

where one can establish

$$\mathcal{J}^{ac}_{x,w}[A_\mu, u] = \frac{\delta f^a[(A^{\tilde{u}}_\mu), x]}{\delta\tilde{u}^c(w)}\bigg|_{\tilde{u}=u} = \frac{\delta f^a[(A^u_\mu), x]}{\delta\, u^c(w)}\,, \tag{25}$$

and

$$\mathcal{Y}^{cb}_{w,y} = \frac{\delta\tilde{u}^c(w,u,\zeta)}{\delta\zeta^b(y)}\bigg|_{\zeta=0}\,. \tag{26}$$

Thus, one can rewrite $\det\mathcal{F}[A^u_\mu]$ as

$$\det\mathcal{F}[A^u_\mu] = \det\mathcal{J}[A_\mu, u]\det\mathcal{Y}[u]\,. \tag{27}$$

Following (27), it is possible to infer that $\det\mathcal{J}[A_\mu, u]$ is identified as the Jacobian of the transformation of integration variables from the $u^a(x)$ to $f^a[A^u_\mu, x]$. Therefore, making a convenient choice for the weight-function $\rho(u) = \frac{1}{\det\mathcal{Y}[u]}$, one can replace this expression in (22) and obtains

$$C[A_\mu] = \int \mathcal{D}u^a(x)B\left[f[A^u_\mu]\right]\det\mathcal{J}[A_\mu, u]\,. \tag{28}$$

Changing the variable integral from u to f, $\det \mathcal{J}[A_\mu, u]$ can be characterized as the Jacobian determinant and the last integral (28) is simplified in the following manner

$$C[A_\mu] = \int \mathcal{D}f^a(x) B[f] = C\,. \tag{29}$$

Finally, it is possible to write that

$$\int \mathcal{D}A_\mu \, \mathcal{T}[A_\mu] = \frac{1}{C}\mathcal{Q}\int \mathcal{D}u^a(x)\rho[u]\,. \tag{30}$$

To compute the expectation value of some gauge-invariant operator, $\mathcal{O}_{inv}$, one can adopt the previous result by choosing $\mathcal{T} = \mathcal{O}_{inv}\exp\{-S_{inv}\}$ and $\mathcal{T}_0 = \exp\{-S_{inv}\}$ to obtain the following expression

$$\langle \mathcal{O}_{inv}\rangle = \frac{\int \mathcal{D}A_\mu \mathcal{O}_{inv}\exp\{-S_{inv}\}}{\int \mathcal{D}A_\mu \exp\{-S_{inv}\}} = \frac{\int \mathcal{D}A_\mu \mathcal{T}[A_\mu]}{\int \mathcal{D}A_\mu \mathcal{T}_0[A_\mu]} = \frac{\int \mathcal{D}A_\mu \mathcal{T}[A_\mu]B[f]\det\mathcal{F}}{\int \mathcal{D}A_\mu \mathcal{T}_0[A_\mu]B[f]\det\mathcal{F}}\,. \tag{31}$$

A convenient choice for $B[f]$ is:

$$B[f] = \exp\left(-\frac{1}{2\alpha}\int d^4x f^a(x) f^a(x)\right), \tag{32}$$

where α is the gauge parameter. Now, let us take the most simple covariant gauge condition, *i.e.*,

$$f^a[A_\mu, x] = \partial_\mu A^a_\mu(x)\,. \tag{33}$$

Thereby, B[f] provides a modification in the action (13) by adding a gauge-fixing term, namely

$$S_{gf} = \frac{1}{2\alpha}\int d^4x \left(\partial_\mu A^a_\mu\right)^2. \tag{34}$$

The expression (31) will be given by

$$\langle \mathcal{O}_{inv}\rangle = \frac{\int \mathcal{D}A_\mu \mathcal{T}[A_\mu]\exp\{-S_{gf}\}\det\mathcal{F}[A_\mu]}{\int \mathcal{D}A_\mu \mathcal{T}_0[A_\mu]\exp\{-S_{gf}\}\det\mathcal{F}[A_\mu]} = \frac{\int \mathcal{D}A_\mu \mathcal{O}_{inv}\exp\{-(S_{inv}+S_{gf})\}\det\mathcal{F}[A_\mu]}{\int \mathcal{D}A_\mu \exp\{-(S_{inv}+S_{gf})\}\det\mathcal{F}[A_\mu]}\,. \tag{35}$$

From (35) one can conclude that the gauge-invariant operator is computed by using or not the gauge-fixing condition. Nonetheless, unfortunately in the continuum the previous integrals have some problems when we do not fix the gauge, *i.e.*, they are ill-defined. Moreover, $\delta(f - \partial_\mu A_\mu)$ can be represented by the well-known Nakanishi-Lautrup field ib^a that implements the gauge condition (90, 91), furthermore one needs to make another trick, i.e, one integrates over the functions $f(x)$ which changes the overall normalization constant, the partition function will be write as

$$
\begin{aligned}
Z &= \int \mathcal{D}f \, \exp\Big(-\frac{1}{2\alpha} Tr \int d^4x \, f(x) f(x)\Big) \, \delta(f - \partial_\mu A_\mu) \int \mathcal{D}A_\mu \, \det\left(\frac{\delta \mathcal{F}}{\delta \zeta}\right) \exp\Big(- S_{YM}[A_\mu]\Big) \\
&= \int \mathcal{D}A_\mu \, \det\left(\frac{\delta f}{\delta \zeta}\right) \exp\Big(- S_{YM}[A_\mu] - \frac{1}{2\alpha} Tr \int d^4x \, (\partial_\mu A_\mu)^2\Big) \\
&= \int \mathcal{D}A_\mu \mathcal{D}b \, \det\left(\frac{\delta f}{\delta \zeta}\right) \exp\left[Tr\left(\frac{1}{2} F_{\mu\nu} F_{\mu\nu} + ib\partial_\mu A_\mu + \frac{\alpha bb}{2}\right)\right] . \qquad (36)
\end{aligned}
$$

One can re-express the Faddeev-Popov determinant by an integral over Grassmann fields. These fields are known as Faddeev-Popov (FP) ghosts $(c^a, \bar{c}^a)$, which are scalar anticommuting fields, *i.e.*,

$$
\det\left(\frac{\delta \mathcal{F}}{\delta \zeta}\right) = \det(\partial_\mu D_\mu) = \int \mathcal{D}c\mathcal{D}\bar{c} \, \exp\left[\int d^4x \Big(\bar{c}^a \, \partial_\mu D^{ab}_\mu c^b\Big)\right] . \qquad (37)
$$

Notice that c^a and $\bar{c}^a$ are not related by Hermitian conjugation but represent two independent Grassmann fields and the covariant derivative is written as $D^{ab}_\mu = \partial_\mu \delta^{ab} + g f^{abc} A^c_\mu$. In this way, one arrives at the following generating functional for pure YM fields,

$$
Z = \int \mathcal{D}A_\mu \mathcal{D}c \, \mathcal{D}\bar{c} \, \mathcal{D}b \, \exp[-S_{YMFP}], \qquad (38)
$$

where the action (11) will be rewritten as

$$
\begin{aligned}
S_{YMFP} &= S_{YM} + S_{FP} \\
&= \int d^4x \, \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + ib^a \partial_\mu A^a_\mu + \frac{\alpha}{2} b^a b^a + \bar{c}^a \partial_\mu D^{ab}_\mu c^b\right) . \qquad (39)
\end{aligned}
$$

Eventually, one obtains that the expectation value of a gauge-invariant quantity $\mathcal{O}_{inv}[A]$, *i.e.*,

$$
\langle \mathcal{O}_{inv}[A] \rangle = \frac{\int \mathcal{D}A_\mu \mathcal{D}c \, \mathcal{D}\bar{c} \, \mathcal{D}b \, \mathcal{O}_{inv}[A] \exp(-S_{YMFP}[A])}{\int \mathcal{D}A_\mu \mathcal{D}c \, \mathcal{D}\bar{c} \, \mathcal{D}b \exp(-S_{YMFP}[A])} . \qquad (40)
$$

The classical field equation for b provides the gauge-fixing condition $f^a = -i\alpha b^a$. From equation (39), for $\alpha = 0$, one has the Landau gauge. When $\alpha = 1$, one obtains the Feynman gauge. These particular choices are interesting because they simplify the gauge boson propagator at tree level, *i.e.*,

$$
\langle A^a_\mu(p) \, A^b_\nu(-p) \rangle = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{1}{p^2} \delta^{ab} + \frac{p_\mu p_\nu}{p^2} \frac{\alpha}{p^2} \delta^{ab} . \qquad (41)
$$

When $\alpha = 0$,

$$
\langle A^a_\mu(p) \, A^b_\nu(-p) \rangle_{Landau} = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{1}{p^2} \delta^{ab} . \qquad (42)
$$

For $\alpha = 1$, one has

$$\langle A^a_\mu(p)\, A^b_\nu(-p)\rangle_{Feynman} = \delta^{ab}\delta_{\mu\nu}\frac{1}{p^2}\,. \tag{43}$$

To finish this section, it is important to make some comments about the ghosts of Faddeev-Popov. For example, they are required to preserve unitarity of the S-matrix elements and precisely cancel the unphysical degrees of freedom of the gauge bosons. For this subject, we refer to Aitchison and Hey (92). Also, they re-establish an invariance of the total action. The gauge invariance (symmetry) is replaced by another adequate invariance (symmetry) — the BRST transformation. This is essential to establish the Ward identities (Slavnov-Taylor identities for nonabelian case) in order to prove the renormalizability of a nonabelian QFT. We shall discuss the BRST transformation in next section.

1.2.2 BRST transformation

The Lorentz invariance of the theory can be manifest with the method of Faddeev-Popov described in the previous section. However, one has to choose the gauge, which hides the gauge invariance of the theory. This invariance constrains the number of terms appearing in the action that are disposable as counterterms to absorb ultraviolet divergences and, as a consequence, it is fundamental to prove the renormalizability of the theory.

Extraordinarily, as briefly underlined in the last section, even after we select a gauge, the path integral (38) does have a residual symmetry called BRST symmetry associated to the gauge invariance. This symmetry was identified by Becchi, Rouet, and Stora (16, 17) and independently by Tyutin (18) in 1975, a few years later the works done by Faddeev-Popov and De Witt. This symmetry can be presented as it was originally discovered, *i.e.* as a by-product of the method of Faddeev and Popov or it can also be regarded as a replacement for the Faddeev-Popov approach.

As is well-known, with the introduction of the gauge-fixing term S_{FP}, the gauge invariance of expression (39) is replaced by the nilpotent BRST transformations using the operator s. This BRST symmetry is kind of a supersymmetry, where it transforms fermions in bosons and reciprocally. In the present case, the BRST transformations take the form

$$
\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu(A)c^b\,, \\
sc^a &= \frac{g}{2}f^{abc}c^bc^c\,, \\
s\bar{c}^a &= ib^a\,, \\
sb^a &= 0\,, \\
s^2 &= 0\,.
\end{aligned} \tag{44}
$$

Consider a theory defined by an action of the form

$$S_{YMFP} = S_0 + s\Psi, \tag{45}$$

with

$$sS_0 = 0\ , \qquad S_0 \neq s(...)\ . \tag{46}$$

Notice that this for the case of S_{YMFP}, one has $S_0 = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$ and $\Psi = \frac{\alpha}{2}b^a\bar{c^a} + \bar{c}^a\partial_\mu A^a_\mu$. Since sA^a_μ has the same form as an ordinary gauge transformation for an infinitesimal gauge parameter c, under which $F^a_{\mu\nu}F^a_{\mu\nu}$ is invariant; and $s^2 = 0$, it is easy to conclude that $sS_{YMFP} = 0$ [6]. If we choose another gauge, the Ψ would be different. Eq. (45) shows that the physical content of any gauge theory is involved in the kernel of the BRST operator, *i.e.*, in a general BRST invariant term, modulo terms in the image of the BRST transformation, which are terms of the form $s\Psi$. The kernel modulo the image of any nilpotent transformation is said to form the cohomology of the transformation. There is another sense in which the physical content of a gauge theory may be identified with the cohomology of the BRST operator.

Two physical states that differ only by a state vector in the image of Q_{BRST}, *i.e.*, of form $Q_{BRST}\,|....\rangle$, have the same matrix element with all other physical states, and are physically equivalent. This explains the physical space is taken as the cohomology of the BRST symmetry, see (89). Also BRST symmetry allows us to prove the renormalizability of the theory to all orders in a loop expansion (93, 94, 95).

[6] In general, the existence of a nilpotent operator characterizes a cohomological structure. The physical states in a perturbative interpretation are in the cohomology of the BRST operator.

1.2.3 Kugo-Ojima's confinement criterion

Kugo and Ojima (KO) in their fundamental work focused on the question of unitarity in Yang-Mills theories (32). In fact, one of the most difficult issues in those kind of theories is to split the physical and nonphysical states. In a confining theory, the fundamental fields are not associated with asymptotic physical states. In general the Fock space is built by considering that particles far apart are not correlated. However, this is problematic in a theory like QCD where the interaction grows at large distances. In the FP approach we have introduced several fields that should not be associated with particles. Similarly, it is expected that the gauge field A has only two polarizations. To accomplish that we need to define the physical subspace, $\mathcal{V}_{phys}$, of the state space of Yang-Mills-Faddeev-Popov (YMFP) theories. Furthermore, a probabilistic interpretation of the quantum theory is established if $\mathcal{V}_{phys}$ is positive semi-definite since the total state space in covariant gauges has an indefinite metric.

The initial point of the investigation done by KO was to define a nilpotent BRST symmetry Q_{BRST} and a ghost charge. Hereafter, they showed that due to the nilpotency property of Q_{BRST} the nonphysical states form the well-known quartets and decouple from the physical spectrum (32). Therefore, one has just physical states surviving under the action of the BRST symmetry. Thus, with this argument they proved that the longitudinal and temporal gauge polarization, the ghost and the antighosts fields can be removed from the physical spectrum. This idea is applied in general cases, as for example, when there is a system with a nilpotent symmetry s.

The picture developed by KO (32) gave a desirable definition of the positive norm states of the subspace, $\mathcal{V}_{phys}$ for confinement scenario. Taking into account the presence of Q_{BRST}, the space of physical states $\mathcal{V}_{phys}$ is characterized by

$$\mathcal{V}_{phys} = |phys\rangle : Q_{BRST}|phys\rangle = 0. \tag{47}$$

$\mathcal{V}_{phys}$ only contains color singlets, *i.e.* $\langle phys|Q^a|phys\rangle = 0$, if we conjecture the existence of a well-defined global color charge Q^a, see (32, 96). Using Landau gauge as the gauge-fixing, the KO confinement criterion can be re-expressed as the condition that the ghost propagator should diverge more strongly than a simple pole at zero momentum (33).

In addition, looking the YMFP action (39) for $\alpha = 0$, KO used the equation of motion for the gauge field and establish that the global color current is

$$J^a_\mu = \partial_\mu F^a_{\mu\nu} + \{Q_{BRST}, D^{ab}_\mu \bar{c}^b\}\,. \tag{48}$$

The color charge is obtained by integrating the zeroth component of J^a_μ, that is

$$Q^a = \int d^3x \left(\partial_i F^a_{0i} + \{Q_{BRST}, D^{ab}_0 \bar{c}^b\} \right) . \tag{49}$$

Thereby, one has two criteria which need to be obeyed for having the color confinement. The first one is based on the assumption that the gauge field propagator cannot have massless poles, thus the first term of (49) vanishes because it is the integral over space of the total derivative of regular function. If the gauge field has massless poles, then the first term will be ill-defined. Thus, to establish the second criterion, from (49) the second term, namely, $\{Q_{BRST}, D^{ab}_0 \bar{c}^b\}$ must be well defined, e.g., for the case (33)

$$u(0) = -1 , \tag{50}$$

where $u(p^2)$ is a function which is defined by the following Green function,

$$\int d^dx \, \exp{(ipx)} \langle D^{ad}_\mu c^d(x) D^{be}_\nu \bar{c}^e(0) \rangle_{FP} = \left(\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) u(p^2) - \frac{p_\mu p_\nu}{p^2} \right) \delta^{ab} , \tag{51}$$

where $\langle O \rangle_{FP} = \langle D^{ad}_\mu c^d(x) D^{be}_\nu \bar{c}^e(0) \rangle_{FP}$ is the expectation value taken with the YMFP action (39). Supposing that the two criteria are accomplished, Q^a is well defined, than color confinement is assured.

Moreover, in (33) the second criterion was connected to the ghost propagator, *i.e.*, one can parametrize the ghost propagator as follows

$$\langle c^a(-p) \bar{c}^b(p) \rangle = \delta^{ab} G(p^2) , \tag{52}$$

with $G(p^2)$ being given by

$$G(p^2) = \frac{1}{p^2(1 + u(p^2) + w(p^2))} . \tag{53}$$

where $w(p^2)$ is another function dependent of the momentum squared and for this expression, it is generally assumed that $w(p^2) = 0$ (condition checked up to two loops (97)), thus, $u(0) = -1$ implies an enhanced ghost propagator which diverges faster than $\frac{1}{p^2}$ at small momentum p.

In QCD using the LCG as a gauge-fixing, the exclusion of nonphysical degrees of freedom in the S-matrix is extensively intricate by the self-interaction of the gauge fields and by the ghost fields that are certainly present in the quantum formulation of these theories (15).

The connection between the BRST and color charge in KO scenario has been under investigation in the studies of Green's functions in Landau gauge because the color charge exhibits an infrared enhanced ghost (28, 98). Actually, as emphasized in (27), following the idea of KO, it still not clear the way of constructing the physical space out of Green's functions when one has the omission of an infrared enhanced ghost which leads to a broken global color charge or nilpotent BRST charge absence in the gauge fixed YM theories (27).

As emphasized before the KO scheme is one particular procedure that establishes the probabilistic interpretation of the quantum theory. Nevertheless, there is another criterion for confinement, namely, violation of positivity which is based on the assumption that if a particular degree of freedom has negative norm contributions there is no Källén–Lehmann (KL) spectral representation for its propagator.

Finally, it is important to make two more comments. Firstly, the KO scenario assumes a globally well-defined nilpotent BRST charge at the perturbative level. In the nonperturbative sector, the existence of such symmetry has been under discussion for the past few years (65, 69, 68, 87) and will be an important part for development of this thesis. Secondly, the above mentioned KO scheme works with Faddeev-Popov's gauge-fixing procedure which neglects certain ambiguities known as Gribov copies (35). This subject is the main topic of this thesis and will be discussed with more details in next section.

1.2.4 Improvement of the Faddeev-Popov gauge-fixing procedure

V.N. Gribov in his seminal work (35) proved that the FP procedure for nonabelian gauge theories was correct only for the perturbative regime. In the infrared sector this well-known gauge-fixing method was incomplete. Indeed, this mechanism works with the premise that the gauge condition admits one and only one solution per gauge orbit, which is not true. In fact, the gauge-fixing cannot remove all the equivalent gauge fields configurations A_μ, connected through gauge transformations. These equivalent configurations are known as Gribov copies and are present for all covariant gauges as pointed out by Singer (49).

The original idea of V.N. Gribov resided on an additional restriction on the integration range in the functional space of nonabelian gauge fields, which consists in integrating only over the fields for which the FP determinant is positive. This new region is the so-called Gribov's region. Such a restriction brings physical effects in the propagators of the theory, e.g., the gauge field propagator in Landau gauge

$$\langle A^a_\mu(k)\, A^b_\nu(-k)\rangle \;=\; \frac{k^2}{k^4+2Ng^2\gamma^4}\left(\delta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\delta^{ab}\,, \tag{54}$$

is suppressed in the low energy sector due to the presence of Gribov's parameter γ defined by the gap equation

$$\int \frac{d^4k}{(2\pi)^4} \frac{3Ng^2/4}{(k^4 + 2Ng^2\gamma^4)} \quad = \quad 1\,. \tag{55}$$

Hence, note that the poles of the propagator are imaginary. As a consequence, the gauge fields are not physical observables and do not belong to the physical spectrum of the theory. Moreover, the trace of Faddeev-Popov's propagator [7] is given by

$$\langle \bar{c}^a(k) c^a(-k) \rangle \approx \frac{\gamma^2}{k^4}, \tag{56}$$

looking (56) more carefully, one can see that the behavior of the propagator is more singular than the perturbative case.

The first steps of the Gribov problem (35) will be reviewed during the next section. Also, it will be discussed important points about the Gribov copies in Faddeev-Popov's quantization (15) and their consequences in the theory.

1.2.5 Gribov's problem

As written previously, the quantization of Yang-Mills action is done through Faddeev-Popov's method. Again, the most consistent path integral to describe this QFT is given by

$$Z \quad = \quad \int \mathcal{D}A_\mu \, \exp[-S_{YM}] \, \det(\mathcal{M}^{ab}) \delta(f^a - \partial_\mu A^a_\mu)\,. \tag{57}$$

Using Landau's gauge-fixing condition

$$\partial_\mu A^a_\mu = 0. \tag{58}$$

Thus, (57) is characterized as

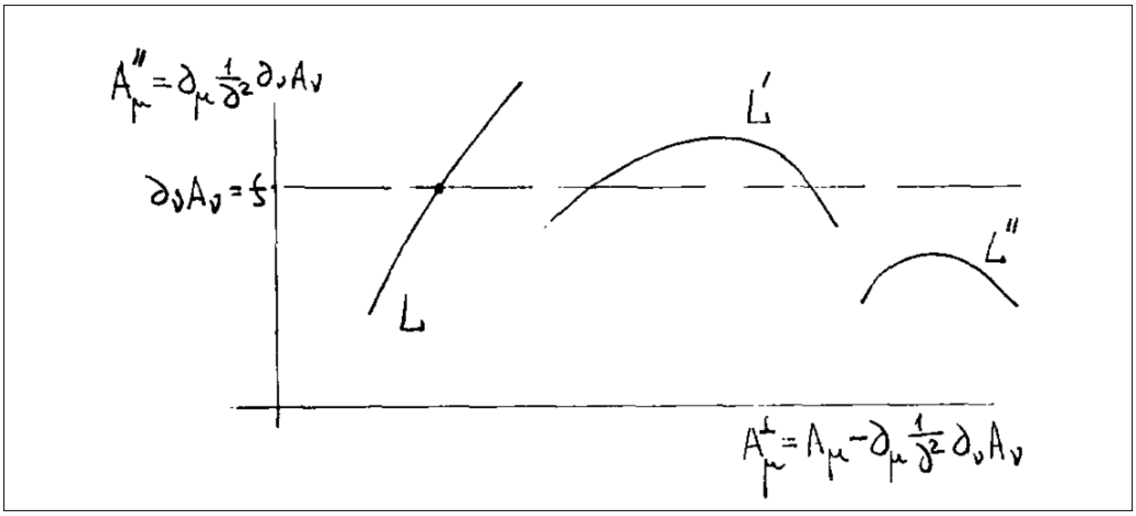
$$Z \quad = \quad \int \mathcal{D}A_\mu \, \exp[-S_{YM}] \, \det(\mathcal{M}^{ab}) \delta(\partial_\mu A^a_\mu)\,, \tag{59}$$

with $\mathcal{M}^{ab}$ being Faddeev-Popov's operator

$$\mathcal{M}^{ab} = -\delta^{ab}\partial^2 + gf^{abc}A^c_\mu \partial_\mu. \tag{60}$$

[7] This result is explicitly obtained in (99).

Figure 1 - The space for the gauge field configurations and the three alternatives of gauge orbits for some gauge-fixing condition.



Source: GRIBOV, 1977, p. 2.

V.N. Gribov showed in (35) that the condition (58) does not fix the gauge completely. In figure (1), V.N. Gribov explained the existence of three alternatives for the gauge orbit crossing a specific gauge condition, e.g., it is possible to intersect once (L), multiple times (L') or without any intersections (L''). If the second case is realized, *i.e* when the gauge field configuration A_μ obeys the condition (58), there is another equivalent configuration A^U_μ which also obeys it. Thus,

$$A^U_\mu = UA_\mu U^\dagger - \frac{i}{g}\left(\partial_\mu U\right)U^\dagger\,, \tag{61}$$

$$\partial_\mu A^U_\mu = 0. \tag{62}$$

The field configuration A^U_μ is known as Gribov's copy associated to the field A_μ. Making the replacement of (61) in (62), one has

$$\partial_\mu U A_\mu U^\dagger + UA_\mu\partial_\mu U^\dagger - \frac{i}{g}\left(\partial^2 U\right)U^\dagger - \frac{i}{g}\left(\partial_\mu U\right)\left(\partial_\mu U^\dagger\right) \;\; = \;\; 0. \tag{63}$$

Let us establish the condition under which there exists a Gribov copy infinitesimally closed to A_μ. We thus linearize the gauge element U, $U = 1 + i\alpha$ e $U^\dagger = 1 - i\alpha$ with $\alpha = \alpha^a T^a$.

At first order in α,

$$\begin{aligned} -\partial_\mu\left(\partial_\mu\alpha + ig[\alpha, A_\mu]\right) &= 0, \\ \mathcal{M}^{ab}\alpha^b &= 0. \end{aligned} \tag{64}$$

The equation (64) implies that the existence of an infinitesimally close Gribov copy is equivalent to stating that the FP operator has a vanishing eigenvalue. Let us study this constraint first for small A_μ, the equation (64) will be

$$-\partial^2\alpha = 0. \tag{65}$$

The eigenvalue equation

$$-\partial^2\psi = \epsilon\psi\,, \tag{66}$$

has positive eigenvalues which implies that no infinitesimal Gribov copy exists in this case. However, this is not guaranteed for larger values of A_μ. Thus, it is expect that one has negative and null eigenvalues for Faddeev-Popov's operator. Thereby, the gauge condition is not sufficient to play the role of just select one configuration in gauge orbit.

At this point, it is important to mention that all this discussion about positivity of the eigenvalues is well-founded because of the hermiticity of Fadeev-Popov's operator in Landau gauge, *i.e.* in this case the eigenvalues have to be real [8].

An important property of the FP gauge-fixing approach is that it is invariant under the computation of expectation values of gauge-invariant operators when the degeneracy of equivalent field configurations is taken out. Therefore, Neuberger (101, 102) investigated with details the expectation value of BRST invariant objects obtained by a BRST invariant action like (45) or

$$S = S_{YM}[A] + \int d^4x\, s\Psi[\phi]\,, \qquad \phi = \{A_\mu, \bar{c}, c, ib\}\,, \tag{67}$$

where $\Psi = \bar{c}^a\partial_\mu A^a_\mu$ for the particular case of Landau gauge. Defining the BRST invariant operator as $\mathcal{O}_{BRST} = \mathcal{O}_{inv} + s\mathcal{O}$ with $\mathcal{O}_{inv}$ gauge-invariant and $\mathcal{O}$ a general operator, Neuberger proved the following expression

$$\int \mathcal{D}\phi \exp\left(-S_{YM} + \int d^4x\, s\Psi[\phi]\right)\mathcal{O}_{BRST}[\phi] = 0\,, \tag{68}$$

in this case, we assume that the Q_{BRST} is unbroken. Moreover, for $\mathcal{O}_{BRST}[\phi] = 1$ one has

[8] This construction can be seen in (100).

$$Z = \int \mathcal{D}\phi \exp\bigg(- S_{YM} + \int d^4x \, s\Psi[\phi]\bigg) = 0\,. \tag{69}$$

Thus, averages of gauge-invariant observables have the indefinite $\frac{0}{0}$ form. The event expected here is the compensation between the numerator and the denominator. Thereby, the absence of a procedure to compute gauge-invariant quantities is the main issue to deal with in this moment and this is directly linked with obtaining a well-defined BRST gauge-fixing method of the form (67). This indefinite expression is related to the nonperturbative aspects which are neglected by the FP quantization. The latter removes only up to a set of discrete equivalent configurations whose contributions in (68) exactly cancel out for gauge group of zero Euler character (19).

After all this discussion it is clear that the Faddeev-Popov quantization is insufficient and it is necessary to correct the gauge-fixing. V.N. Gribov was the first to attempt to solve the problem of copies as already mentioned several times and proposed in 1977 (35) a restriction to a region of integration in the field space, the so-called Gribov region Ω, which obeys the Landau gauge condition and is defined as

$$\Omega = \{\ A^a_\mu |\ \partial_\mu A^a_\mu = 0,\ \mathcal{M}^{ab} > 0\ \}\ . \tag{70}$$

The Faddeev-Popov operator $\mathcal{M}^{ab}$ given by (60) is positive definite. Looking the figure (2) below the border of the region Ω, namely, $\delta\Omega$ is known as the first Gribov horizon and at this border the first nontrivial eigenvalue of Faddeev-Popov's operator vanishes. Crossing this horizon, this eigenvalue becomes negative. Similarly, one can illustrate the other horizons, as drawn on the picture (2). Nonetheless, this image is just a simple illustration and does not represent very well the space of gauge fields.

1.2.6 An alternative formulation for Gribov's region

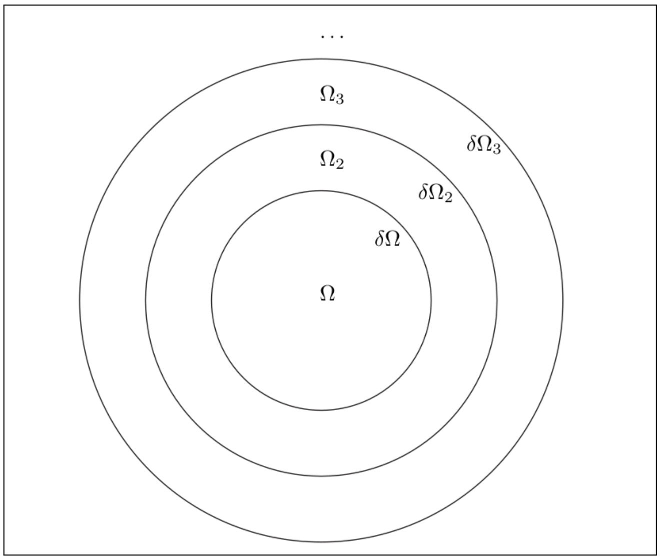
The Gribov region can also be defined as a set of local minima (with respect to U) of the following functional in Landau gauge [9] [10],

$$\mathcal{H}_{Landau}[A,U] = Tr\int d^4x\ \ A^U_\mu(x)A^U_\mu(x) = \frac{1}{2}\int d^4x\ \ A^{aU}_\mu(x)A^{aU}_\mu(x)\,. \tag{71}$$

[9] The reader can find the original formulation in (103, 104, 105).

[10] The Serreau-Tissier framework is based on this construction.

Figure 2 - A pictorial figure representing the various regions for the hyperspace $\partial A = 0$.



Source: VANDERSICKEL, 2012, p. 51.

Thus, one has to make an infinitesimal transformation of $\mathcal{H}_{Landau}$ assuming a field A_μ which minimizes the functional (71) in order to have an extremum. Thereby,

$$\begin{aligned}\delta\mathcal{H}_{Landau}[A] &= \delta\left(\frac{1}{2}\int d^4x\ A^a_\mu(x)A^a_\mu(x)\right) = \int d^4x\ \left(\delta A^a_\mu(x)\right)A^a_\mu(x) \\ &= -\int d^4x\ \left(D^{ab}_\mu\vartheta^b(x)\right)A^a_\mu(x) = -\int d^4x\ \left(\partial_\mu\vartheta^a(x)\right)A^a_\mu(x) \\ &= \int d^4x\ \vartheta^a(x)\partial_\mu A^a_\mu(x) = 0\,. \end{aligned} \tag{72}$$

Where the infinitesimal transformation (61) is given by

$$\delta A^a_\mu = -D^{ab}_\mu\vartheta^b, \tag{73}$$

with $\vartheta(x)$ being a parameter associated to the infinitesimal transformation. Also, the equation (72) must be null for all $\vartheta(x)$, in this manner, $\partial_\mu A^a_\mu(x) = 0$. This construction is used to perform gauge-fixed lattice simulations because there are efficient numerical algorithms to minimize a function and it selects in each gauge orbit the field configurations that minimize $\mathcal{H}_{Landau}$. Finally the equation (72) shall be minimized to determine the stability of $\mathcal{H}_{Landau}$. Taking the second derivative of $\mathcal{H}_{Landau}$, we have

$$\delta^2\mathcal{H}_{Landau} = \int d^4x\ \vartheta^a(x)\partial_\mu\left(\delta A^a_\mu(x)\right) = \int d^4x\ \vartheta^a(x)\left(-\partial_\mu D^{ab}_\mu\right)\vartheta^b(x) > 0. \tag{74}$$

In case, the operator $-\partial_\mu D^{ab}_\mu = \mathcal{M}^{ab}$ must be positive definite. Another subject to point out is the fact that there is more than one minimum per gauge orbit and this definition agrees with Gribov's region.

1.2.7 Properties of Gribov's region for Landau gauge

There are important properties about the region Ω defined previously. Some details of each property will be given in the following (106, 107, 100):

Property I: *The region Ω is convex.*
This means that if $(A^1_\mu, A^2_\mu) \in \Omega$ then also $A_\mu = \alpha A^1_\mu + (1-\alpha)A^2_\mu$ with $0 < \alpha < 1$ belongs to Ω.

Proof: First of all one notices that $\partial_\mu A_\mu = \alpha\partial_\mu A^1_\mu + (1-\alpha)\partial_\mu A^2_\mu = 0$. Let us now evaluate $\mathcal{M}^{ab}(A_\mu)$ to check that this is indeed a minimum,

$$\begin{aligned} \mathcal{M}^{ab}(A_\mu) &= -\left(\partial^2\delta^{ab} + \alpha g f^{acb}(A^1_\mu)\partial^c_\mu + (1-\alpha) g f^{acb}(A^2_\mu)^c\partial_\mu\right) \\ &= -\left(\alpha\partial^2\delta^{ab} + (1-\alpha)\partial^2\delta^{ab} + \alpha g f^{acb}(A^1_\mu)\partial^c_\mu + (1-\alpha) g f^{acb}(A^2_\mu)^c\partial_\mu\right) \\ &= \alpha\mathcal{M}^{ab}(A^1_\mu) + (1-\alpha)\mathcal{M}^{ab}(A^2_\mu) > 0. \end{aligned} \tag{75}$$

Thus $A_\mu \in \Omega$. If a field can be written as a linear interpolation between two other fields that belong to the Gribov region then, this field is also inside of Gribov's region.

Property II: *The region Ω is bounded in every direction in field space.*
In order to prove this statement one shows that if $A^a_\mu \in \Omega$, then for sufficiently large constant λ, the configuration λA^a_μ is located outside of Ω.

Proof: Let us consider A^a_μ a field belonging to Ω; $\mathcal{M}^{ab}(A_\mu) > 0$ and $\partial_\mu A^a_\mu = 0$. From,

$$\begin{aligned} \mathcal{M}^{ab}(A_\mu) &= -\partial^2\delta^{ab} - g f^{acb} A^c_\mu\partial_\mu \\ &= -\partial^2\delta^{ab} + \mathcal{M}^{ab}_1(A_\mu)\,, \end{aligned} \tag{76}$$

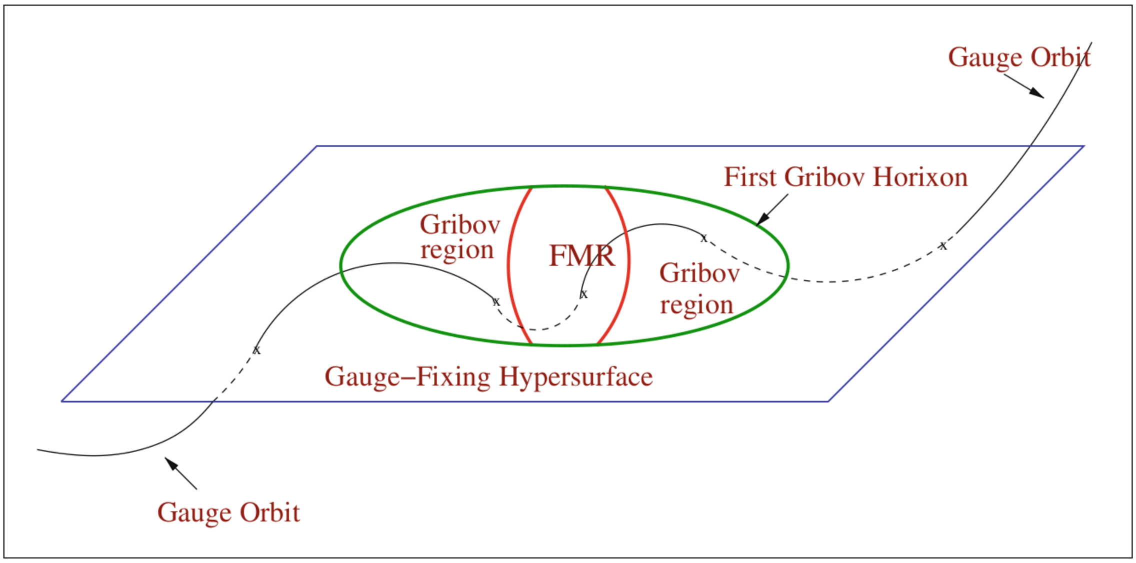
with $-\partial^2\delta^{ab}$ being always positive and

$$\mathcal{M}^{ab}_1(A_\mu) = -g f^{acb} A^c_\mu \partial_\mu\,. \tag{77}$$

Due to the presence of the structure constants f^{abc} the trace (sum of eigenvalues) of $\mathcal{M}^{ab}_1$ vanishes, *i.e.*, $Tr\mathcal{M}^{ab}_1 = \mathcal{M}^{aa}_1 = 0$, which implies that $\mathcal{M}^{ab}_1$ has positive and negative eigenvalues. Let us consider one of the negative eigenvalues of $\mathcal{M}_1$, *i.e.*, $-k$, $k > 0$, and χ^a the corresponding eigenvector. Since $A^a_\mu \in \chi \to \int d^4x\, \chi^a\mathcal{M}^{ab}(A)\chi^b > 0$. Then,

$$\int d^4x\, \chi^a \mathcal{M}^{ab}(A)\chi^b = \int d^4x\, \chi^a(-\partial^2)\chi^a - k\int d^4x\, \chi^a\chi^a\,. \tag{78}$$

Figure 3 - The subspace of all gauge field configurations which satisfy a particular gauge condition is represented by the rectangular region. A gauge orbit can intersect this subspace several times. The "x" crossing points are known as Gribov copies or equivalent gauge field configurations. The Gribov region is bounded by the first Gribov horizon and it is possible to characterize a subregion, namely, Fundamental Modular region (FMR).



Source: GREENSITE, 2011, p. 134.

Considering now the re-scaled field λA^a_μ, with λ constant,

$$\begin{aligned}\int d^4x\,\chi^a\mathcal{M}^{ab}(\lambda A)\chi^b &= \int d^4x\,\chi^a(-\partial^2)\chi^a + \lambda\int d^4x\,\chi^a\mathcal{M}^{ab}_1(A)\chi^b \\ &= \int d^4x\,\chi^a(-\partial^2)\chi^a - \lambda k\int d^4x\,\chi^a\chi^a\,, \end{aligned} \tag{79}$$

with $\chi^a(-\partial^2)\chi^a > 0$, $\lambda k < 0$ and $\chi^a\chi^a > 0$. Thus, for sufficiently large λ

$$\int d^4x\,\chi^a\mathcal{M}^{ab}(\lambda A)\chi^b \;<\; 0\,. \tag{80}$$

This means that λA^a_μ is located outside of the region Ω. This shows that Ω is bounded in all directions in field space. The boundary $\delta\Omega$ of Ω is the region in which the first vanishing eigenvalues of $\mathcal{M}^{ab}(A)$ shows up.

Property III: *Every gauge orbit crosses the region Ω at least once.*
This is an elementary property of the region Ω. It means that a gauge configuration located outside of the region Ω is a copy of a configuration located inside Ω. The gauge orbit is understood as a variation of the field A_μ along the U elements of the group defined in (61). Thereby, all the field configurations A_μ living outside Gribov's region have an equivalent configuration inside Ω. This property is demonstrated in (107).

From the properties described before for the Gribov region Ω, it was a natural way of improving the FP gauge-fixing, which consists in limiting the path integral to the Gribov:

$$\int[\mathcal{D}\mu]\exp\Bigg(-S_{YMFP}\Bigg)\rightarrow\int_{\Omega}[\mathcal{D}\mu]\exp\Bigg(-S_{YMFP}\Bigg), \tag{81}$$

where $\mathcal{D}\mu$ represents in an economic way all the fields related to the Faddeev-Popov procedure. The expression (81) should be the right way to quantize YM theory. However, there are still copies inside Ω, *i.e.*, there is a set of local minima over the gauge orbit which also belongs to the Gribov region and these local minima are also considered Gribov copies, this was first discussed in (104) and this could be a problem (see next subsection for more details), also see figure (3). A way out of this ambiguity would consist in restricting further the path integral. To do so, observe that, among all local minima, we could retain only one: the absolute minimum. Nonetheless, there exists a smaller region $\Lambda\subset\Omega$, known as fundamental modular region (FMR), which is totally free from Gribov copies. However, up to date, we do not know anything about this region. In this thesis, the author will work with all the Gribov regions in Serreau-Tissier approach and in the frontier between the first Gribov region and the FMR for the Gribov-Zwanziger framework.

1.2.8 The Fundamental Modular Region

As written before from the functional (71) it is possible to define the fundamental modular region (Λ) which is even more restrictive than Gribov's region (Ω). To do so one selects just one field configuration for each gauge orbit which is an absolute minimum. It is important to observe that the absolute minimum of the functional (71) can only be reached by a suitable global gauge transformation. As fixing the gauge does not break the global gauge symmetry and it is thus difficult to observe how to impose this constraint in terms of a local field theory, one has to perform a global gauge transformation P independent from the space time coordinate x_μ, then the expression (71) does not change. Therefore, one has

$$\mathcal{H}[A,U]_P \;=\; Tr\int d^4x\;\; PA^U_\mu(x)P^{+}PA^U_\mu(x)P^{+}=Tr\int d^4x\;\; A^U_\mu(x)A^U_\mu(x)=\mathcal{H}[A,U]\,. \tag{82}$$

An important aspect to remark is that the region Λ will give the correct gauge-fixing if it has a nondegenerated global minimum of the functional (71). Though, there is a proof about the degeneracy of the minimum occurs just at the boundary of Λ, *i.e.*, at $\delta\Lambda$ (37). Thus, in order to establish one way to have a correct quantization for YM theories, one must restrict the domain of integration to the region Λ, where just one field configuration will be choose for each gauge orbit. However, it is not possible to make practical calculations in the continuous for the region Λ. Thereby, one has to work at Gribov's first region for this framework. V.N. Gribov proposed a semiclassical method and D. Zwanziger constructed an action which was able to restrict the path integral to the first Gribov region Ω. D. Zwanziger in his works (108, 109) used as hypothesis that all the important configurations are in the frontier between $\delta\Lambda$ and $\delta\Omega$ of the regions Ω and Λ. Thus, the extra copies inside Ω do not play any significant role.

Finally, for the GZ framework it is sufficient to restrict to the region Ω. On the contrary, the Serreau-Tissier approach is based on to take into account all Gribov copies, in this case each copy has a different weight in the path-integral such that its degeneracy is lifted. Applied to the Landau and nonlinear covariant gauges, this alternative method provides a gauge fixed action which deals explicitly with Gribov copies. We will discuss with more details both frameworks in the next chapter.

2 THE GRIBOV-ZWANZIGER AND SERREAU-TISSIER APPROACHES

At the beginning of this manuscript we have motivated the needs of improvement of the Faddeev-Popov gauge-fixing procedure to take into account the Gribov copies effects. All this effort is justified through the insufficiency of the FP quantization procedure to explain the infrared characteristics of the YM correlation functions due to the Neuberger problem. Though the lattice analysis has solid results for the YM correlation functions, this occurs because only one Gribov copy is picked in some gauges, e.g., the Landau one due to the lattice gauge-fixing method. Unfortunately, no one knows the equivalent version for the continuum limit and it is impossible to choose just one copy using a local QFT.

As previously mentioned in section (1.2.8) D. Zwanziger in his works (108, 109) showed that all the important configurations are in the boundary between $\delta\Lambda$ and $\delta\Omega$ of the regions Ω and Λ. Therefore, for the GZ framework it is sufficient to restrict to the region Ω and, thus, it is possible to obtain a good agreement with lattice simulations in the case of the RGZ (39, 40, 110, 111, 112).

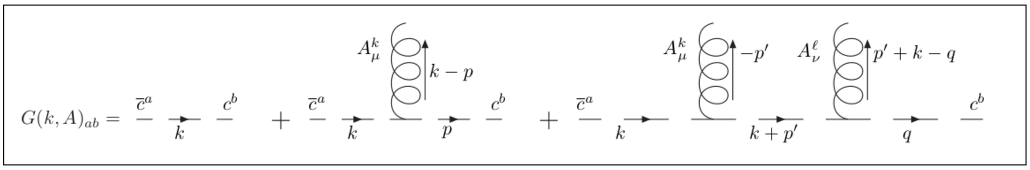
In this thesis, we also study another point of view for Gribov's problem developed by J. Serreau and M. Tissier (77, 78). This approach leads to a gauge fixed action which deals explicitly with Gribov copies and it is possible to investigate this model with the elementary perturbative mechanism and it has likewise the RGZ framework a good agreement with the lattice results. Finally, a problem previously detected in the work (77) for the alternative approach will be solved. The issue was related to the procedure used to write under the structure of a local QFT the Serreau-Tissier scheme, *i.e.*, a number n of extra auxiliary fields must be added while averages of physical observables shall be calculated in the limit $n \to 0$. In order to answer this problem, we finally give a good explanation for the generation of the gauge field mass in this framework. This subject will be detailed in chapter (3).

2.1 **The Gribov-Zwanziger framework**

2.1.1 The no-pole condition

In his seminal work V.N. Gribov (35) had the main objective to restrict the path integral to the first Gribov region Ω and presented, as a first attempt, a semiclassical solution which consisted in adding a new term $\mathcal{V}(\Omega)$ in the generating functional (59).

Figure 4 - This image shows the ghost propagator coupled to a external gauge field up to second order.



Source: VANDERSICKEL, 2012, p. 58.

This new functional was defined as

$$\begin{aligned} Z &= \int_\Omega \mathcal{D}A \, \exp[-S_{YM}] \\ &= \int \mathcal{D}A \delta(\partial_\mu A^a_\mu) \det(\mathcal{M}^{ab}) \mathcal{V}(\Omega) \, \exp[-S_{YM}] \,, \end{aligned} \tag{83}$$

where the functional $\mathcal{V}(\Omega)$ was determined through the ghost fields propagator, namely

$$\begin{aligned} \langle \bar{c}^a(x) c^b(y) \rangle &= \int \mathcal{D}A\mathcal{D}\bar{c}\mathcal{D}c \delta(\partial_\mu A^a_\mu) \mathcal{V}(\Omega) \det(\mathcal{M}^{ab}) \, \exp[-S_{YM} + \int d^4x \, \bar{c}^a \partial_\mu D^{ab}_\mu c^b] \\ &= \int \mathcal{D}A \delta(\partial_\mu A^a_\mu) \det(-\partial_\mu D_\mu) \, \mathcal{V}(\Omega) \, [(\partial_\mu D_\mu)^{-1}]^{ab}_{xy} \, \exp[-S_{YM}] \,. \end{aligned} \tag{84}$$

The restriction to the Gribov region is equivalent to the condition that the Fourier transform $\Im(k^2; A)$ of the inverse of Faddeev-Popov's operator does not develop poles except for vanishing momentum. Thus, one has the following Gribov's horizon

$$\Im(k^2; A) = \frac{1}{N^2-1} \langle k | [\mathcal{M}^{ab}]^{-1} | k \rangle = \frac{1}{k^2}(1 + \sigma(k, A)), \tag{85}$$

with $\sigma(k, A)$ being the Gribov form factor depending on the external momentum k_μ of the ghost and the internal insertions of gauge fields. In addition, $[\mathcal{M}^{ab}]^{-1}$ is the inverse of the Faddeev-Popov operator and N the number of colors. The form factor related to the correlation function of the ghost can be computed in perturbation theory at second order on the gauge coupling g (35) [11]. The Gribov form factor at second order will be expressed as

$$\Im(k^2; A) = \frac{1}{k^2} + \frac{1}{\vartheta} \frac{1}{k^4} \frac{g^2 N}{N^2-1} \int \frac{d^d q}{(2\pi)^d} \frac{(k-q)_\mu q_\nu}{(k-q)^2} A^a_\mu(q) A^a_\nu(-q) + \mathcal{O}(g^3) \,, \tag{86}$$

[11] The main contributions are showed in figure (4).

where ϑ represents the volume of space-time. Therefore, the equation (85) can be rewritten as

$$\Im(k^2;A) \approx \frac{1}{k^2(1-\sigma(k,A))}, \tag{87}$$

Then, the requirement that the FP operator has no zero modes is given by

$$\sigma(k,A) < 1\,. \tag{88}$$

One can simplify the expression (88) of this condition in the following way. For Landau gauge, one has $q_\mu A_\mu = 0$ with $A^a_\mu A^a_\nu$ being transverse,

$$\langle A^a_\mu(-q)A^a_\nu(q)\rangle = \omega(A)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) = \omega(A)P_{\mu\nu}\,, \tag{89}$$

additionally, multiplying the last expression by $\delta_{\mu\nu}$, one finds that $\omega(A) = \frac{1}{d-1}\langle A^a_\mu A^a_\nu\rangle$, with d the number of dimensions. Thus, it is possible to simplify σ,

$$\sigma(k,A) = \frac{1}{\vartheta}\frac{1}{d-1}\frac{Ng^2}{N^2-1}\frac{k_\mu k_\nu}{k^2}\int\frac{d^dq}{(2\pi)^d}A^a_\gamma(-q)A^a_\gamma(q)\frac{1}{(k-q)^2}P_{\mu\nu}\,. \tag{90}$$

It is well-known that $\sigma(k,A)$ decreases with increasing k^2 with $\langle A^a_\gamma(-q)A^a_\gamma(q)\rangle$ being positive, therefore the condition (88) will be

$$\sigma(0,A) < 1\,. \tag{91}$$

Applying the limit $k^2 \to 0$ in $\sigma(k,A)$, one has

$$\begin{aligned}\sigma(0,A) &= \frac{1}{\vartheta}\frac{1}{d-1}\frac{Ng^2}{N^2-1}\lim_{k^2\to 0}\frac{k_\mu k_\nu}{k^2}\frac{d-1}{d}\delta_{\mu\nu}\int\frac{d^dq}{(2\pi)^d}A^a_\gamma(-q)A^a_\gamma(q)\frac{1}{q^2}\\ &= \frac{1}{\vartheta}\frac{1}{d}\frac{Ng^2}{N^2-1}\int\frac{d^dq}{(2\pi)^d}A^a_\gamma(-q)A^a_\gamma(q)\frac{1}{q^2}\,.\end{aligned} \tag{92}$$

Finally, the no-pole condition is

$$\mathcal{V}(\Omega) = \theta(1-\sigma(0,A)), \tag{93}$$

with θ being the Heaviside function. From the condition (92), we have the following expression in four dimensions for the no-pole condition,

$$\mathcal{V}(\Omega) = \exp\left[-Ng^2\gamma^4\int\frac{d^4q}{(2\pi)^4}\frac{A^a_\mu(q)A^a_\mu(-q)}{q^2} + 4(N^2-1)\gamma^4\right], \tag{94}$$

where γ is known as the Gribov parameter determined via gap equation (55). Moreover, this parameter is not free and it has the dimension of a mass. The functional $\mathcal{V}(\Omega)$ coupled to the generating functional (59) and the gap equation (55) ensures that the integration is done just for gauge field configurations that belong to the Gribov region. In the next subsection, we discuss the method developed by D. Zwanziger who improved the proposal established by V.N. Gribov.

2.1.2 Horizon function

The solution proposed by Gribov and introduced in the last subsection is self-consistent, however it has the restriction of implementing the no-pole condition just at leading order. An all order implementation is desirable to understand in more details the effects of the restriction to Ω. The first attempt to make an improvement of the solution established by V. N. Gribov was proposed by D. Zwanziger in (36). In his original work, D. Zwanziger applied the restriction to Ω with a different viewpoint. Instead of dealing with the ghost propagator he worked directly in the Faddeev-Popov operator spectrum. Therefore, let us begin writing the eigenvalues equation of the Faddeev-Popov operator

$$\mathcal{M}^{ab}\omega^{b} = \epsilon\omega^{a}. \tag{95}$$

This equation can be considered as a Schrödinger like equation with A^a_μ playing the role of a generic potential. Moreover, the eigenvalues were determined to all orders with a good approximation and the spectrum of the operator $\mathcal{M}^{ab}$ was obtained by imposing the restriction that the gauge field did not produce bound states. Nevertheless, this procedure omits some important features and the solution is approximate. Thus, D. Zwanziger developed in (113, 36, 114) a new formalism to deal with the Gribov copies problem via an effective action which was implemented in a practical manner at perturbative theory viewpoint. Then, the improvement proposed by D. Zwanziger gave a result more precise than the one obtained by V.N. Gribov (94). The condition for this proposal is given by

$$8\left(N^2-1\right) - 4NH > 0, \tag{96}$$

where H is the so-called horizon function,

$$H[A] \;=\; g^2\int d^4x d^4y\;\; f^{abc}A^b_\mu(x)\left(\mathcal{M}^{-1}\right)^{ad}(x,y) f^{dec}A^e_\mu(y). \tag{97}$$

The l.h.s of the expression (96) is related to the eigenvalues of the Faddeev-Popov operator, thus the functional $\mathcal{V}(\Omega)$ can be rewritten in the following way

$$\mathcal{V}(\Omega) \;=\; \exp\left[\gamma^4 g^2 \int d^4x d^4y \;\, f^{abc} A^b_\mu(x) \left(\mathcal{M}^{-1}\right)^{ad}(x,y) f^{dec} A^e_\mu(y) - 4(N^2-1)\gamma^4\right]. \tag{98}$$

Thereby, the gap equation responsible for determining the Gribov parameter will be

$$\langle H[A]\rangle = 4(N^2-1). \tag{99}$$

The equation (99) is also known as the horizon condition. The restrictions (98) and (99) ensure that the path integral is effectively restricted to the Gribov region. Then, the gauge field configurations near the horizon give the dominant contribution in the path integral (83). This occurs because the inverse FP operator in functional H starts to diverge as a consequence of the presence of the inverse of the Faddeev-Popov operator ($\mathcal{M}^{-1}$). Thus, this term gives the dominant contribution to the low energy properties of the path integral (83). Moreover, the exponential of the expression (98) can be replaced by an integral representation of Dirac's delta [12]. Thus, the functional $\mathcal{V}(\Omega)$ will be written as

$$\mathcal{V}(\Omega) \;=\; \delta\left[g^2 \int d^4x d^4y \;\, f^{abc} A^b_\mu(x) \left(\mathcal{M}^{-1}\right)^{ad}(x,y) f^{dec} A^e_\mu(y) - 4\vartheta(N^2-1)\right]. \tag{100}$$

An important remark to make is that if we approximate $\mathcal{M}$ by ∂^2, we recover the original result of V.N. Gribov.

2.1.3 The Gribov-Zwanziger action

The path integral (83) can be written as

$$Z \;=\; \int \mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}b \, \exp[-S_{GZ}], \tag{101}$$

where S_{GZ} is the nonlocal Gribov-Zwanziger action and it is established in the following way

$$S_{GZ} = S_{YM} + S_{FP} + \gamma^4 g^2 \int d^4x d^4y \;\, f^{abc} A^b_\mu(x) \left(\mathcal{M}^{-1}\right)^{ad}(x,y) f^{dec} A^e_\mu(y) - 4\vartheta(N^2-1)\gamma^4 \,.$$

[12] For more explanations about the change of the exponential for the Dirac's delta see (36).

(102)

The YM gauge fixed action, *i.e.*, $S_{YM}+S_{FP}$ is given by (39) for $\alpha=0$. Besides the presence of the functional (98), the action (102) must be stable at the quantum level. Therefore, this action shall be local and renormalizable by power counting, which is perfectly possible. To define this renormalizable action, the algebraic renormalization procedure (76) establishes that the BRST symmetry must exist as discussed in chapter (1). The BRST symmetry is respected when one takes an extended version of (102), which reduces to this expression in some physical limit in the general action [13]. Additionally, the action (102) can be cast in a local way by introducing auxiliary fields [14]. Thus, the horizon function present in (102) is localized through the following expression

$$\begin{aligned}\exp\left[-\gamma^4 H\right] \;\; &= \;\; \int \mathcal{D}\varphi\,\mathcal{D}\bar{\varphi}\,\mathcal{D}\omega\,\mathcal{D}\bar{\omega}\,\exp\Big[-\bar{\omega}^{ac}_{\mu}\mathcal{M}^{ab}\omega^{bc}_{\mu}+\bar{\varphi}^{ac}_{\mu}\mathcal{M}^{ab}\varphi^{bc}_{\mu} \\ &\quad -\gamma^2 g f^{abc}A^a_{\mu}\left(\varphi^{bc}_{\mu}+\bar{\varphi}^{bc}_{\mu}\right)+4\vartheta(N^2-1)\gamma^4\Big]\,, \end{aligned} \tag{103}$$

where the auxiliary fields $(\varphi,\bar{\varphi})$ and $(\omega,\bar{\omega})$ are respectively bosons and fermions (in the sense that the latter are represented by Grassmann fields while the former involve usual real or complex fields) [15]. As a result, the local GZ action known as the physical action is given by

$$S_{phys}=S_{YM}+S_{FP}+S_{aux}+S_{\gamma}\,, \tag{104}$$

with the auxiliary fields action defined as

$$S_{aux}=\int d^4x\left(\bar{\omega}^{ac}_{\mu}\mathcal{M}^{ab}\omega^{bc}_{\mu}-\bar{\varphi}^{ac}_{\mu}\mathcal{M}^{ab}\varphi^{bc}_{\mu}\right), \tag{105}$$

and the action which depends on the Gribov parameter will be

$$S_{\gamma}=\int d^4x\left[\gamma^2 g f^{abc}A^a_{\mu}\left(\varphi^{bc}_{\mu}+\bar{\varphi}^{bc}_{\mu}\right)-4\vartheta(N^2-1)\gamma^4\right]. \tag{106}$$

To finish this subsection some comments are essential. First the GZ action was proven to be renormalizable to all orders in a loop expansion (36). Moreover, with this fundamental property it is possible to compute the correlation functions of the theory and compare its results with the available lattice data. The first attempt to verify the

[13] The BRST invariant action for this approach will be discussed with more details in chapter (4).

[14] The properties of locality and renormalizability can be seen with more details in (36, 114, 50, 63).

[15] It is important to remark that the fields $(\omega,\bar{\omega})$ are ghosts with integer spin and fermionic statistic. Thus, they are nonphysical fields, *i.e.*, it is possible to integrate and remove them in the path integral, the same occurs for $(\varphi,\bar{\varphi})$.

validity of GZ approach consisted in computing the correlation functions for the gauge field and the ghosts. These results were presented in (54) and (56). Nevertheless, they were incompatible when compared to the current lattice data. This discrepancy motivated further methodological developments that we present in the thus next subsection.

2.1.4 Refined Gribov-Zwanziger's action

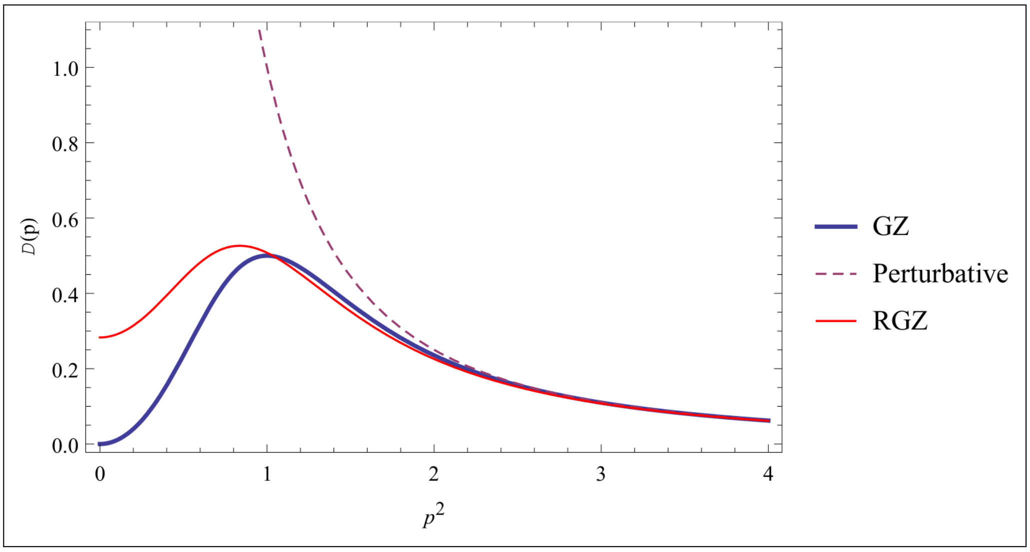
As already pointed out, in the last decades the discussion about the gauge field and ghost propagators, equations (54) and (56) were the main focus, in particular in Landau gauge. With novel results for those quantities obtained from the lattice data (115, 45, 116), the improvement for the GZ approach was required. In 2008, S.P. Sorella and collaborators proposed a refinement for the GZ action, for more details see (39, 40, 41). This progress consisted in taking into account the existence of dimension two condensates to adjust the results obtained by the GZ framework compared to the lattice data. In fact, during a long period it was believed that the gauge field propagator was strongly suppressed in the deep infrared regime, vanishing in the zero-momentum limit (44, 117). The original analysis based on Dyson-Schwinger equations (DSE) or the renormalization group (RG) exact equations also indicated the same behavior of the GZ picture, as discussed in (118, 19, 119, 120).

However, as mentioned before, in 2007, a novel lattice data (115) provided different results for the propagators behavior in the infrared for 3 and 4 dimensions with larger volume. It was unambiguously established that the propagators had finite values see figure (5), and that the gauge field propagator presented violation of positivity. Additionally, the ghost propagator showed a soft behavior (115, 116). Thus, the GZ approach was inconsistent with these lattice results. Let us pose for a moment and discuss with more details the objects known as condensates. In order to explain the behavior of the gauge field and ghost propagator in GZ approach, one needs to take into account the presence of other nonperturbative effects. The sources for these effects in gauge theories are condensates, i.e., the vacuum expectation values of certain local operators. As we are going to see also in the Serreau-Tissier framework, the dimension two condensate $\langle A^2 \rangle$ has a large interest for the confinement community since using the minimum of the functional (71), defined as A^2_{min}, it is clear that by construction $\langle A^2_{min} \rangle$ is a gauge-invariant quantity [16]. Furthermore, when one introduces the local composite operator A^2_μ in GZ action the renormalizability is preserved [17]. Following the results obtained in (121) one has the

[16] For more details see appendix (A).

[17] For more details of the proof, see (121).

Figure 5 - Qualitative description comparing the behavior of the gauge field form factor in the Gribov-Zwanziger, its refined version and the perturbative actions.



Source: PEREIRA, 2016, p. 69.

following action which takes into account the restriction of the path integral domain to Ω and the nonperturbative effects invoked by the operator A_μ^2 to the GZ action

$$S_0 = S_{GZ} + \int d^4x \left(\frac{J}{2} A_\mu^a A_\mu^a + \frac{\theta}{2} J^2 \right), \tag{107}$$

with J a new source invariant under the BRST transformation s, which attains the following physical value $J\Big|_{phys} = m^2$, m being a mass parameter and θ a new parameter [18]. From the last action (107), the tree-level gauge field propagator is specified by

$$\langle A_\mu^a(p) A_\nu^b(p) \rangle \;=\; \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{p^2}{(p^2 + m^2 + 2g^2\gamma^4 N)}. \tag{108}$$

For the ghost propagator at one-loop order,

$$\lim_{p\to 0} \frac{\delta^{ab}}{N^2-1} \langle \bar{c}^a(p) c^b(-p) \rangle \approx \frac{4}{3Ng^2\mathcal{K}p^4}\,, \tag{109}$$

[18] The author emphasizes here that θ is not the Grassmann coordinate that appears in the supersymmetric formulation of the Serreau-Tissier framework which will be presented in the next section.

where $\mathcal{K}$ is given by

$$\mathcal{K} = \int \frac{d^4p}{2\pi^d} \frac{1}{p^2(p^4 + m^2p^2 + 2g^2N\gamma^4)} \,, \tag{110}$$

which is real and finite for $d = 4$. The result established by (108) is not qualitatively different from the standard GZ action. Thus, the gauge field propagator remains suppressed at the infrared vanishing at zero momentum and positivity violating (121). Moreover, at one-loop order the ghost propagator continues its enhancement for $p \approx 0$. Such results entail that GZ action with the operator A^2_μ is insufficient to reproduce the decoupling behavior obtained by lattice gauge-fixing simulations (115, 45, 116). Thus, it was proposed to introduce other $d = 2$ condensates. One of the candidates was the local composite operator $\left(\bar{\varphi}^{ab}_\mu(x)\varphi^{ab}_\mu(x) - \bar{\omega}^{ab}_\mu(x)\omega^{ab}_\mu(x)\right)$. Previously the main role of these auxiliary fields was to localize the horizon function, however they also develop their own quantum dynamics and condensate. Therefore, this operator will be included in the action (107) as

$$S_{\varphi\omega} = \int d^4x \left(s(-\tilde{J}\bar{\omega}^{ab}_\mu\varphi^{ab}_\mu) \right) , \tag{111}$$

with $\tilde{J}$ an external source invariant under BRST transformations $s\tilde{J} = 0$, which attains the physical value $\tilde{J}\Big|_{phys} = M^2$, with M being a mass parameter. The introduction of this term does not spoil the renormalizability of S_0. The term $\frac{\theta}{2}J^2$ removes ultraviolet divergences proportional to J^2. Nonetheless, in the case of $\left(\bar{\varphi}^{ab}_\mu(x)\varphi^{ab}_\mu(x) - \bar{\omega}^{ab}_\mu(x)\omega^{ab}_\mu(x)\right)$ these divergences do not emerge in the correlation functions and can be ignored. Finally, in 4 dimensions the refined Gribov-Zwanziger action is

$$S_{RGZ} = S_{GZ} + \frac{m^2}{2} \int d^4x \, A^a_\mu A^a_\mu + M^2 \int d^4x \left(\bar{\varphi}^{ab}_\mu\varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu\omega^{ab}_\mu \right) . \tag{112}$$

Just as the Gribov parameter, γ^2, the new parameters (m^2, M^2) are not free ones and they are dynamically generated at quantum level. These parameters result in the existence of the dimension two condensates $\langle A^a_\mu(x) \, A^a_\mu(x)\rangle$ and $\langle\bar{\varphi}^{ab}_\mu\varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu\omega^{ab}_\mu\rangle$ which are obtained through the minimization of the vacuum energy.

Thereby, when γ^2 is nonzero then $\langle\bar{\varphi}\varphi - \bar{\omega}\omega\rangle$ and $\langle A^a_\mu A^a_\mu\rangle$ also have nonzero value. As written before, the refined version of the Gribov-Zwanziger action at tree-level has a good agreement with the current lattice data. For example, the gauge field propagator in this improved approach is characterized by

$$\langle A^a_\mu(p) \, A^b_\nu(p)\rangle \quad = \quad \delta^{ab} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \mathcal{D}_{RGZ}(p^2), \tag{113}$$

where

$$\mathcal{D}_{RGZ} = \frac{p^2+M^2}{(p^2+m^2)(p^2+M^2)+2g^2\gamma^4 N}\,. \tag{114}$$

2.1.5 Soft breaking of BRST symmetry

The GZ action (104) is not invariant under BRST transformation, characterized by the following transformations:

$$\begin{aligned}
sA^a_\mu &= -D^{ab}_\mu c^b,\\
sc^a &= \frac{g}{2} f^{abc} c^b c^c,\\
s\bar{c}^a &= ib^a,\\
sb^a &= 0,\\
s\bar{\omega}^{ac}_\mu &= \bar{\varphi}^{ac}_\mu,\\
s\bar{\varphi}^{ac}_\mu &= 0,\\
s\varphi^{ac}_\mu &= \omega^{ac}_\mu,\\
s\omega^{ac}_\mu &= 0\,.
\end{aligned} \tag{115}$$

The reason for this breaking of invariance is represented by the expression

$$\Delta_\gamma = sS_{GZ} = sS_\gamma = -g\gamma^2 \int d^4x\, f^{abc}(A^a_\mu \omega^{bc}_\mu - (D^{am}_\mu c^m)(\bar{\varphi}^{bc}_\mu + \varphi^{bc}_\mu))\,. \tag{116}$$

The presence of the Gribov parameter is responsible for this soft breaking in the RGZ action. Furthermore, in the old context of RGZ formulation, which was presented in this chapter, the BRST soft breaking happens exclusively because of the presence of the horizon function. Being more specific, the restriction to the first Gribov region in the generating functional breaks the BRST symmetry, this occurs since when one moves into the functional space by infinitesimal gauge transformations, eventually one crosses the horizon. Let us quantify this argument by following the work done in (40) exhibiting two simple cases,

- **The gauge field A^a_μ is not located near the horizon ($\partial\Omega$).** Given a field configuration $A^a_\mu \in \Omega$, the infinitesimal gauge field $\mathcal{A}^a_\mu$ defined by $\mathcal{A}^a_\mu = A^a_\mu + D^{ab}_\mu \omega^b$, where ω^a is an infinitesimal parameter, cannot belong to the region Ω.

  Proof. Assuming $A^a_\mu \in \Omega$ and the Landau gauge-fixing condition,

  $$\partial_\mu \mathcal{A}^a_\mu = \partial_\mu A^a_\mu - \partial_\mu D^{ab}_\mu(A)\omega^b = 0 \Rightarrow -\partial_\mu D^{ab}_\mu(A)\omega^b = 0\,, \tag{117}$$

  which makes contradictory the hypothesis about $A_\mu \in \Omega$ and $-\partial_\mu D^{ab}_\mu(A)\omega^b > 0$.

For complementing this case, see (35).

- **The gauge field A_μ is located near $\partial\Omega$.** If the configuration A^a_μ is near the horizon $\partial\Omega$, *i.e.*, $A^a_\mu = r^a_\mu + T^a_\mu$, where $T^a_\mu \in \partial\Omega$ and r^a_μ is a small perturbation. Thus, there is an equivalent configuration $\mathcal{A}^a_\mu$ located outside Ω.

  Proof. From $\partial_\mu T^a_\mu = \partial_\mu r^a_\mu = 0$, one has

$$\mathcal{A}^a_\mu = T^a_\mu + r^a_\mu + D^{ab}_\mu(T)\omega^b + ... \tag{118}$$

  for the copy $\mathcal{A}^a_\mu$ at lowest order in r^a_μ and ω^a. Thereby, T^a_μ belongs to the horizon and ω being a zero mode related to T^a_μ, it is possible to have

$$\partial_\mu \mathcal{A}^a_\mu = \partial_\mu D^{ab}_\mu(T)\omega^b = 0\,, \tag{119}$$

  with the last expression, one can state that $\mathcal{A}^a_\mu$ is transverse and it is situated to the boundary of the horizon. Nonetheless, it is located outside of the Gribov region Ω and on the other side of $\partial\Omega$ when one compares with the gauge field A^a_μ.

Therefore from the last case, one can conclude that the BRST transformation of a gauge field which belongs to Ω produces a copy outside Ω. As already known the RGZ action is restricted to region Ω, thus, the soft breaking of the BRST symmetry is obligatory. Even with this soft breaking, the model is still renormalizable, the reason for that is the preservation of the Slavnov-Taylor identity (38), *i.e.*, the term Δ_γ has mass dimension two, and it is irrelevant in the ultraviolet regime, where the invariance of the cohomology of the BRST symmetry is recovered. In this thesis, we are going to review and extend for matter fields a new RGZ formalism which is BRST-invariant. For more details, see chapter (4).

Before changing the philosophy of the Gribov problem presenting the Serreau-Tissier framework, let us summarize what we have written about the RGZ scenario [19] to clarify the ideas for the reader: The GZ action was developed by considering the presence of infinitesimal Gribov ambiguities in the quantization procedure of YM theories. This action was presented in a nonlocal formulation because of the horizon function and showed a novel massive parameter known as the Gribov parameter, which is determined by the gap equation (99). Additionally, extra auxiliary fields were introduced to localize the horizon function presented in (102). In this form, the GZ formalism is local and renormalizable to all orders in a loop expansion. Moreover, up to now in this chapter we have commented that even at perturbative level, there are infrared instabilities for this action. They are

[19] The version of this approach with both fermionic and bosonic local gauge-invariant composite fields will be discussed in more details together with its renormalizability in chapters (4), (6) and (5).

associated with the formation of dimension two condensates $\langle A^a_\mu(x)\, A^a_\mu(x)\rangle$ and $\langle \bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu\rangle$. Finally, two extra massive terms were included in the original GZ action (102) to consider those instabilities. This gives rise to the RGZ action (112). As commented before, we will discuss the GZ scenario in chapter (4), where important improvements in the recent years will be of great importance to extend this study to linear covariant gauges with both local fermionic and bosonic gauge-invariant composite fields.

2.2 **The Serreau-Tissier framework**

2.2.1 The Curci-Ferrari model

As explained above, lattice simulations performed in (115) unambiguously showed that the gluon propagator saturates to a finite value in the zero momentum limit. Based on this observation M. Tissier and N. Wschebor proposed to use, as a phenomenological model, the Curci-Ferrari (CF) model (122) to describe these lattice data. The CF model is an extension of the FP action where a mass term is added for the gluons. The model (26) was shown to reproduce with a good accuracy several properties observed in lattice simulations. This idea modified the theory in the infrared regime, however the standard FP conjecture for momenta $p \gg m$ at all orders of perturbation theory was conserved. In such case, it was realized that this model reproduces the lattice forecasts up to two loops with good accuracy (26, 123, 124, 125), the mass term does not destroy the renormalizability and they also displayed that the spectral function of the gluons is not positive definite, which is in conform to other studies (19, 44, 126).

Therefore, they considered the YMFP Euclidean action gauge fixed in Landau gauge with a massive gauge field term:

$$S_{CF} = \int d^4x \left(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b + \frac{m^2}{2}A^a_\mu A^a_\mu\right), \tag{120}$$

where m is defined as the mass parameter and the original YMFP action in Landau gauge is obtained when $m = 0$. The mass term induces a soft breaking of BRST symmetry (44) as in the original RGZ case mentioned previously. Therefore, for the reasons described above, this model is renormalizable in four dimensions (122, 127) and the renormalization factors were figured out down to three loops in a $\overline{MS}$ (modified minimal subtraction) scheme in (128). In fact, the model with the CF action (120) has a pseudo-BRST symmetry, which is non-nilpotent and has the same characteristics as the standard BRST (44), except that, the Lautrup-Nakanishi field b variation is given by $sb^a = -im^2c^a$. Therefore, the action (120) still displays a large set of symmetries.

M. Tissier and N. Wschebor did not change in their seminal work the field content of the theory, e.g., the ghost sector remained the same since the ghost propagator is infared divergent in lattice simulations. Moreover, they did not modify the interactions in the action to maintain the original ultraviolet nature of the theory and the prognosis of perturbative QCD for momenta higher than Λ_{QCD}. As a result, the only possible terms which are renormalizable, local and do not modify the ultraviolet regime one can introduce in the YMFP action are massive terms.

At this point, the calculation of the gluon propagator in this phenomenological scheme was pushed to two loops (125), which shows a remarkable agreement with lattice simulations. This proves that the CF model is indeed a good starting point to describe lattice data in the quenched approximation. It is possible that if one adds more loops, one could get some correlation functions that converge to results very far away from the lattice results. Luckily, (125) ruled out this hypotheses. The convergence of perturbation theory in the infrared sector is controlled. From this phenomenological system, one can notice that most of the nonperturbative dynamics are precisely picked up by the effective gauge field mass and the residual dynamics can then be treated perturbatively. Obviously, it would be more convincing to see this mass emerging from first principles, instead of merely adding it on phenomenological grounds. It was propose in (77) that the origin for this mass could be related to the Gribov copies. We shall review in the sequence the formalism which was used to attack this issue.

2.2.2 The Serreau-Tissier proposal for dealing with Gribov copies

The first attempt to explain the model based on Curci-Ferrari of M. Tissier and N. Wschebor was through the first principles model developed by J. Serreau and M. Tissier (77). This framework is an alternative attempt to the (R)GZ approach to deal correctly with the Gribov ambiguities in the gauge-fixing procedure originally created by Faddeev-Popov, as exhaustively referred in this manuscript. The authors J. Serreau and M. Tissier proposed in their seminal works (77, 78) firstly in Landau and then extended to nonlinear covariant gauges [20] a method which deals explicitly with the Gribov copies and can be implemented within perturbative calculations. This proposal consists on taking a specific average over Gribov copies of each gauge field configuration. These are good gauge-fixings in the sense that gauge-invariant objects are independent of the gauge-fixing procedure. A similar averaging procedure, however not restricted to Gribov copies and nonrenormalizable in the Landau gauge was proposed in (132, 66, 133).

[20] In this case, the gauge-fixing term is the so-called Curci-Ferrari-Delbourgo-Javis (122, 129). This gauge condition was studied with special attention in the following works (127, 130, 131).

When one compares this framework with the (R)GZ approach one notices that, the latter is based on the requirement that in the presence of Gribov ambiguities, one shall restrict the path integral and consider just the gauge field configurations in the domain between $\delta\Omega$ and $\delta\Lambda$, as already written in subsection (1.2.8). Instead, in the Serreau-Tissier model all Gribov copies contribute, with a certain weight. This lifts the degeneracy of the copies, therefore avoiding the 0/0 Neuberger problem. In Landau and nonlinear covariant gauges, this framework is a good alternative. In this thesis, the author will focus just on the development of this model in Landau gauge. The reason will be very clear in the next chapter.

Now, let us turn our attention to the original work done by J. Serreau and M. Tissier (77). In order to compute usual YM correlators in the presence of Gribov copies, one first makes a (pseudo) average over these ambiguities with a nonuniform statistical weight for each given gauge field configuration A_μ, obviously belonging to the same gauge orbit and then perform an average over the gauge field configurations with the usual YM weight. The average over Gribov copies for any operator $\mathcal{O}[A]$ of a given A_μ field is

$$\langle\mathcal{O}[A]\rangle \;=\; \frac{\Sigma_i\mathcal{O}[A^{U_i}]s(i)\exp(-S_W[A^{U_i}])}{\Sigma_i\, s(i)\exp(-S_W[A^{U_i}])}\,, \tag{121}$$

with $s(i)$ being the sign of the Faddev-Popov operator (60) taken at $A = A^{U_i}$ and the weight factor S_W is defined as

$$S_W[A] \;=\; \beta_0\int d^4x\,\mathcal{H}[A,U_i]\,. \tag{122}$$

Where $\mathcal{H}[A,U_i]$ was given in equation (82) and $\beta_0 > 0$ is a free gauge parameter that has mass dimension 2. From expressions (121) and (122) one observes that the sum runs over minima, saddle and maxima of $S_W[A]$, such that all Gribov copies are considered. The weight $\exp\left(\beta_0\mathcal{H}[A,U_i]\right)$ lifts the degeneracy of the copies conforming with the perspective of the extrema of $\mathcal{H}$. For β_0 not too small, the equivalent gauge field configurations outside the region Ω are suppressed by S_W. Indeed, for $\beta_0\to\infty$ the absolute minimum $U = U_{abs}$ is selected and the averaging method coincides to the absolute Landau gauge, namely

$$\lim_{\beta_0\to\infty}\langle\mathcal{O}[A]\rangle = \mathcal{O}[A^{U_{abs}}]\,. \tag{123}$$

Looking at the opposite limit $\beta_0\to 0$, all Gribov copies have the same contribution in the average (121) besides the sign factor $s(i)$. Since there are as many contributions for each sign, the denominator in (121) vanishes: $\Sigma_i s(i) = 0$. This is at the origin of the Neuberger 0/0 problem. For any $\beta_0 > 0$, the degeneracy is lifted and the denominator in (121) is typically nonzero which solves the Neuberger zero problem. However, J. Serreau and M. Tissier assumed that some field configurations might yield a null denominator and they stated that such configurations have to be of zero measure.

Once this first average over Gribov copies is performed, one averages over all gauge field configurations A_μ with the Yang-Mills weight, *i.e.*,

$$\overline{\mathcal{O}[A]} = \frac{\int \mathcal{D}A\mathcal{O}[A]\exp(-S_{YM}[A])}{\int \mathcal{D}A\exp(-S_{YM}[A])}\,. \tag{124}$$

Then, to obtain the observable $\mathcal{O}$ in this approach, one needs first average over Gribov copies and for the last average over gauge field configurations, being more specifically

$$\overline{\langle\mathcal{O}[A]\rangle}\,. \tag{125}$$

To finalize this subsection, an important remark needs to be written, *i.e.*, the gauge-invariant operators $\mathcal{O}_{inv}[A^U] = \mathcal{O}_{inv}[A]$ are unaltered by the average (121): $\langle\mathcal{O}_{inv}[A]\rangle = \mathcal{O}_{inv}[A]$. This ensures that this gauge-fixing method keeps unchanged the physical observables as it should. This is only possible because of the presence of the denominator in (121). Therefore,

$$\overline{\langle\mathcal{O}_{inv}[A]\rangle} = \overline{\mathcal{O}_{inv}[A]}\,. \tag{126}$$

2.2.3 Functional integral formalism

The importance of the denominator in the Serreau-Tissier gauge-fixing procedure in (121) was emphasized at the end of the last subsection. Nonetheless, this denominator results in the nonlocality of (125). Thereby, if one intends to have a local field theory an additional effort shall be required. Following the development of (77), one has the identity

$$\sum_i \mathcal{F}[A^{U_i}]s(i) \;=\; \int \mathcal{D}U\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}b\,\mathcal{F}[A^U]\exp\{-S_{FP}[A^U,c,\bar{c},b]\}\,, \tag{127}$$

where (127) is the average of Gribov ambiguities rewritten as a functional integral over a $SU(N)$ matrix field U and the already well-known Faddev-Popov and Lautrup-Nakanishi fields. Moreover, the action S_{FP} is given in (39) with $\alpha = 0$ and without the Yang-Mills sector ($\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$). We shall see later that the set of fields U, c, $\bar{c}$ and b can be merged into a superfield, that we shall call V. Employing the identity (127) for $\mathcal{F}[A^U] = \mathcal{O}[A^U]\exp(-\beta_0\mathcal{H}[A,U])$ or $\mathcal{F}[U] = \exp(-\beta_0\mathcal{H}[A,U])$, the average (121) over equivalent gauge field configurations can be rewritten as

$$\langle\mathcal{O}[A]\rangle \;=\; \frac{\int \mathcal{D}V\,\mathcal{O}[A^U]\exp\{-S_{FP}[A,V]-\beta_0\mathcal{H}[A,U]\}}{\int \mathcal{D}V\,\exp\{-S_{FP}[A,V]-\beta_0\mathcal{H}[A,U]\}}\,. \tag{128}$$

We can rewrite this expression in a more compact form:

$$\langle \mathcal{O}[A] \rangle = \frac{\int \mathcal{D}V \mathcal{O}[A^U] \exp\{-S_{gf}[A,V]\}}{\int \mathcal{D}V \exp\{-S_{gf}[A,V]\}}\,, \tag{129}$$

with

$$\begin{aligned} S_{gf}[A,V] &= S_{FP}[A^U, c, \bar{c}, b] + S_W[A^U] \\ &= S_{FP}[A^U, c, \bar{c}, b] + \int d^4x\, \beta_0 \mathcal{H}[A,U]\,. \end{aligned} \tag{130}$$

This construction strongly resembles the strategy used in statistical systems in the presence of a quenched disorder (80, 81). One then needs to compute the thermal average of some physical quantity, that is denoted by $\langle\rangle$, and which involves the same ratio as in eq. (121). As a second step, one then averages over the disorder configurations. The same idea was adapted for YM theories by J. Serreau and M. Tissier, as written before, the first (thermal) average corresponds to the one over Gribov copies, (121), for fixed gauge field A_μ. Then, this gauge field plays the role of the disorder field to be averaged over in the second step. These two-step averages are difficult to implement in analytical calculations. However, using a method known as replica trick (80, 81) it is possible to solve this issue. This trick consists in writing the denominator (121) as

$$\frac{1}{\int \mathcal{D}V \exp\{-S_{gf}[A,V]\}} = \lim_{n\to 0} \int \prod_{k=1}^{n-1} (\mathcal{D}V_k \exp\{-S_{gf}[A,V_k]\})\,, \tag{131}$$

where $n-1$ independent copies of V_k are implemented and labeled by the replica index k. The limit in (131) is to be interpreted as the value of the function of n on the r.h.s when $n \to 0$. Then, the average over the disorder gauge field A_μ is

$$\overline{\langle \mathcal{O}[A] \rangle} = \lim_{n\to 0} \frac{\int \mathcal{D}A (\prod_{k=1}^{n} \mathcal{D}V_k) \mathcal{O}[A^{U_1}] \exp\{-S[A,\{V\}]\})}{\int \mathcal{D}A \exp\{-S_{YM}[A]\}}\,, \tag{132}$$

The action in this approach will be expressed as

$$S[A,V] = S_{YM}[A] + \sum_{k=1}^{n} S_{gf}[A,V_k]\,. \tag{133}$$

The above expression enjoys the following property

$$\int \mathcal{D}A \exp\{-S_{YM}[A]\} = \lim_{n\to 0} \int \mathcal{D}A \prod_{k=1}^{n} \mathcal{D}V_k \exp\{-S[A,V]\}\,, \tag{134}$$

as observed when $\mathcal{O}[A] = 1$ in equation (132). Then, the Serreau-Tissier gauge-fixing is cast in the form of a local field theory, with the action (133), in the following way

$$\overline{\langle \mathcal{O}[A] \rangle} = \lim_{n\to 0} \frac{\int \mathcal{D}A (\prod_{k=1}^{n} \mathcal{D}V_k) \mathcal{O}[A^{U_1}] \exp\{-S[A,\{V\}]\}}{\int \mathcal{D}A (\prod_{k=1}^{n} \mathcal{D}V_k) \exp\{-S[A,\{V\}]\}}\,, \tag{135}$$

where the choice of the replica $k = 1$ is arbitrary due the permutation symmetry among the replicas. Moreover, it is useful to factor out the volume of the gauge group $\int \mathcal{D}U$ and this can be done by making the change of variables $A \to A^{U_1}$ and $U_k \to U_k U_1^{-1}$, $\forall k > 1$ in (135). Once this is done, no explicit dependence on U_1 appears and the integral over this field factorizes. To simplify the notation, we establish the following replacement $(c_1, \bar{c}_1, b_1) \to (c, \bar{c}, b)$. Therefore, one gets

$$\overline{\langle \mathcal{O}[A] \rangle} \;=\; \lim_{n\to 0} \frac{\int \mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}b(\prod_{k=2}^{n}\mathcal{D}V_k)\mathcal{O}[A]\exp\{-S[A,c,\bar{c},b,\{V\}]\}}{\int \mathcal{D}A\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}b(\prod_{k=2}^{n}\mathcal{D}V_k)\exp\{-S[A,c,\bar{c},b,\{V\}]\}}. \tag{136}$$

Then, the Serreau-Tissier action in Landau gauge reads

$$\begin{aligned} S[A,c,\bar{c},b,\{V\}] \;&=\; \int d^4x \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu + \bar{c}^a\partial_\mu D^{ab}_\mu c^b + \frac{\beta_0}{2}A^a_\mu A^a_\mu \right. \\ &+ \left. \sum_{k=2}^{p}(S_W[A^{U_k}] + S_{FP}[A^{U_k}, c_k, \bar{c}_k, b_k])\right]. \end{aligned} \tag{137}$$

The gauge fixed action (137) gives rise to an alternative of the refined GZ framework for continuum implementation of the Landau gauge. Furthermore, the Serreau-Tissier approach turns out to be renormalizable in four dimensions (77). In particular, this method is a nice gauge-fixing because the averages of gauge-invariant observables are equal to those obtained with just the YM weight. The $n \to 0$ limit is decisive to keep the validation of the last property as it accounts for the existence of the denominator in (121). Another comment to make is that the only hypothesis made so far was the change of order between the limit $n \to 0$ with the path-integral over A_μ. Then, the Serreau-Tissier framework can be summarized by first calculate averages for fixed n with the action (137) and in the perturbative computations analytic functions of n are obtained, after this achievement one takes the $n \to 0$ limit

$$\langle \mathcal{O}_{inv}[A] \rangle = \lim_{n\to 0}[\mathcal{O}[A](n)]\,. \tag{138}$$

Finally, the traditional Curci-Ferrari model is recovered when $n = 1$, *i.e.*,

$$\langle \mathcal{O}_{inv}[A] \rangle_{CF} = [\mathcal{O}[A](n=1)]\,. \tag{139}$$

This emphasizes that the phenomenological model cannot agree with the gauge fixed version of YM theories when the Gribov copies are considered in the Serreau-Tissier framework. Moreover, the limit $n \to 0$ is mandatory for the independence of gauge-invariant quantities with respect to the gauge-fixing procedure.

2.2.4 The supersymmetric formulation of the Serreau-Tissier approach

In the previous subsections we started to review with more details the alternative approach proposed by Serreau-Tissier for the gauge-fixing procedure in Landau gauge taking into account the Gribov copies effects. This gauge-fixing procedure can be cast into a local QFT by adding the replica fields. For having access to averages in the Serreau-Tissier framework one needs to perform two steps which are summarized as follows: first compute them with the Serreau-Tissier action (137), and second apply the $n \to 0$ limit. As a consequence, the calculations are done for n fixed.

In this subsection, we turn our attention to review the supersymmetric version of the Serreau-Tissier action (137) [21]. This supersymmetric extension shows nontrivial symmetries and as a result the proof of the renormalizability of the model in four dimensions, as commented before, turns out possible (77). Let us begin defining the symbol V as a $SU(N)$ supermatrix field, namely

$$V(x,\theta,\bar{\theta}) \;=\; \exp\{ig[\bar{\theta}c + \bar{c}\theta + \bar{\theta}\theta\tilde{b}]\}U\,, \tag{140}$$

where the superfield V lives on a superspace created by the standard Euclidean space (x) with d dimensions and by Grasmannian coordinates $(\theta, \bar{\theta})$ which satisfies $\theta^2 = \bar{\theta}^2 = \theta\bar{\theta} + \bar{\theta}\theta = 0$. Furthermore, V is a superfield which takes value in the gauge group under consideration and thereby, for SU(N), it satisfies $V^{\dagger}.V = \mathbb{I}$. The Lautrup-Nakanishi field is rewritten as $\tilde{b}^a = ib^a + \frac{g}{2}f^{abc}\bar{c}^b c^c$ and the dependence from the coordinate x is present through the fields $U, c, \bar{c}$ and b. Moreover, the Grassmann space is curved and it has a line element defined as $ds^2 = g_{MN}dNdM = 2g_{\theta\bar{\theta}}d\bar{\theta}d\theta$. The Grassmann metric g_{MN} from (131) is

$$\begin{aligned} g_{\bar{\theta}\theta} &= -g_{\theta\bar{\theta}} = \beta_0\bar{\theta}\theta + 1\,, \\ g^{\bar{\theta}\theta} &= -g^{\theta\bar{\theta}} = \beta_0\bar{\theta}\theta - 1\,. \end{aligned} \tag{141}$$

The invariant integration measure is characterized by

$$\int d\theta d\bar{\theta}\, g^{\frac{1}{2}}(\theta\bar{\theta}) = \int d\theta d\bar{\theta}\,(\beta_0\bar{\theta}\theta - 1) = \beta_0\,. \tag{142}$$

The super gauge transformation A^V reads

$$A^V_\mu \;=\; VA_\mu V^{-1} + \frac{i}{g}V\partial_\mu V^{-1}\,, \tag{143}$$

[21] The CF action (120) can be rewritten in a supersymmetric description and it was already studied in (131). Thus, the Serreau-Tissier action also presents such formulation (77, 78).

the expression (143) is analogous to the simple gauge transform given by (1.1). In superspace, the weight (132) takes the form

$$S_W[A^U] + S_{FP}[A^U, c, \bar{c}, b] \;=\; \int d^d x d\theta d\bar{\theta}(\beta_0 \bar{\theta}\theta - 1) Tr(A^V_\mu)^2 \,. \tag{144}$$

The description of (144) as a supersymetric nonlinear sigma model characterized on the superspace $(x, \theta, \bar{\theta})$ is expressed as

$$Tr(A^V_\mu)^2 \;=\; Tr(A_\mu - \frac{i}{g} V^{-1}\partial_\mu V)^2 = -\frac{1}{g^2} Tr(V^{-1} D_\mu V)^2 \,, \tag{145}$$

where the covariant derivative for this case is

$$D_\mu V \;\equiv\; \partial_\mu V + igVA_\mu \,. \tag{146}$$

The action (133) using (132) and (144)-(145), defines a set of n gauged supersymmetric nonlinear sigma models. Also, it is invariant under the super gauge transformation $A \to A^V$ and $V_k \to V_k V^{-1}$, $\forall k = 1, ..., n$. This symmetry is broken when one replica is selected to factor out the volume of the gauge group. Thereby, the action (137) represents a theory which is gauge fixed for $n-1$ gauged supersymmetric nonlinear sigma models by (132), thus (137) can be rewritten as

$$\begin{aligned} S[A, c, \bar{c}, b, \{V\}] \;&=\; S_{YM}[A] + S_W[A] + S_{FP}[A, c, \bar{c}, b] \\ &- \; \frac{1}{g^2}\sum_{k=2}^{p} \int d^d x d\theta d\bar{\theta}(\beta\bar{\theta}\theta - 1) Tr(V_k^{-1} D_\mu V_k)^2 \,. \end{aligned} \tag{147}$$

The proof of the renormalizability of Serreau-Tissier framework (137)-(145) in Landau gauge for $d = 4$ was done in (77). Some remarks about this proof will be given below, e.g., due to the presence of nonlinear sigma models [22], which are known to be renormalizable in $d = 2$. The proof was based on the principles established by (89) and the idea was to recognize local terms with mass dimension less or equal to four [23] in the effective action Σ compatible with the symmetries of the theory.

Additionally, the demonstration was done for any n, however the limit $n \to 0$ was not employed and because of that the renormalization factors depended on n. Also, The gluon and ghost two-point vertex functions were computed at one-loop in the CF model (25, 26) with dimensional regularization. The corresponding one-loop expressions for this model can be determined by replacing the bare gluon mass $m_0^2 \to n\beta_0$. However, when

[22] The nonlinear sigma model is a model with an O(N) symmetry and the field is a N-vector of fixed length.

[23] This comes from the theorem of Weinberg which states that the free propagators decrease fast enough at large momentum.

one takes the limit $n \to 0$, one can observe that the gluon mass goes to zero recovering the original Yang-Mills-Faddeev-Popov action (39) and not the particular CF action (120) established by (26). Therefore, the Serreau-Tissier framework is based on just when the limit $n \to 0$ is applied, thus, the renormalization procedure should be done only in this limit. Nonetheless, when the computation of averages are realized, it is fundamental to work in a finite n theory and then take the limit $n \to 0$. Moreover, the perturbation theory is defined for finite n. Also, it is possible to study the Serreau-Tissier approach by computing various one-loop contributions for two point vertex functions, etc. And the results reproduce well the lattice data. The possible problems when it takes the limit $n \to 0$ are the questionable order of limits presented before and the generation of a gluon mass, this one will be well-explained in the next chapter (3). These efforts were to generate the gauge field mass with Serreau-Tissier's gauge-fixing method, however up to this thesis none of them were totally convincing (77, 79).

3 GENERATING THE GLUON MASS IN THE SERREAU-TISSIER APPROACH

In this chapter, we are going to establish a good explanation for the generation of gauge field (gluon) mass added in the particular CF phenomenological model proposed by M. Tissier and N. Wschebor (26) using the first principles model developed by J. Serreau and M. Tissier (77, 78, 79). To accomplish that, we must discuss the similarities between the nonlinear sigma models (NLσ) in two space-time dimensions and QCD.

3.1 The analogies between NLσ model and Yang-Mills theories

The NLσ and YM theories have as a common characteristic the asymptotic freedom behavior (9, 10, 134). Moreover, their low energy excitations are gaped, while their microscopic versions contain massless fields. For instance in the NLσ model, this visible transformation of the spectrum can be seen as an effect of the symmetry restoration phenomenon, which we will review using the $\mathrm{O}(N)/\mathrm{O}(N-1)$ example (135). One of the possible viewpoints for the NLσ model is through the description of the ordered phase of a N component vectorial model, where the radial fluctuations are frozen and it contains $N-1$ pseudo Goldstone bosons. Nonetheless, the theorem established by Mermin and Wagner declares that no such ordered phase exists in two spacetime dimensions (136). The real spectrum of the theory involves instead N degenerate massive modes, with an exponentially small mass.

From the Wilson functional renormalization group framework (137, 138) one can understand in a better manner the phenomenon of symmetry restoration, this occurs since it is possible to compute the dependence of the effective potential as a function of a renormalization-group scale k. For k of the order of the inverse lattice scale, the potential is strongly peaked around a nonzero value, which guarantees that the radial modes are frozen and that only the transverse pseudo Goldstone modes contribute. When k decreases, the minimum of the potential decreases and eventually vanishes below some renormalization group scale k_r. Below this scale, the symmetry in internal space is restored and all modes are massive, with a common mass. On the other hand, as already discussed in subsection (2.1.4) in Yang-Mills theories, there is now strong evidence that the gluon field is massive, in the sense that the gluon propagator saturates at finite values when the momentum tends to zero. It is also claimed that the gluon mass is generated by using nonperturbative methods such as Dyson-Schwinger equations, functional renormalization group (139, 27, 23, 140, 141, 24).

The main goal to be achieved in this chapter is to show that the similarities between

Yang-Mills theory and the NLσ are more robust than what was presented above. This study is realized in Landau gauge, where the Gribov ambiguity is considered by weighting the different Gribov copies, in a simple generalization of the procedure introduced in (77) and what was reviewed in the last chapter (2). As emphasized before, this particular gauge-fixing can be formulated in terms of a field theory in the continuum limit, which involves a set of NLσ fields. The first aspect to pursuit is the question of whether the phenomenon of symmetry restoration discussed above is realized or not in this particular model. This may seem surprising at first view because the theory lives in four space-time dimensions while the phenomenon of symmetry restoration in the NLσ model occurs in two dimensions less. Therefore, we show that this phenomenon occurs in our case thanks to supersymmetries which, in an effective way "reduce" the number of space-time dimensions by two. This phenomenon is similar to the dimensional reduction of Parisi and Sourlas (142, 143). Moreover, a fundamental consequence of this symmetry restoration is that the gluons acquire a mass. This makes contact with the results obtained in large volume lattice simulations (115, 116, 144, 145, 112).

As already written, the authors J. Serreau and M. Tissier focused on discover the most reasonable explanation for the generation of the gluon mass with the gauge-fixing described in (77) for the particular CF model proposed by M. Tissier and N. Wschebor in the last decade, however before this thesis none of the attempts were totally satisfactory. In their seminal work, a gluon mass was generated under the doubtful conjecture that two limits can be inverted as commented in subsection (2.2.3). In (79), the gluon mass was generated due to the presence of collective effect in an extension of the gauge-fixing to nonlinear Curci-Ferrari-Delbourgo-Jarvis gauge. Unfortunately, in the limit where this gauge coincides with the Landau gauge, the mass tends to zero. Thus, the present chapter finally establishes a reasonable solution to the generation of the mass in this context.

3.2 A novel weight function for the Serreau-Tissier procedure

In this section we revisit some definitions presented in the first two chapters of this thesis using them in the gauge-fixing method developed by J. Serreau and M. Tissier with some novelties to discuss how their framework can be expressed in terms of a field theory in a modified way when one compares with previous analysis. As usually for this approach we focus on Landau gauge (14), where the lattice simulations provided a lot of results. In practice, one obtains from Eq.(62) the gauge transformation U when a gauge field configuration $A_\mu = A_\mu^a T^a$ is selected, moreover the finite gauge transformation reads the same as (1.1). Remembering the expression (71) the gauge can be equivalently fixed by imposing that U extremizes

$$\mathcal{H}[A,U] = \int d^dx\, \mathrm{Tr}(A^U_\mu)^2\,, \tag{148}$$

at fixed A_μ. As unveiled in the original work of Gribov (35), and as discussed at length in the previous chapters, and considering the exhaustively discussion in this thesis, there exist Gribov copies, U_i, to this gauge-fixing problem. Following the strategy put forward in (77) and sum over the different Gribov copies, with a weight function $\mathcal{P}[U]$. The gauge-fixing procedure applied to some operator $\mathcal{O}$ will be rewritten as

$$\langle\mathcal{O}[A]\rangle = \frac{\sum_i \mathcal{O}[A^{U_i}]\mathcal{P}[U_i]}{\sum_i \mathcal{P}[U_i]}\,, \tag{149}$$

where the sums run over all Gribov copies as emphasized in section (2.2).

In this chapter, we establish as a weight function the following expression

$$\mathcal{P}[U] = \frac{\det\Big(\mathcal{M}^{ab}[A^U;x,y] + \varrho_0\delta_{ab}\delta(x-y)\Big)}{|\det\Big(\mathcal{M}[A^U]\Big)|}\exp\Big(-\beta_0\mathcal{H}[A,U]\Big)\,, \tag{150}$$

with $\mathcal{M}^{ab}[A;x,y]$ the well-known Faddeev-Popov operator in Landau gauge determined by (60); as a novelty compared to the model presented in subsection (2.2.2) we have ϱ_0, which is a gauge-fixing parameter of mass dimension squared. Observe that for $\varrho_0 = 0$, this gauge-fixing identifies with the one proposed in (77) or (122). For finite ϱ_0, the ratio of determinants favors the Gribov copies which are near to the Gribov horizons, where the Faddeev-Popov operator has a zero mode. In the last decade works (77, 78, 79), this parameter was not considered because it leads to a breaking of some symmetries, which makes the algebra more cumbersome. Nevertheless, there is no evidence that this symmetry is indeed realized in lattice simulations and it is a priori authentic to introduce it.

As shown in the subsection (2.2.3) this gauge-fixing procedure can be rewritten in terms of a continuum field theory and involves the introduction of auxiliary fields. Thus, the numerator of (149) is described as

$$\sum_i \mathcal{O}[A^{U_i}]\mathcal{P}[U^i] = \int \mathcal{D}U\mathcal{D}c\mathcal{D}\bar{c}\mathcal{D}b\,\mathcal{O}[A^U]\exp\Big(-S_{gf}[A^U,c,\bar{c},b]\Big)\,, \tag{151}$$

where the integral over U involves the Haar measure over the group. Therefore,

$$S_{gf}[A,c,\bar{c},b] = \int d^dx\left[ib^a\partial_\mu A^a_\mu + \varrho_0\,\bar{c}^a c^a + \frac{\beta_0}{2}(A^a_\mu)^2 + \partial_\mu\bar{c}^a(\partial_\mu c^a + gf^{abc}A^b_\mu c^c)\right]. \tag{152}$$

From the Serreau-Tissier formalism one can cast the fields appearing in the path integral (151) in a conveniently superfield, $V(x,\bar{\theta},\theta)$, eq.(140). The gauge-fixing action simply rewrites

$$\widetilde{S}_{gf}[A,V] = \frac{1}{g^2}\int d^dx \int d\theta d\bar{\theta}(\beta_0\bar{\theta}\theta - 1)\mathrm{Tr}\Big[(D_\mu V)^\dagger(D_\mu V) + 2\varrho_0\theta\bar{\theta}\partial_{\bar{\theta}}V^\dagger\partial_\theta V\Big], \tag{153}$$

where the covariant derivative is given by (146). In our conventions, one has the expression (142), also $\int d\theta d\bar{\theta}\ \bar{\theta}\theta = 1$, and $\int d\theta d\bar{\theta}(\beta_0\bar{\theta}\theta - 1)\bar{\theta}\theta = -1$.

In order to encode the denominator appearing in eq.(149), we use the same replica trick already presented in subsection (2.2.3), *i.e*, we introduce $p-1$ copies of the superfield V and take the limit $p \to 0$ at the end of the calculation. Therefore, we have p copies of the supersymmetric field V_k, as usually, one for the numerator and $p-1$ for the denominator.

Following the order of the steps established in section (2.2), it is the moment to average over the gauge configurations with the Yang-Mills weight:

$$\overline{\langle\mathcal{O}\rangle} = \frac{\int \mathcal{D}A\,\langle\mathcal{O}\rangle\exp\{-S_{\mathrm{YM}}\}}{\int \mathcal{D}A\ \exp\{-S_{\mathrm{YM}}\}}\,. \tag{154}$$

At this level, the dynamics is given by the action:

$$\begin{aligned} S[A,c,\bar{c},b,V] &= \int d^dx\Big[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu + \varrho_0\,\bar{c}^a c^a + \frac{\beta_0}{2}(A^a_\mu)^2 \\ &+ \partial_\mu\bar{c}^a(\partial_\mu c^a + gf^{abc}A^b_\mu c^c)\Big] + \sum_{k=2}^{p}\widetilde{S}_{gf}[A,V_k]\,. \end{aligned} \tag{155}$$

The theory with $\varrho_0 = 0$ was shown to be renormalizable (77). Adding a operator with mass dimension squared should not break the perturbative renormalizability of the theory. As shown in (77), at this particular point, the (super)symmetries of the model guaranties that all closed loops of replica fields vanish.

3.3 **Symmetry restoration phenomenon**

The field theory characterized in the last section involves fields V living in a superspace (with 4 bosonic and 2 Grassmann coordinates) and which take values in the gauge group. These fields look like closely the constrained fields of a nonlinear sigma model in 2 dimensions, which are known to exhibit the phenomenon of symmetry restoration.

In order to simplify the discussion, let us focus in the remaining of this chapter on the SU(2) gauge group and rewrite the elements of this group in terms of a unit 4-component vector field of unit norm (n^A), with $A \in \{0,1,2,3\}$): $V_k = n^A_k\Sigma^A$ where $\Sigma^A = \{\mathbb{I}, i\sigma^a\}$. Here and below, latin indices run from 1 to 3 and the capital latin indices at the beginning of the alphabet run from 0 to 3 and are associated with the SU(2) group structure. We then rewrite the action in terms of an unconstrained field and impose the constraint on the norm of n through a new auxiliary field ς. To deal with this constraint, let us first open up the eq. (153), then the term $Tr\Big[(D_\mu V)^\dagger D_\mu V\Big]$ will be given by

$$\begin{aligned}
\frac{1}{g^2}\int d^dx\int d\theta d\bar{\theta}\Big(\beta_0\bar{\theta}\theta-1\Big)\,Tr\left[(D_\mu V)^\dagger(D_\mu V)\right] \;&=\; \frac{1}{g^2}\int d^dx\int d\theta d\bar{\theta}\Big(\beta_0\bar{\theta}\theta-1\Big)\Big[2(\partial_\mu n^0)^2\\
&-\;\frac{g^2}{2}A^a_\mu A^a_\mu+2g\left[\partial_\mu\Big(n^a\Big)\right]n^0A^b_\mu\\
&-\;2g\varepsilon^{abc}A^a_\mu\left[\partial_\mu\Big(n^b\Big)\right]n^c\\
&+\;2\Big(\partial_\mu n^a\Big)\Big(\partial_\mu n^b\Big)\delta^{ab}\\
&-\;\left(\frac{g^2A^2}{2}\right)n^an^b\delta^{ab}\Big]\,. \qquad (156)
\end{aligned}$$

Moreover, the term $Tr\Big[2\varrho_0\theta\bar{\theta}\partial_{\bar{\theta}}V^\dagger\partial_\theta V\Big]$ will be rewritten as

$$\begin{aligned}
\frac{1}{g^2}\int d^dx\int d\theta d\bar{\theta}\Big(\beta_0\bar{\theta}\theta-1\Big)Tr\left[2\varrho_0\theta\bar{\theta}\partial_{\bar{\theta}}V^\dagger\partial_\theta V\right] \;&=\; \frac{2}{g^2}\int d^dx\int d\theta d\bar{\theta}\Big(\beta_0\bar{\theta}\theta-1\Big)\\
&.\;\left[2\varrho_0\theta\bar{\theta}\partial_{\bar{\theta}}n^A\partial_\theta n^A\right]\,.
\end{aligned}\qquad (157)$$

Thus, adding the constraint term ς and the results (156), (157) the action Eq. (153) is replaced by

$$\begin{aligned}
\tilde{S}_{gf}[A,n,\varsigma] \;&=\; \frac{2}{g^2}\int d^dx\int d\theta d\bar{\theta}(\beta_0\bar{\theta}\theta-1)\Big\{(\partial_\mu n^A)^2+gf^{aAB}A^a_\mu n^A\partial_\mu n^B+\frac{g^2}{4}(n^A)^2(A^a_\mu)^2\\
&+\;2\varrho_0\theta\bar{\theta}\partial_{\bar{\theta}}n^A\partial_\theta n^A+i\varsigma\left[(n^A)^2-1\right]\Big\}\,, \qquad (158)
\end{aligned}$$

where $f^{aAB}=-\frac{i}{4}\mathrm{Tr}[\sigma^a((\Sigma^A)^\dagger\Sigma^B-(\Sigma^B)^\dagger\Sigma^A)]$ is antisymmetric in its last two indices and is fully characterized by $f^{a0b}=\delta^{ab}$ and $f^{abc}=\epsilon^{abc}$. The second term from eq.(153) can be computed using the supersymmetric formalism, furthermore this one explicitly breaks the supersymmetry establishing a new term when one compares to the work (77) therefore in the gauge-fixing action, one has for fixed $k = 1$ replica

$$\begin{aligned}
\frac{1}{g^2}\int d^dx \int d\theta d\bar{\theta}\Big(\beta_0\bar{\theta}\theta - 1\Big)Tr\left[2\varrho_0\theta\bar{\theta}\partial_{\bar{\theta}}V^{\dagger}\partial_{\theta}V\right] &= \frac{2\varrho_0}{g^2}\int d^dxTr\left[\partial_{\bar{\theta}}V^{\dagger}\partial_{\theta}V\right] \\
&= -\frac{2\varrho_0}{g^2}\int d^dxTr\left[g^2c\bar{c}\right] \\
&= \int d^dx\ \left[\varrho_0\bar{c}^ac^a\right] . \qquad (159)
\end{aligned}$$

The constraint on the norm of the field n must be implemented correctly, to achieve that the auxiliary field ς shall be integrated on the imaginary axis. From the review in subsection (2.2.4) we remind that the matrix field V has a replica index, which implies that both the 4-vector field n and the auxiliary field ς also have a replica index, even though not expressly described above. Henceforward, all expectation values are taken with the action given by eq. (155) or in more detailed way eq. (158) and we shall denote this average with $\langle\rangle$.

At this moment, let us comment what we have earned with this new proposal. In the description of the theory in terms of a $SU(2)$ field V, or equivalently, in terms of a unit norm vector n^A, both actions S_{gf} and $\widetilde{S}_{gf}$ offer a contribution to the mass squared of the gluons which is equal to β_0. From eq. (158) one can easily observe the contribution of $\widetilde{S}_{gf}$ to the square mass of the gluon, see the third term with $(n^A)^2 = 1$. Since the action $\widetilde{S}_{gf}$ appears $(p-1)$ times in the action eq. (155), the total gluon square mass is established by $\beta_0 + (p-1)\beta_0$, which tends to zero in the limit of vanishing number of replica. Nevertheless, one can think in another scenario, *i.e*, the tree-level equation of motion for the n field which is $in^A_k\varsigma_k = 0$ allows as a solution $n = 0$ if $\varsigma \neq 0$. This would correspond to a symmetry restoration for the NLσ fields. Thus, for this viewpoint the third term in eq. (158) does not contribute to the tree-level square mass of the gluons although represents a 4-point vertex. Using this conjecture, the total gluon square mass is given by β_0. This observation is crucial to comprehend whether or not the average value of the field n vanishes. Let us now try to understand the symmetry restoration of the n field. To accomplish that, we are going to adopt the method developed in (146, 147) which resides in investigating the equation of motion for the ς field. In terms of the effective action $\Gamma[\hat{\varsigma}_k]$ for the classical field $\hat{\varsigma}_k = \langle i\varsigma_k\rangle$, the minimization equation is presented as

$$\frac{\delta\Gamma[\hat{\varsigma}_k]}{\delta\hat{\varsigma}_k(x)} = \left\langle\frac{\delta S}{\delta i\varsigma_k(x)}\right\rangle . \qquad (160)$$

Note that even if the fluctuating field is integrated over the imaginary axis, the classical field $\hat{\varsigma}$ is real. Moreover, since the action is linear in ς, we obtain the exact nonperturbative to all order equation:

$$\int d\theta d\bar{\theta}(\beta_0\bar{\theta}\theta - 1)\left\langle(n^A_k)^2 - 1\right\rangle = 0\,. \qquad (161)$$

The second term in the previous equation is easy to compute. By using the definition of the measure in Grassmann space, $\int d\theta d\bar{\theta}(\beta_0\bar{\theta}\theta - 1) = \beta_0$. The first term can be interpreted as an integral over momenta of the propagator for the field n, integrated over the Grassmann coordinates $\int d\theta d\bar{\theta}(\beta_0\bar{\theta}\theta - 1)$ and summed over the SU(2) index A. In this thesis, we limit ourselves to the calculation of this propagator at leading order (tree level). Therefore, in the next subsection we are going to compute these tree-level propagators with some details.

3.3.1 Tree-level propagators for the Serreau-Tissier action

In this subsection, we are going to show the tree-level propagators obtained from the Serreau-Tissier action in Landau gauge since these results are important for the next subsection. Thus, to accomplish that, let us first represent the unit norm vector n^A written in terms of the Grassmann and space-time coordinates with $(k = 2, ..., p)$ replicas, *i.e.*,

$$n_k^A(x,\theta,\bar{\theta}) \;=\; N_k^A + i\bar{\theta}c_k^A + i\theta\bar{c}_k^A + \theta\bar{\theta}\tilde{b_k}^A\,. \tag{162}$$

The first step consists in decomposing the action (158) in components to make all the action (155) dependent only of the space-time coordinates. Therefore, the starting action for us to work is

$$\begin{aligned} S \;=\; & \int d^dx\Bigg\{\Bigg[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu - \bar{c}^a\partial_\mu D^{ab}_\mu c^b + \frac{\beta_0}{2}A^a_\mu A^a_\mu + \varrho_0\bar{c}^a c^a\Bigg] \\ + & \sum_{k=2}^{p}\int d^dx\int d\theta d\bar{\theta}\Big(\beta_0\bar{\theta}\theta - 1\Big)\Bigg[\frac{(n_k^A)^2}{2}A^a_\mu A^a_\mu + \frac{2}{g^2}(\partial n_k^A)(\partial n_k^A) \\ + & \frac{2}{g}f^{aAB}A^a_\mu n_k^A\partial_\mu n_k^B + \frac{2i}{g^2}\varsigma_k\Big((n_k^A)^2 - 1\Big)\Bigg] \\ + & \frac{4\varrho_0}{g^2}\sum_{k=2}^{p}\int d^dx\int d\theta d\bar{\theta}\Big(\beta_0\bar{\theta}\theta - 1\Big)\theta\bar{\theta}\partial_{\bar{\theta}}n_k^A\partial_\theta n_k^A\Bigg\}\,. \end{aligned} \tag{163}$$

Making a suitable change in the variables, one has

$$\begin{aligned} \frac{4\varrho_0}{g^2} &= \varrho_0\,, \\ \frac{4}{g^2} &= 1\,. \end{aligned} \tag{164}$$

Thereby, establishing the replacement of (157) and (164) in (163), we have

$$
\begin{aligned}
S &= \int d^dx\Bigg\{\Bigg[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^a\partial_\mu A^a_\mu - \bar{c}^a\partial_\mu D^{ab}_\mu c^b + \frac{\beta_0}{2}A^a_\mu A^a_\mu + \varrho_0\bar{c}^a c^a\Bigg] \\
&+ \sum_{k=2}^{p}\int d^dx\int d\theta d\bar{\theta}\Bigg(\beta_0\bar{\theta}\theta - 1\Bigg)\Bigg[\frac{(n^A_k)^2}{2}A^a_\mu A^a_\mu + \frac{1}{2}(\partial_\mu n^A_k)(\partial_\mu n^A_k) + \frac{g}{2}f^{aAB}A^a_\mu n^A_k\partial_\mu n^B_k \\
&+ i\varsigma_k\Bigg((n^A_k)^2 - 1\Bigg)\Bigg] + \varrho_0\sum_{k=2}^{p}\int d^dx\Bigg(\bar{c}^A_k c^A_k\Bigg)\Bigg\}\,. \qquad (165)
\end{aligned}
$$

Using (162) in the expression (165), the gauge-fixing sector which mix the gauge field and the unit vector with replicas will be

$$
\begin{aligned}
\sum_{k=2}^{p}\int d^dx\int d\theta d\bar{\theta}\Bigg(\beta_0\bar{\theta}\theta - 1\Bigg)\Bigg(\frac{(n^A_k)^2}{2}A^a_\mu A^a_\mu\Bigg) &= \sum_{k=2}^{p}\int d^dx\Bigg[\frac{\beta_0}{2}(N^A_k)^2\Bigg(A^a_\mu A^a_\mu\Bigg) \\
&+ \bar{c}^A_k c^A_k\Bigg(A^a_\mu A^a_\mu\Bigg) + \tilde{b}_k^{\;A} N^A_k\Bigg(A^a_\mu A^a_\mu\Bigg)\Bigg]\,. \qquad (166)
\end{aligned}
$$

$$
\begin{aligned}
\sum_{k=2}^{p}\int d^dx\int d\theta d\bar{\theta}\Bigg(\beta_0\bar{\theta}\theta - 1\Bigg)\Bigg[\frac{g}{2}f^{aAB}A^a_\mu n^A_k\partial_\mu n^B_k\Bigg] &= \sum_{k=2}^{p}\int d^dx\Bigg(\frac{g\beta_0}{2}f^{aAB}A^a_\mu N^A_k\partial_\mu N^B_k \\
&- \frac{g}{2}f^{aAB}A^a_\mu c^A_k\partial_\mu\bar{c}^B_k + \frac{g}{2}f^{aAB}A^a_\mu\bar{c}^A_k\partial_\mu c^B_k\Bigg)\,. \qquad (167)
\end{aligned}
$$

For the unit vector term, one has

$$
\begin{aligned}
\sum_{k=2}^{p}\int d^dx\int d\theta d\bar{\theta}\Bigg(\beta_0\bar{\theta}\theta - 1\Bigg)\Bigg[\frac{1}{2}(\partial_\mu n^A_k)(\partial_\mu n^A_k)\Bigg] &= \sum_{k=2}^{p}\int_x\Bigg[\frac{\beta_0}{2}\partial_\mu N^A_k\partial_\mu N^A_k \\
&+ (\partial_\mu\bar{c}^A_k)\partial_\mu c^A_k + \partial_\mu\tilde{b}_k^{\;A}\partial_\mu N^A_k\Bigg]\,. \qquad (168)
\end{aligned}
$$

Looking the auxiliary ς field sector, one has

$$
\begin{aligned}
\sum_{k=2}^{p}\int d^dx\int d\theta d\bar{\theta}\Bigg(\beta_0\bar{\theta}\theta - 1\Bigg)\Bigg[i\varsigma_k\Bigg((n^A_k)^2 - 1\Bigg)\Bigg] &= \sum_{k=2}^{p}\int d^dx\Bigg\{\frac{i\beta_0}{2}\varsigma_k N^A_k N^A_k + i\varsigma_k\bar{c}^A_k c^A_k \\
&+ i\varsigma_k N^A_k\tilde{b}_k^{\;A} - \frac{i\beta_0}{2}\varsigma_k\Bigg\}\,. \qquad (169)
\end{aligned}
$$

Therefore, the Serreau-Tissier action in components is written as

$$
\begin{aligned}
S &= \int d^dx \Bigg\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i b^a \partial_\mu A^a_\mu - \bar{c}^a \partial_\mu D^{ab}_\mu c^b + \frac{\beta_0}{2} A^a_\mu A^a_\mu + \varrho_0 \bar{c}^a c^a \\
&+ \sum_{k=2}^{p} \Bigg[\frac{\beta_0}{2} (N^A_k)^2 \Big(A^a_\mu A^a_\mu \Big) - \bar{c}^A_k c^A_k \Big(A^a_\mu A^a_\mu \Big) + \tilde{b}^A_k N^A_k \Big(A^a_\mu A^a_\mu \Big) \\
&+ \frac{\beta_0}{2} \partial_\mu N^0_k \partial_\mu N^0_k + \frac{\beta_0}{2} \partial_\mu N^a_k \partial_\mu N^a_k + (\partial_\mu \bar{c}^0_k) \partial_\mu c^0_k + (\partial_\mu \bar{c}^a_k) \partial_\mu c^a_k \\
&+ \partial_\mu \tilde{b}^0_k \partial_\mu N^0_k + \partial_\mu \tilde{b}^a_k \partial_\mu N^a_k + \frac{g\beta_0}{2} f^{aAB} A^a_\mu N^A_k \partial_\mu N^B_k + g f^{abc} A^a_\mu \bar{c}^b_k (\partial_\mu c^c_k) \\
&+ \frac{i\beta_0}{2} \varsigma_k N^0_k N^0_k + \frac{i\beta_0}{2} \varsigma_k N^a_k N^a_k + i\varsigma_k \bar{c}^0_k c^0_k + i\varsigma_k \bar{c}^a_k c^a_k + i\varsigma_k N^0_k \tilde{b}^0_k \\
&+ i\varsigma_k N^a_k \tilde{b}^a_k - \frac{i\beta_0}{2} \varsigma_k + \varrho_0 \Big(\bar{c}^0_k c^0_k \Big) + \varrho_0 \Big(\bar{c}^a_k c^a_k \Big) \Bigg] \Bigg\} \,.
\end{aligned}
\tag{170}
$$

From the quadratic part of the action (170), also making the replacement of $i\varsigma_k \rightarrow \hat{\varsigma} k$ and using the equality (160), one can establish all the propagators of this model, *i.e.*,

$$
\begin{aligned}
\Big\langle A^a_\mu(q)\, A^b_\nu(-q) \Big\rangle &= \delta^{ab} \left(\frac{\wp_{\mu\nu}}{q^2 + \beta_0} \right) , \\
\Big\langle A^a_\mu(q)\, b^b(-q) \Big\rangle &= 0 \,, \\
\Big\langle A^a_\mu(q)\, N^a_l(-q) \Big\rangle &= 0 \,, \\
\Big\langle b^a_k(q)\, b^b_l(-q) \Big\rangle &= \delta^{ab} \left(-\frac{\beta_0}{q^2 + \hat{\varsigma}_k} \right) , \\
\Big\langle b^A_k(q)\, N^B_l(-q) \Big\rangle &= -\frac{\delta^{AB}\delta_{kl}}{\Big(q^2 + \hat{\varsigma}_k \Big)} \,,
\end{aligned}
\tag{171}
$$

$$
\begin{aligned}
\Big\langle N^A_k(q)\, N^B_l(-q) \Big\rangle &= 0 \,, \\
\langle c^0_k(q) \bar{c}^0_l(-q) \rangle &= \frac{1}{\Big(q^2 + \hat{\varsigma}_k \Big)} \left[\delta_{kl} - \frac{\varrho_0}{q^2 + \hat{\varsigma}_k + \varrho_0} \right] , \\
\langle c^a_k(q) \bar{c}^b_l(-q) \rangle &= \frac{\delta^{ab}}{\Big(q^2 + \hat{\varsigma}_k \Big)} \left[\delta_{kl} - \frac{\varrho_0}{q^2 + \hat{\varsigma}_k + \varrho_0} \right] .
\end{aligned}
\tag{172}
$$

3.3.2 Exact nonperturbative equation

Let us now establish the exact nonperturbative to all order equation (161) by taking into account the equation (160), the results presented in (172) also the real averages (super)fields $\hat{\varsigma}_k \equiv \langle i\varsigma_k \rangle$ and $\hat{n}_k^A \equiv \langle n_k^A \rangle$, we have up to first order in this computation, $\hat{\varsigma}_k$ and $\hat{\varsigma}_k + \varrho_0$ appearing as square masses of propagators and are therefore restricted to be positive. This yields:

$$\frac{\beta_0}{2\bar{g}^2}\left(\hat{n}^2 - 1\right) + \mu^{2\epsilon} \int \frac{d^d q}{(2\pi)^d} \left[\frac{1}{q^2 + \hat{\varsigma}_k} - \frac{1}{q^2 + \hat{\varsigma}_k + \varrho_0}\right] \;=\; 0\,, \tag{173}$$

here we have introduced the dimensionless coupling $\bar{g} = g\mu^{-\epsilon}$ and a momentum scale μ. As expected, the loop contribution given by the right-hand side of the previous equation vanishes at $\varrho_0 = 0$. This implies that the loop-divergence is proportional to ϱ_0 and thus only logarithmic. This is a manifestation of the dimensional reduction mentioned above. Observe finally that all replica fields $\hat{\varsigma}_k$ satisfy the same equation, a consequence of the symmetry under permutation of the replica fields. In the following, to alleviate notations, we remove the replica index k.

At the same order of approximation, we go back to the discussion of the equation

$$\hat{\varsigma}\hat{n}^A = 0\,, \tag{174}$$

which has two solutions. The one with $\hat{\varsigma} = 0$ corresponds to the phase of broken O(4) symmetry with the hard constraint $\hat{n}^2_{\text{brok}} = \text{const}$. As already mentioned, $\hat{\varsigma}$ plays the role of a square mass for the bosonic components of the superfield n_k^A, which are nothing but the Goldstone modes. In that phase, we have from (173),

$$\hat{n}^2_{\text{brok}} = 1 + \frac{2\bar{g}^2}{\beta_0}\mu^{2\epsilon} \int \frac{d^d q}{(2\pi)^d} \left[\frac{1}{q^2 + \varrho_0}\right]\,. \tag{175}$$

As mentioned previously, the case $\varrho_0 > 0$ allows other solution to (174), with $\hat{n}^A = 0$, corresponding to a radiatively restored O(4) symmetry. The latter is characterized by massive modes with square mass $\hat{\varsigma} = \hat{\varsigma}_{\text{sym}} > 0$, solution of the gap equation (173) rewritten as

$$\frac{\beta_0}{2\bar{g}^2} \;=\; \frac{1}{16\pi^2}\left[\frac{\varrho_0}{\epsilon} + \varrho_0 + (\hat{\varsigma}_{r,\text{sym}} + \varrho_0)\ln\frac{\bar{\mu}^2}{\hat{\varsigma}_{r,\text{sym}} + \varrho_0} - \hat{\varsigma}_{r,\text{sym}}\ln\frac{\bar{\mu}^2}{\hat{\varsigma}_{r,\text{sym}}}\right]\,. \tag{176}$$

where we have used dimensional regularization with $d = 4 - 2\epsilon$ and $\bar{\mu}^2 = 4\pi e^{-\gamma}\mu^2$ where γ is the Euler constant. Before discussing the solutions of (175) and (176), it is necessary to renormalize them. Therefore, we introduce the renormalized fields and parameters as

$${n_k}^A = \sqrt{Z_n}\, n^A_{k,r}, \quad \varsigma_k = \sqrt{Z_\varsigma}\,\varsigma_{k,r}, \quad \beta_0 = Z_\beta \beta_r, \quad \varrho_0 = Z_\varrho \varrho_r, \quad \bar{g}^2 = Z_{\bar{g}^2}\bar{g}_r^2. \tag{177}$$

Note that the first (classical) term on the left-hand side of (173) receives a overall factor $\sqrt{Z_\varsigma}$. Also, at the present order of approximation, we can set all renormalization factors to 1 in the tadpole (one-loop) integrals. It is possible to remove the UV divergence in

(173) with the choices

$$\sqrt{Z_\varsigma} Z_\beta Z_{\bar{g}^2}^{-1} = Z_n^{-1} = 1 + \frac{\bar{g}_r^2}{8\pi^2}\frac{\varrho_r}{\beta_r}\left(\frac{1}{\epsilon} + 1\right). \tag{178}$$

The broken phase solution (175) rewrites

$$\hat{n}_{r,\mathrm{brok}}^2 = 1 - \frac{\bar{g}_r^2}{16\pi^2}\frac{\varrho_r}{\beta_r}\ln\frac{\bar{\mu}^2}{\varrho_r}, \tag{179}$$

therefore the gap equation (173) in the symmetric phase becomes

$$\frac{8\pi^2\beta_r}{\bar{g}_r^2} = (\hat{\varsigma}_{r,\mathrm{sym}} + \varrho_r)\ln\frac{\bar{\mu}^2}{\hat{\varsigma}_{r,\mathrm{sym}} + \varrho_r} - \hat{\varsigma}_{r,\mathrm{sym}}\ln\frac{\bar{\mu}^2}{\hat{\varsigma}_{r,\mathrm{sym}}}. \tag{180}$$

The right-hand-side is a monotonously decreasing function of $\hat{\varsigma}_{r,\mathrm{sym}}$ so there exists a unique solution if

$$\frac{8\pi^2\beta_r}{\bar{g}_r^2} \leq \varrho_r \ln\frac{\bar{\mu}^2}{\varrho_r} \leq \frac{\bar{\mu}^2}{e}, \tag{181}$$

where the second inequality is a bound for all values of the parameter ϱ_r. If this bound is not fulfilled, our perturbative treatment does not lead to a consistent solution (with $\hat{\varsigma}_{r,\mathrm{sym}} \geq 0$). Depending on the sign of $\beta_r/\bar{g}_r^2$, we obtain different types of solutions.

It remains to be checked which phase [24] is favoured by the dynamics. With this mind, we compute the effective potential $\mathbb{V}(\hat{\varsigma}, \hat{n})$, defined from the effective action evaluated at constant fields as $\Gamma[\hat{\varsigma}, \hat{n}] = \int d^dx\, \mathbb{V}(\hat{\varsigma}, \hat{n})$. At the order of approximation considered here, the latter reads[25]

$$\mathbb{V}(\hat{\varsigma}, \hat{n}) = (p-1)\left[\frac{2\beta_0}{g^2}\hat{\varsigma}\left(\hat{n}^2 - 1\right) + 4\int\frac{d^dq}{(2\pi)^d}\ln\frac{q^2+\hat{\varsigma}}{q^2+\hat{\varsigma}+\varrho}\right] + \text{const.} \tag{182}$$

$$\mathbb{V}(\hat{\varsigma}, \hat{n}) = \mathbb{V}(0,0) + (p-1)\left\{\frac{2\beta_r}{\bar{g}_r^2}\hat{\varsigma}_r\left(\hat{n}_r^2 - 1\right) + \frac{1}{8\pi^2}\left[\varrho_r\hat{\varsigma}_r + (\hat{\varsigma}_r + \varrho_r)^2\ln\frac{\bar{\mu}^2}{\hat{\varsigma}_r + \varrho_r}\right]\right\}. \tag{183}$$

In the last expression we have used the renormalization factors (178) and a field-independent divergence has been absorbed in $\mathbb{V}(0,0)$ in (183), where the limit $\epsilon \to 0$ can be safely

[24] The phases are defined as broken or restored O(4) symmetry.

[25] One easily checks that the one-loop equations of motion (173) and $in_k^A\varsigma_k = 0$ correspond to $\partial\mathbb{V}/\partial\hat{\varsigma} = \partial\mathbb{V}/\partial\hat{n}^A = 0$.

taken. We now compare the values of the potential for the two solutions of the equations of motion corresponding to the broken and the symmetric phases. The former is characterized by $\hat{\varsigma} = 0$ and one easily checks that $\mathbb{V}(0, \hat{n}) = \mathbb{V}(0,0)$ is independent of $\hat{n}$. The symmetric phase is defined by $\hat{n}^A = 0$ and $\hat{\varsigma} = \hat{\varsigma}_{\text{sym}}$ which solves the gap equation (180). Using the latter to express $\beta_r/\bar{g}_r^2$ in terms of $\hat{\varsigma}_{r,\text{sym}}$, we obtain

$$\mathbb{V}(\hat{\varsigma}_{\text{sym}}, 0) - \mathbb{V}(0, \hat{n}_{\text{brok}}) = \frac{p-1}{8\pi^2}\left[\varrho_r^2 \ln \frac{\varrho_r}{\hat{\varsigma}_{r,\text{sym}} + \varrho_r} - \hat{\varsigma}_{r,\text{sym}}^2 \ln \frac{\hat{\varsigma}_{r,\text{sym}}}{\hat{\varsigma}_{r,\text{sym}} + \varrho_r} + \varrho_r \hat{\varsigma}_{r,\text{sym}}\right]. \quad (184)$$

The term in bracket is always positive for $\varrho_r, \hat{\varsigma}_{r,\text{sym}} > 0$. We conclude that $\mathbb{V}(\hat{\varsigma}_{\text{sym}}, 0) < \mathbb{V}(0, n_{\text{brok}})$ in the limit $p \to 0$, *i.e.*, the symmetric phase, whenever permitted by the parameters (see above), is the favoured one.

4 THE BRST INVARIANT FORMULATION OF THE GZ MODEL

The construction of the GZ action, which is responsible for generalize to all orders the no-pole condition of Gribov, generates an effective action to characterize the infrared regime for the YM theories in a local and renormalizable way, however up to now just using particular aspects of Landau gauge. The extension of this formalism to other gauges is a highly nontrivial task. Moreover, there is the lost of the BRST symmetry, which is primordial to control the independence of physical quantities. As already written in the subsection (2.1.5), one can understand the soft breaking by looking the region Ω and seeing the correspondent gauge field may be outside this region (40). This argument leads us to consider possible nonperturbative effects coming from the horizon function, which would be responsible for influencing the BRST operator.

One way to deal with this problem is to investigate the BRST restoration case for the RGZ action through directly changes in the transformations done by the operator s in the fields of the model. However, in order to keep both Lorentz and color invariance, the theory turns out to be nonlocal, looses its spectrum and the nilpotency (53, 148, 149). Another idea it would consider the construction of a horizon function in an invariant way. In the next section, we will review as a first step the recent construction of the action of the GZ approach in a BRST invariant form (65, 68). This novel method implies in a restriction in the functional integral domain like Gribov's approach, although equivalent to a BRST invariant horizon function. This assumption makes possible the extension for LCG and the inclusion of fermionic matter fields in a renormalizable way, which is our original work for this part of the manuscript and the main objective for the next two chapters. An important remark needs to be established, this form of thinking invokes a double nonlocality, the first one coming from, as usually, the horizon function and the other one come from the own BRST transformation. This issue is solved by introducing a set of auxiliary fields including a field like Stückelberg, which turns out possible to localize the GZ action even being nonpolynomial.

4.1 The BRST invariant horizon function in Landau gauge

In the sequel, let us develop a nonperturbative restitution mechanism for the BRST symmetry of the GZ action in Landau gauge (102). As a first step, one has to consider the A^2_{min} [26]. This operator operator is achieved through the minimization procedure of

[26] The detailed construction of this operator is in appendix (A).

the functional $Tr \int d^4x\, A^U_\mu A^U_\mu$ along the gauge orbit of A_μ (100, 66, 106, 107), *i.e.*,

$$A^2_{min} \equiv \min_{\{U\}} Tr \int d^4x\, A^U_\mu A^U_\mu\,, \quad A^U_\mu = U^\dagger A_\mu U + \frac{i}{g} U^\dagger \partial_\mu U\,. \tag{185}$$

The stationary points of this functional are established by the transverse gauge field configurations $A^h_\mu = A^{h,a}_\mu T^a$, where h is one of the possibilities for U and for some gauge field configuration A_μ, one obtains the transverse gauge field $A^{h,a}_\mu$, which obeys $\partial_\mu A^h_\mu = 0$ and can be express as an infinite series of the gauge field A_μ (66), thus

$$\begin{aligned} A^h_\mu &= \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\phi_\nu\,, \qquad \partial_\mu A^h_\mu = 0\,, \\ \phi_\nu &= A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right] + \mathcal{O}(A^3)\,. \end{aligned} \tag{186}$$

Explicitly, $A^{h,a}_\mu$ is given by

$$A^h_\mu = \left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu\frac{1}{\partial^2}\partial A\right] + \mathcal{O}(A^3)\right)\,, \tag{187}$$

with A^h_μ being BRST-invariant,

$$sA^h_\mu = 0\,, \qquad sA^a_\mu = -D^{ab}_\mu(A)c^b\,. \tag{188}$$

Therefore, using (186) in (185), one has

$$\begin{aligned} A^2_{min} &= Tr \int d^4x\, A^h_\mu A^h_\mu \\ &= \frac{1}{2}\int d^4x \left[A^a_\mu\left(\delta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\partial^2}\right)A^a_\nu - g f^{abc}\left(\frac{\partial_\nu}{\partial^2}A^a\right)\left(\frac{1}{\partial^2}A^b\right)A^c_\nu\right] + \mathcal{O}(A^4)\,. \end{aligned} \tag{189}$$

Rewriting the functional (186) in terms of the field strength applying the gauge-invariant nature of the expression (189), one has the following equation which was already demonstrated in (66)

$$\begin{aligned} A^2_{min} &= -\frac{1}{2} Tr \int d^4x \left(F_{\mu\nu}\frac{1}{D^2}F_{\mu\nu} + 2i\frac{1}{D^2}F_{\lambda\mu}\left[\frac{1}{D^2}D_\kappa F_{\kappa\lambda}, \frac{1}{D^2}D_\nu F_{\nu\mu}\right]\right. \\ &\quad \left. -2i\frac{1}{D^2}F_{\lambda\mu}\left[\frac{1}{D^2}D_\kappa F_{\kappa\nu}, \frac{1}{D^2}D_\nu F_{\lambda\mu}\right]\right) + \mathcal{O}(F^4)\,. \end{aligned} \tag{190}$$

The gauge invariance is apparent in (190). Moreover, in Landau gauge the operator A^2_{min} is the well-known functional $\mathcal{H} = A^2$, thus

$$A^2_{min} = \frac{1}{2}\int d^4x\, A^a_\mu A^a_\mu\,. \tag{191}$$

Turning our attention to the nonlocal gauge field (186), one can see that the term ∂A is present in all the terms with higher g order and as a consequence the horizon function $H(A)$ can be rewritten in terms of A^h_μ as

$$H(A) = H(A^h) - R(A)(\partial A)\,, \tag{192}$$

where $R(A)(\partial A) = \int d^4x d^4y R^a(x,y)(\partial A^a)_y$ is an infinite nonlocal series in powers of A_μ. Therefore, the GZ action is written omitting the color indices in the following way

$$\begin{aligned} S_{GZ} &= S_{YM} + \int d^4x\,(ib\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A) \\ &= S_{YM} + \int d^4x\,(ib\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A^h) - \gamma^4 R(A)(\partial A) \\ &= S_{YM} + \int d^4x\,\left(ib^h\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h)\,, \end{aligned} \tag{193}$$

with the new Nakanishi-Lautrup b^h field given by

$$b^h = b - i\gamma^4 R(A)\,. \tag{194}$$

The use of the field b^h offers the possibility of writing the BRST transformations exactly. The GZ action will be rewritten by the local Zwanziger fields $(\bar{\varphi}, \varphi, \omega, \bar{\omega})$ as

$$\begin{aligned} S_{GZ} &= S_{YM} + \int d^4x\,\left(ib^h\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) \\ &- \int d^4x\,\left(\bar{\varphi}\mathcal{M}(A^h)\varphi + \bar{\omega}\mathcal{M}(A^h)\omega - \gamma^2 A^h(\bar{\varphi}+\varphi)\right). \end{aligned} \tag{195}$$

where the FP operator will be rewritten as,

$$\mathcal{M}(A^h) = -\partial_\mu D^{ab}_\mu(A^h) = -\delta^{ab}\partial^2 + gf^{abc}A^{h,c}_\mu\partial_\mu\,, \tag{196}$$

with $\partial_\mu A^h_\mu = 0$. The action (195) is invariant by the following nilpotent BRST transformations

$$\begin{aligned} &s_{\gamma^2}A^a_\mu = -D^{ab}_\mu c^b\,, \qquad s_{\gamma^2}c^a = \frac{g}{2}f^{abc}c^bc^c\,, \qquad s_{\gamma^2}\bar{c}^a = ib^{h,a}\,, \qquad s_{\gamma^2}b^{h,a} = 0\,, \\ &s_{\gamma^2}\varphi^{ab}_\mu = \omega^{ab}_\mu\,, \qquad s_{\gamma^2}\omega^{ab}_\mu = 0\,, \qquad s_{\gamma^2}\bar{\omega}^{ab}_\mu = \bar{\varphi}^{ab}_\mu - g\gamma^2 f^{cdb}\int d^4y\, A^{h,c}_\mu(y)\left[\mathcal{M}^{-1}(A^h)\right]^{da}_{yx}\,, \\ &s_{\gamma^2}\bar{\varphi}^{ab}_\mu = 0\,. \end{aligned} \tag{197}$$

This set of transformations is responsible for changing the BRST symmetry of the GZ action, thus

$$s_{\gamma^2}S_{GZ} = 0\,. \tag{198}$$

This new operator is the s_{γ^2} is nonperturbative BRST operator and obeys some properties, e.g.,

$$s_{\gamma^2} = s + \delta_{\gamma^2}\,, \quad s^2_{\gamma^2} = s = \delta^2_{\gamma^2} = 0\,, \quad \Rightarrow \{s, \delta_{\gamma^2}\} = 0\,, \tag{199}$$

where s is the traditional BRST operator (46), while the operator δ_{γ^2} has the following transformations

$$\begin{aligned} \delta_{\gamma^2}\bar{c}^a &= -\gamma^4 R^a(A)\,, \quad \delta_{\gamma^2} b^a = \gamma^4 s R^a(A)\,, \\ \delta_{\gamma^2}\bar{\omega}^{ac}_\mu &= g\gamma^2 f^{kbc} A^{h,k}_\mu \left[\mathcal{M}^{-1}(A^h)\right]^{ba}\,, \quad \delta_{\gamma^2}(rest) = 0. \end{aligned} \tag{200}$$

An important comment to make at this point is that when one sets the Gribov parameter γ^2 to zero, the traditional BRST operator s is recovered, this a fundamental characteristic for looking the consistency of the GZ action, *i.e.*, in the limit $\gamma \to 0$ the usual YMFP theory must be restored. The proof for $s^2_{\gamma^2} = 0$ is directly from the condition $s_{\gamma^2} A^h_\mu = 0$. Thus, the conditions wrote in (199) and the transformations (197) established an exact BRST symmetry for the GZ action, a remarkable improvement compared to the soft breaking of BRST symmetry presented in the section (2.1.5).

Therefore, at this moment the BRST symmetry depends on the Gribov parameter. This evidence assures the reduction for the usual BRST transformations in the ultraviolet regime. Moreover, this new kind of transformation brings us knowledge about the infrared sector, however the set established in (197) is nonlocal. Another thing to observe is that,

$$s_{\gamma^2}\frac{\partial S_{GZ}}{\partial \gamma^2} = f^{abc} A^{h,a}_\mu \omega^{bc}_\mu \neq 0 \Rightarrow \frac{\partial S_{GZ}}{\partial \gamma^2} \neq s_{\gamma^2}(...)\,, \tag{201}$$

this means that the Gribov parameter is not a gauge parameter, which is the same result obtained from the BRST soft breaking case. If one desires to have more details about the physical meaning of the parameters introduced in the BRST soft breaking case, we indicate (54).

Finally, to outline this subsection in a clear way, we have a nonlocal exact nonperturbative BRST symmetry, which turns out the GZ action invariant when the operator s_{γ^2} acts on it, moreover, this is possible since the transverse gauge-invariant field A^h_μ is added. The expression (195) is nonlocal due to the presence of the horizon function and A^h_μ, the later is represented by an infinite power series. Thus, the main objective from now on will be to localize and renormalize the GZ action including the dimension two condensates ($\langle\bar{\varphi}\varphi - \bar{\omega}\omega\rangle$ and $\langle A^a_\mu A^a_\mu\rangle$), *i.e.*, the RGZ case by the algebraic renormalization method (76). This action is going to be investigated by extending it in two manners. The first one is the extension to LCG, *i.e.*, $\alpha \neq 0$, which an earlier attempt was done in (63), and the second one is to include gauge-invariant fermionic composite fields, which are represented by the Dirac spinor fields $(\psi^h, \bar{\psi}^h)$ also being nonlocal and represented by an infinite power series. All of that will be done by using the new formalism for the GZ approach, *i.e.*, the employment of the exact nonperturbative BRST symmetry, which will be also cast in a local form .

4.2 Extending the nonperturbative BRST-invariant formalism to LCG

Now, let us discuss the properties of BRST nilpotency and a construction for the Gribov region to study the Gribov-Zwanziger approach in BRST-invariant way in LCG. The first attempt was developed originally in (71, 72, 63), however this formulation presented some problems and it will be clear in the following steps. Anyway, on this type of gauge-fixing, the FP operator is not Hermitian, therefore, the original geometric establishment of the Gribov region with this operator strictly positive loses the meaning. For instance, if we could use the intuition, it will be possible to define a horizon function w.r.t. a transversal invariant gauge field configuration, $A^{h,a}_{\mu}$, being able to deal with the non-Hermitian problem. However, it is required to prove that this construction is at least equivalent in some properties with the Gribov region in the Landau gauge.

Let us spend few more words about the first attempt, e.g., there exists infinitesimal Gribov copies when the FP operator $\mathcal{M}(A) = -\partial_{\mu} D^{ab}_{\mu}$ develops zero-modes. Though, for non-vanishing α, such an operator is not Hermitian as was previously mentioned and this problem makes difficult to use the standard GZ analysis for the removal of such zero-modes from the path integral measure. This hard task has been investigated for Gribov copies with any values of α. Moreover, the works (71, 72, 63) used the projection of the operator $\mathcal{M}(A)$ onto the transverse component of the gauge field. In this viewpoint, the projected operator was Hermitian and the traditional procedure for the construction of a horizon function was available. This structure was developed in (71, 72) and its renormalizability was analyzed in details in (63). However, this development has some inconveniences, like the Landau gauge is not fully recovered for standard RGZ action and it breaks the BRST symmetry softly, turning difficult the control of the gauge parameter (in)dependence of the correlation functions of gauge-invariant quantities. These difficulties have been supressed by the construction of the BRST-invariant formulation of the RGZ action in Landau and LCG, as discussed previously in this Thesis and originally in (65).

Before defining a Gribov region for LCG, we must remember the equation (39) which is the standard Faddeev-Popov framework, the gauge fixed Yang-Mills action in this class of gauge, written now as a function of the nonperturbative BRST operator s_{γ^2}

by

$$
\begin{aligned}
S_{YMFP} &= \int d^4x \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + ib^{h,a}\partial_\mu A^a_\mu + \frac{\alpha}{2}b^{h,a}b^{h,a} + \bar{c}^a\partial_\mu D^{ab}_\mu c^b\right] \\
&= \int d^4x \left[\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + s_{\gamma^2}\left(\bar{c}^a\partial_\mu A^a_\mu - \frac{i\alpha}{2}\bar{c}^a b^{h,a}\right)\right] . \qquad (202)
\end{aligned}
$$

From the last expression, one can observe that the gauge parameter is coupled to an exact quantity s_{γ^2}, this means that the expectation values of quantities that are invariant by s^2_γ do not depend on α. Thus, the GZ action is described as

$$
S^{LCG}_{GZ} = S_{YMFP} - g^2\gamma^4 \int d^4x d^4y\, f^{abc} A^{h,b}_\mu(x) \left[\mathcal{M}^{-1}(A^h)\right]^{ad}(x,y) f^{dec} A^{h,e}_\mu(y) \,. \qquad (203)
$$

An important remark to make at this moment is that the shifted Lautrup-Nakanishi field $b^{h,a}$ has a trivial Jacobian determinant and it does not modify the physical content of the theory, i.e., the correlation functions remains the same. Therefore, using this argument the label h is unnecessary, thus there is no difference of treatment for the fields b^a and $b^{h,a}$, and let us write everything from now on in terms of b^a. The action (203) is an usual extension of the Landau gauge case ($\alpha = 0$), additionally it is invariant under the nonperturbative BRST operator

$$
s_{\gamma^2} S^{LCG}_{GZ} = 0 \,, \qquad (204)
$$

Thereby, a candidate for the Gribov region in LCG presented in (65) is

$$
\Omega_{LCG} = \left\{ A^a_\mu \;\middle|\; \partial_\mu A^a_\mu = i\alpha b^a,\ \partial_\mu A^{h,a}_\mu = 0,\ -\partial_\mu D^{ab}_\mu(A^h) > 0 \right\} . \qquad (205)
$$

This region Ω_{LCG} has the same important properties of the Gribov region Ω in the Landau gauge, for example, it is convex and it is bounded in all directions (107). These properties are connected directly with the linearity of the operator $-\partial_\mu D^{ab}_\mu(A^h)$. By the knowledge of the author of this manuscript up to now the property referred to the existence of a gauge field configuration located outside the Gribov region which is a copy of other one located inside Ω was proved just in Landau gauge (104, 107). Therefore, the restriction to Ω_{LCG} is obtained through the following constraint on the original Faddeev-Popov path integral,

$$
\begin{aligned}
Z &= \int_{\Omega_{LCG}} \mathcal{D}A_\mu \mathcal{D}b \mathcal{D}c \mathcal{D}\bar{c} \exp\{-(S_{YM} + S_{FP})\} \\
&= \int \mathcal{D}A_\mu \mathcal{D}b \mathcal{D}c \mathcal{D}\bar{c} \exp\{-(S_{YM} + S_{FP} + \gamma^4 H(A^h) - 4\vartheta\gamma^4(N^2-1))\} \,, \qquad (206)
\end{aligned}
$$

with the horizon function being written in terms of the gauge-invariant field A^h_μ.

$$H(A^h) = g^2 \int d^4x d^4y\ f^{abc} A^{h,b}_\mu(x) \left[\mathcal{M}^{-1}(A^h)\right]^{ad} (x,y) f^{dec} A^{h,e}_\mu(y)\,, \tag{207}$$

is the horizon function written in terms of the composite bosonic field A^h, with Faddeev-Popov's operator described now as $[\mathcal{M}^{-1}(A^h)]^{ab} = -\partial_\mu D^{ab}_\mu(A^h)$, also as a remind ϑ is the spacetime volume and N is the number of colors or internal degrees of freedom of the gauge group $SU(N)$. As observed in section (2.1.2), the Gribov parameter γ is not free, although fixed through the gap equation (this time depending on A^h),

$$\langle H(A^h)\rangle = 4\vartheta(N^2-1)\,, \tag{208}$$

this is a manifestly gauge-invariant expression that implements the gauge independence of γ and, thus, assigns to it an authentic physical meaning. As an effect, it can enter expectation values of gauge-invariant operators $\mathcal{O}_{g-inv}$, which LCG will obey $s_{\gamma^2} O_{g-inv} = 0$. From (207), as highly emphasized before, the horizon function is a nonlocal expression and, consequently, we have a nonlocal action which, thanks to the introductory properties of the Gribov region Ω_{LCG}, takes into account a large set of Gribov copies.

4.3 **Localization procedure for the GZ approach in LCG**

4.3.1 Localization of the GZ action in LCG

The procedure for localizing the horizon function in LCG is analogous as considered in Landau one. First, let us localize the horizon function using the already well-known Zwanziger's ghosts $(\bar{\varphi}, \varphi, \omega, \bar{\omega})$. In LCG, the action will be

$$\begin{aligned} S^{LCG}_{GZ} &= S_{YMFP} - \int d^4x \left(\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h)\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \mathcal{M}^{ab}(A^h)\omega^{bc}_\mu\right) \\ &+ \gamma^2 \int d^4x\ g f^{abc} (A^h)^a_\mu (\varphi+\bar{\varphi})^{bc}_\mu\,. \end{aligned} \tag{209}$$

Unfortunately, as exhaustively mentioned, this procedure is not complete, *i.e.* the gauge-invariant field, $A^{h,a}_\mu$, is nonlocal itself, thereby, we need to localize it. With this in mind, let us introduce through auxiliary fields formalism, the well-known Stückelberg-like field ξ^a, like (67, 150, 151, 152, 153)

$$h = e^{ig\xi} = e^{ig\xi^a T^a}\,, \tag{210}$$

where the matrices $\{T^a\}$ are the generators of the internal symmetry group $SU(N)$ as presented in chapter (1). Then, we define A^h_μ by the equation

$$A^h_\mu \equiv A^{h,a}_\mu T^a = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h\,. \tag{211}$$

The expression (211) is local, nonetheless nonpolynomial as the equation below shows, i.e.

$$(A^h)^a_\mu = A^a_\mu - \partial_\mu \xi^a + g f^{abc} A^b_\mu \xi^c - \frac{g^2}{2} f^{abc} \xi^b \partial_\mu \xi^c + \text{higher orders}\,. \tag{212}$$

For more details of the procedure to reach the gauge-invariant bosonic field A^h_μ in terms of ξ, we refer the appendix (A), where to understand the method one just need to replace ξ for the gauge parameter ϕ. Furthermore, under a gauge transformation with group element U, one has

$$A_\mu \to A^U_\mu = U^\dagger A_\mu U + \frac{i}{g} U^\dagger \partial_\mu U\,, \quad h \to h^U = U^\dagger h\,, \quad h^\dagger \to (h^U)^\dagger = h^\dagger U\,, \tag{213}$$

thus, it is possible to verify the gauge invariance of A^h_μ, i.e.

$$A^h_\mu \to (A^U_\mu)^h = A^h_\mu\,. \tag{214}$$

The localization procedure is not finished yet, in the next subsection we are going to localize the nonperturbative BRST transformations. Moreover, it is important to remember that a new Lagrange multiplier, τ^a , must be added in the action because of the transversality condition obeyed by A^h, *i.e.* $\partial_\mu A^h_\mu = 0$; as well as the corresponding Jacobian, which is controlled by a new pair of ghosts, namely, $(\eta, \bar{\eta})$. An important remark to make at this moment is that, the procedure mentioned here is analogous to the standard FP method detailed in chapter (1). However, now we are introducing a gauge-invariant constraint, i.e., we are avoiding problems with Gribov ambiguities. Thus, the local Gribov-Zwanziger action in LCG is written as

$$\begin{aligned} S^{LCG}_{GZ} &= S_{YMFP} - \int d^4x \left(\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h) \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \mathcal{M}^{ab}(A^h) \omega^{bc}_\mu \right) \\ &+ \gamma^2 \int d^4x \; g f^{abc} (A^h)^a_\mu (\varphi + \bar{\varphi})^{bc}_\mu + \int d^4x \; \tau^a \partial_\mu (A^h)^a_\mu \\ &- \int d^4x \; \bar{\eta}^a \mathcal{M}^{ab}(A^h) \eta^b\,. \end{aligned} \tag{215}$$

Even the equation (215) being local, it is nonpolynomial. However, this fact does not affect the renormalization procedure, this occurs since there are similar nonpolynomial models as Super YM in the superspace and nonlinear sigma models which are renormalizable as well.

4.3.2 Localizing the nonperturbative BRST transformations

The action (209) was localized in the previous subsection, let us turn our attention to the localization procedure of the BRST operator (199), which brings the inverse of Faddeev-Popov's operator in the transformation of $\bar{\omega}$ (197). Therefore, the Stückelberg field ξ is present in the GZ action inside definition (210). Moreover, the BRST symmetry for the field h is written as

$$sh^{ij} = -igc^a (T^a)^{ik} h^{kj}\,, \quad sA^{h,a}_\mu = 0\,, \tag{216}$$

where the indices (i, j, k) belong to the fundamental representation of the group $SU(N)$.

The BRST transformation of the Stückelberg field ξ^a is obtained iteratively from the transformation of h^{ij}, yielding

$$s\xi^a = g^{ab}(\xi)c^b\,, \tag{217}$$

with a power series in ξ^a, namely

$$g^{ab}(\xi) = -\delta^{ab} + \frac{g}{2} f^{abc}\xi^c - \frac{g^2}{12} f^{amr} f^{mbq} \xi^q \xi^r + O(\xi^3)\,. \tag{218}$$

Furthermore, it is possible to check BRST invariance of the bosonic composite field A^h_μ, i.e., using (210) and (197) into (211), one has (omitting the color indices)

$$\begin{aligned} sA^h_\mu &= igh^\dagger c A_\mu h + h^\dagger(-\partial_\mu c + ig[A_\mu, c])h - igh^\dagger A_\mu c h - h^\dagger c\partial_\mu h + h^\dagger \partial_\mu (ch) \\ &= igh^\dagger c A_\mu h - h^\dagger \partial_\mu c h + igh^\dagger A_\mu c h - igh^\dagger c A_\mu h - igh^\dagger A_\mu c h - h^\dagger c \partial_\mu h \\ &+ h^\dagger (\partial_\mu c) h + h^\dagger c \partial_\mu h \\ &= 0\,. \end{aligned} \tag{219}$$

Now, let us localize the BRST symmetry itself. First, we need to make a trick, i.e.,

$$\exp\Big(-\gamma^4 H(A^h)\Big) = \exp\Big(-\frac{\gamma^4}{2} H(A^h)\Big)\exp\Big(-\frac{\gamma^4}{2} H(A^h)\Big)\,. \tag{220}$$

The second detail is to employ Zwanziger's localization method once again, however, at this time for each factor of (220) independently,

$$\begin{aligned} \exp\Big(-\frac{\gamma^4}{2}H(A^h)\Big) \;\sim\; & \int \mathcal{D}\varphi\mathcal{D}\bar{\varphi}\mathcal{D}\omega\mathcal{D}\bar{\omega}\exp\Big[-\int d^4x\Big(-\bar{\varphi}^{ac}_\mu \mathcal{M}^{ab}(A^h)\varphi^{bc}_\mu \\ &+ \bar{\omega}^{ac}_\mu \mathcal{M}^{ab}(A^h)\omega^{bc}_\mu + \frac{g\gamma^2}{\sqrt{2}} f^{abc} A^{h,a}_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu)\Big)\Big] \end{aligned} \tag{221}$$

and

$$\exp\Bigg(-\frac{\gamma^4}{2}H(A^h)\Bigg) \sim \int \mathcal{D}\pi\mathcal{D}\bar{\pi}\mathcal{D}\phi\mathcal{D}\bar{\phi}\exp\Bigg[-\int d^4x\Bigg(-\bar{\pi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\pi^{bc}_{\mu} + \bar{\phi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\phi^{bc}_{\mu} - \frac{g\gamma^2}{\sqrt{2}}f^{abc}A^{h,a}_{\mu}(\pi^{bc}_{\mu}+\bar{\pi}^{bc}_{\mu})\Bigg)\Bigg], \tag{222}$$

where $(\pi,\bar{\pi})$ is a pair of bosonic complex fields and $(\phi,\bar{\phi})$ are fermionic fields. Making a product between (221) and (222), one has the following alternative action

$$\begin{aligned}
S^{LCG}_{GZ} &= S_{YMFP} - \int d^4x\,\left(\bar{\varphi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\omega^{bc}_{\mu}\right) \\
&- \int d^4x\,\left(\bar{\pi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\pi^{bc}_{\mu} - \bar{\phi}^{ac}_{\mu}\mathcal{M}^{ab}(A^h)\phi^{bc}_{\mu}\right) + \frac{g\gamma^2}{\sqrt{2}}\int d^4x\; f^{abc}(A^h)^a_{\mu}(\varphi+\bar{\varphi})^{bc}_{\mu} \\
&- \frac{g\gamma^2}{\sqrt{2}}\int d^4x\, f^{abc}A^{h,a}_{\mu}(\pi^{bc}_{\mu}+\bar{\pi}^{bc}_{\mu}) + \int d^4x\;\tau^a\partial_{\mu}(A^h)^a_{\mu} - \int d^4x\;\bar{\eta}^a\mathcal{M}^{ab}(A^h)\eta^b\,.
\end{aligned} \tag{223}$$

The actions (215) and (223) have the same physical content. Nonetheless, in the last action we gain the possibility of redefining the BRST transformations in a local way, i.e.,

$$s_l S^{LCG}_{GZ} = 0\,, \tag{224}$$

where s_l is the localized BRST operator. Therefore, the BRST transformations will be given by

$$\begin{aligned}
s_l A^a_{\mu} &= -D^{ab}_{\mu}c^b\,, & s_l c^a &= \frac{g}{2}f^{abc}c^bc^c\,, \\
s_l\bar{c}^a &= ib^a\,, & s_l b^a &= 0\,, \\
s_l\varphi^{ab}_{\mu} &= \omega^{ab}_{\mu}\,, & s_l\omega^{ab}_{\mu} &= 0\,, \\
s_l\bar{\omega}^{ab}_{\mu} &= \bar{\varphi}^{ab}_{\mu} + \bar{\pi}^{ab}_{\mu}\,, & s_l\bar{\varphi}^{ab}_{\mu} &= 0\,, \\
s_l h^{ij} &= -igc^a(T^a)^{ik}h^{kj}\,, & s_l A^{h,a}_{\mu} &= 0\,, \\
s_l\tau^a &= 0\,, & s_l\bar{\eta}^a &= 0\,, \\
s_l\eta^a &= 0\,, & s_l\bar{\pi}^{ab}_{\mu} &= 0\,, \\
s_l\pi^{ab}_{\mu} &= \phi^{ab}_{\mu}\,, & s_l\bar{\phi}^{ab}_{\mu} &= 0,\,, \\
s_l\bar{\pi}^{ab}_{\mu} &= 0\,, & s_l^2 &= 0\,.
\end{aligned} \tag{225}$$

Thus, to recover the nonlocal version s_{γ^2}, one needs to integrate over the auxiliary field $\bar{\pi}$ using the equations of motion of π. Furthermore, we notice that the initial nonlocal action (209), with the set of fields $(\tau,\eta,\bar{\eta})$ integrated, is written in terms of γ^4, while (223) is described only by terms depending on γ^2, which suggests a discrete symmetry related to the invariance of the model, i.e., $\gamma^2 \to -\gamma^2$ and implies in the invariance of the local action by the following transformations

$$
\begin{aligned}
&\varphi_\mu^{ab} \to -\pi_\mu^{ab}\,, \quad \bar{\varphi}_\mu^{ab} \to -\bar{\pi}_\mu^{ab}\,, \quad \pi_\mu^{ab} \to -\varphi_\mu^{ab}\,, \quad \bar{\pi}_\mu^{ab} \to -\bar{\varphi}_\mu^{ab}\,, \\
&\omega_\mu^{ab} \to -\phi_\mu^{ab}\,, \quad \bar{\omega}_\mu^{ab} \to -\bar{\phi}_\mu^{ab}\,, \quad \phi_\mu^{ab} \to -\omega_\mu^{ab}\,, \quad \bar{\phi}_\mu^{ab} \to -\bar{\omega}_\mu^{ab}\,.
\end{aligned} \tag{226}
$$

At this point, we have a set of nontrivial fields which are BRST-singlets and belonging to the BRST cohomology. From (225), one has

$$
s_l(\varphi_\mu^{ab} + \pi_\mu^{ab}) = 2\omega_\mu^{ab}\,, \quad s_l(\varphi_\mu^{ab} - \pi_\mu^{ab}) = 0\,. \tag{227}
$$

Establishing a convenient combination, i.e.,

$$
\iota_\mu^{ab} = \frac{1}{\sqrt{2}}(\varphi_\mu^{ab} + \pi_\mu^{ab})\,, \quad \upsilon_\mu^{ab} = \frac{1}{\sqrt{2}}(\varphi_\mu^{ab} - \pi_\mu^{ab})\,. \tag{228}
$$

Then, the action (223) will be written as

$$
\begin{aligned}
S_{GZ}^{LCG} &= S_{YMFP} + \int d^4x \left[\tau^a \partial_\mu (A^h)_\mu^a - \bar{\eta}^a \mathcal{M}^{ab}(A^h)\eta^b - \bar{\iota}_\mu^{ac}\mathcal{M}^{ab}(A^h)\iota_\mu^{bc} + \bar{\omega}_\mu^{ac}\mathcal{M}^{ab}(A^h)\omega_\mu^{bc}\right. \\
&- \left.\bar{\upsilon}_\mu^{ac}\mathcal{M}^{ab}(A^h)\upsilon_\mu^{bc} + \bar{\phi}_\mu^{ac}\mathcal{M}^{ab}(A^h)\phi_\mu^{bc} + \frac{g\gamma^2}{\sqrt{2}}\, f^{abc}(A^h)_\mu^a(\upsilon + \bar{\upsilon})_\mu^{bc}\right] \\
&= S_{YMFP} + \int d^4x \left[\tau^a \partial_\mu (A^h)_\mu^a - \bar{\eta}^a \mathcal{M}^{ab}(A^h)\eta^b - \bar{\upsilon}_\mu^{ac}\mathcal{M}^{ab}(A^h)\upsilon_\mu^{bc}\right. \\
&- \left. s_l\left(\frac{1}{\sqrt{2}}\bar{\omega}_\mu^{ac}\mathcal{M}^{ab}(A^h)\iota_\mu^{bc}\right) + \bar{\phi}_\mu^{ac}\mathcal{M}^{ab}(A^h)\phi_\mu^{bc} + \frac{g\gamma^2}{\sqrt{2}}\, f^{abc}(A^h)_\mu^a(\upsilon + \bar{\upsilon})_\mu^{bc}\right],
\end{aligned} \tag{229}
$$

while the full set of local nonperturbative BRST transformations of (225) is updated by

$$
\begin{aligned}
s_l A_\mu^a &= -D_\mu^{ab}c^b\,, & s_l c^a &= \frac{g}{2}f^{abc}c^b c^c\,, \\
s_l \bar{c}^a &= ib^a\,, & s_l b^a &= 0\,, \\
s_l \iota_\mu^{ab} &= \sqrt{2}\omega_\mu^{ab}\,, & s_l \omega_\mu^{ab} &= 0\,, \\
s_l \bar{\omega}_\mu^{ab} &= \sqrt{2}\bar{\iota}_\mu^{ab}\,, & s_l \bar{\iota}_\mu^{ab} &= 0\,, \\
s_l h^{ij} &= -igc^a(T^a)^{ik}h^{kj}\,, & s_l A_\mu^{h,a} &= 0\,, \\
s_l \tau^a &= 0\,, & s_l \bar{\eta}^a &= 0\,, \\
s_l \eta^a &= 0\,, & s_l \bar{\phi}_\mu^{ab} &= 0\,, \\
s_l \upsilon_\mu^{ab} &= 0\,, & s_l \bar{\upsilon}_\mu^{ab} &= 0,\,, \\
s_l \bar{\phi}_\mu^{ab} &= 0\,, & s_l^2 &= 0\,.
\end{aligned} \tag{230}
$$

From (230), it is easy to see which fields are BRST singlets, i.e., $(A^h, \upsilon, \bar{\upsilon}, \phi, \bar{\phi}, \tau, \eta, \bar{\eta})$.

Moreover, the set of trivial fields $(\iota, \bar{\iota}, \omega, \bar{\omega})^{ab}_{\mu}$ form BRST-doublets, i.e., they do not constitute physical quantities, their perturbative contribution is null in the level of Feynman diagrams (they cancel each other) and can be integrated out in the path integral. Therefore, it is possible to rewrite the action in a local way by auxiliary fields which are BRST singlets. Redefining in a convenient manner $(\upsilon, \bar{\upsilon}, \phi, \bar{\phi}) \to (\varphi, \bar{\varphi}, \omega, \bar{\omega})$, one has the following local nonperturbative BRST-invariant GZ action

$$
\begin{aligned}
S^{LCG}_{GZ} &= S_{YMFP} - \int d^4x \left(\bar{\varphi}^{ac}_{\mu} \mathcal{M}^{ab}(A^h) \varphi^{bc}_{\mu} - \bar{\omega}^{ac}_{\mu} \mathcal{M}^{ab}(A^h) \omega^{bc}_{\mu} \right) \\
&+ \gamma^2 \int d^4x \; g f^{abc} (A^h)^a_{\mu} (\varphi + \bar{\varphi})^{bc}_{\mu} + \int d^4x \; \tau^a \partial_{\mu} (A^h)^a_{\mu} \\
&- \int d^4x \; \bar{\eta}^a \mathcal{M}^{ab}(A^h) \eta^b \,,
\end{aligned}
\tag{231}
$$

where we go back to the original notation developed by Zwanziger for the auxiliary fields, i.e., a bosonic pair $(\varphi, \bar{\varphi})$ and a fermionic pair $(\omega, \bar{\omega})$, now as BRST-singlets. To end this section, the Gribov parameter γ belongs to the cohomology of the BRST operator s_l, thus

$$
s_l \frac{\partial S^{LCG}_{GZ}}{\partial \gamma^2} = s_l \int d^4x \left[g f^{abc} A^{h,a}_{\mu} (\varphi + \bar{\varphi})^{bc}_{\mu} \right] = 0 \Rightarrow \frac{\partial S^{LCG}_{GZ}}{\partial \gamma^2} \neq s_{\gamma^2}(...) \,.
\tag{232}
$$

Therefore, γ is a physical parameter of the theory.

5 THE RGZ ACTION AND THE HORIZON FUNCTION FOR THE MATTER

In the last chapter we presented a gauge-invariant formulation of the GZ action. It allow us to study the Gribov problem in different gauges, not only in the Landau gauge as originally proposed by Gribov. Now we are able to generalize the RGZ model in a BRST invariant formulation.

Furthermore, we propose in this thesis an extension of the investigation done in (65, 68, 69, 70, 71, 72, 63, 73, 74, 75) about the renormalizability properties of a local and BRST invariant RGZ action including, at this time, a gauge-invariant local composite Dirac field in the fundamental representation and its own horizon-like function. Therefore, in this chapter we will present the model and its symmetry content, in the next chapter we are going to prove by using the algebraic renormalization method (76), the all orders renormalizability of such model, a subject which was still absent in those preceding works.

5.1 The gauge-invariant RGZ action

As already explained in subsection (2.1.4), once we have a nonvanishing Gribov parameter γ, it is also possible to show that the vacuum expectation value of the dimension two local operators $(A^a_\mu(x))^2$ and $(\bar{\varphi}^{ab}_\mu(x)\varphi^{ab}_\mu(x) - \bar{\omega}^{ab}_\mu(x)\omega^{ab}_\mu(x))$ do exist in the theory. In other words, these operators condense and such condensates affect in a nontrivial way the behavior of the gauge field and ghost propagators. Now, in the BRST invariant approach, the operator $(A^a_\mu)^2$ is replaced by $(A^{a,h}_\mu)^2$. Notice that these two operators coincide in the Landau gauge. However, for different gauges $(A^a_\mu)^2$ is not invariant while $(A^{a,h}_\mu)^2$ remains gauge-invariant by construction. Thus, the operator $(A^{a,h}_\mu)^2$ is the correct operator to be introduced for general gauges[27]. Also, the localizing auxiliary Zwanziger fields are, in the new BRST invariant formulation, BRST singlets, then the operator $(\bar{\varphi}^{ab}_\mu\varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu\omega^{ab}_\mu)$ is automatically invariant. With these condensates, the local and BRST-invariant RGZ action in the LCG is written as

[27] In the case of the maximal Abelian gauge (MAG), where the internal group symmetry is broken, there might be a difference between the diagonal and off-diagonal condensates, i.e. a breaking of degeneracy is expected.

$$
\begin{aligned}
S_{RGZ}^{LCG} &= S_{YMFP} + \int d^4x \left[\tau^a \partial_\mu A_\mu^{h,a} - \bar{\eta}^a \mathcal{M}^{ab}(A^h)\eta^b \,. \right. \\
&- \bar{\varphi}_\mu^{ac} \mathcal{M}^{ab}(A^h)\varphi_\mu^{bc} + \bar{\omega}_\mu^{ac}\mathcal{M}^{ab}(A^h)\omega_\mu^{bc} - \gamma^2 \, g f^{abc} A_\mu^{h,a} (\varphi + \bar{\varphi})_\mu^{bc} \\
&+ \left. \frac{m^2}{2} A_\mu^{h,a} A_\mu^{h,a} - M^2 \left(\bar{\varphi}_\mu^{ab}\varphi_\mu^{ab} - \bar{\omega}_\mu^{ab}\omega_\mu^{ab} \right) \right], \qquad (233)
\end{aligned}
$$

where S_{YMFP} is the Euclidean Yang-Mills action in the LCG, given by eq. (39), and

$$
\begin{aligned}
m^2 &\propto \langle A_\mu^{h,a} A_\mu^{h,a} \rangle \,, \qquad (234) \\
M^2 &\propto \langle \bar{\varphi}_\mu^{ab}\varphi_\mu^{ab} - \bar{\omega}_\mu^{ab}\omega_\mu^{ab} \rangle \qquad (235)
\end{aligned}
$$

are mass parameters emerging from the condensation of the dimension two operators mentioned above. Also, It can be verified without difficulties that action (233) is invariant under the following BRST transformations

$$
\begin{aligned}
sA_\mu^a &= -D_\mu^{ab}(A)c^b \,, & sc^a &= \frac{g}{2} f^{abc} c^b c^c \,, \\
s\bar{c}^a &= ib^a \,, & sb^a &= 0 \,, \\
s\varphi_\mu^{ab} &= 0 \,, & s\omega_\mu^{ab} &= 0 \,, \\
s\bar{\omega}_\mu^{ab} &= 0 \,, & s\bar{\varphi}_\mu^{ab} &= 0 \,, \\
sh^{ij} &= -igc^a \, T^{a,ik} \, h^{kj} \,, & sA_\mu^{h,a} &= 0 \,, \\
s\tau^a &= 0 \,, & s\bar{\eta}^a &= 0 \,, \\
s\eta^a &= 0 \,, & s^2 &= 0 \,, \qquad (236)
\end{aligned}
$$

where we consider from now on the replacement $s_l \to s$, remembering that s_l was the notation adopted in the last chapter for the local, nonperturbative and nilpotent BRST transformations (230).

Besides the BRST invariance, action (233) enjoys some additional properties, see (68, 69, 70), that can be remarked here: (i) the correlation functions of gauge invariant quantities, the mass parameters (γ, m, M) and the transversal component of the gauge field propagator are independent of the gauge parameter α. (ii) The longitudinal component of the gauge field propagator has the same result as in tree-level. (iii) Finally, action (233) definitely implements a restriction of the path integral for the RGZ model, as well as action (231) implements the restriction to the region Ω_{LCG} given by eq. (205).

5.2 Remarks on the gauge-fixing and infrared regularization

Let us point out that the Stückelberg field ξ is a massless field, whose propagator behaves like $\langle \xi(p)\xi(-p)\rangle = \alpha/p^4$, a feature which might eventually lead to undesired spurious infrared divergences in some Green functions. Though, as shown in (64), the gauge-fixing term S_{FP} in (39) can be suitable modified in order to account for a renormalizable BRST invariant infrared regularization for the field ξ. More precisely, one can introduce a regularizing infrared mass μ^2 through the exact BRST term

$$\begin{aligned} S_{FP}^{(\mu)} &= \int d^4x \; s\left(\bar{c}^a(\partial_\mu A_\mu^a - \mu^2\xi^a) - i\frac{\alpha}{2}\bar{c}^a b^a\right) \\ &= \int d^4x \left(ib^a\,\partial_\mu A_\mu^a + \frac{\alpha}{2}b^a b^a + \bar{c}^a\,\partial_\mu D_\mu^{ab}(A)c^b - i\mu^2 b^a\xi^a + \mu^2\bar{c}^a g^{ab}(\xi)c^b\right). \end{aligned} \tag{237}$$

The gauge parameter μ^2 plays the role of an infrared regulator for the ξ field, whose propagator gets now the infrared safe form $\langle \xi(p)\xi(-p)\rangle = \alpha/(p^2+\mu^2)^2$. Moreover, eq. (237) shows that μ^2 appears in a BRST exact term, as well as the parameter α. As a consequence, it is a pure gauge parameter which will not affect the correlation functions of the gauge-invariant operators. However, as we are actually interested in the possible ultraviolet (UV) divergences, we will proceed by choosing the the particular case $\mu^2 = 0$, which is no more than the LCG already adopted in (39).

5.3 The gauge-invariant composite fermionic field

Within the local nonperturbative BRST framework introduced in the last chapter, we can now discuss the construction of a gauge-invariant local fermionic composite field $\psi^h(x)$ and its correspondent Dirac adjoint spinor $\bar{\psi}^h = (\psi^h)^\dagger\gamma_4$ in the same way we have constructed $A_\mu^h(x)$, see eqs. (210), (211) and (212). Let us start by the following definition[28]

$$\psi_\alpha^{h,i} \equiv h^\dagger \psi_\alpha^i\,, \tag{238}$$

[28] According to the notations adopted here, the Greek indices $\{\mu,\nu,\rho,\sigma\}$ are the vector indices of the Euclidean space, while the Greek indices $\{\alpha,\beta,\gamma,\delta\}$ are the Dirac spinor indices. The Latin indices $\{a,b,c,d,e\}$, running from 1 to N^2-1, are the indices of the adjoint representation of $SU(N)$; while the Latin indices $\{i,j,k,l\}$, running from 1 to N, represent the indices of the fundamental representation of the group. The Dirac gamma matrices γ_μ in Euclidean space are given by

$$\gamma_4 = \begin{pmatrix} 0 & \mathbb{I} \\ \mathbb{I} & 0 \end{pmatrix}, \qquad \gamma_k = -i\begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & 0 \end{pmatrix}, \qquad \gamma_5 = \gamma_4\gamma_1\gamma_2\gamma_3 = \begin{pmatrix} \mathbb{I} & 0 \\ 0 & -\mathbb{I} \end{pmatrix},$$

where $\mathbb{I}$ is the 2×2 identity matrix, $k = 1,2,3$ and σ_k are the well-known Pauli matrices.

where ψ^i_α is the Dirac spinor field and h is given by (210). As the Dirac field transforms as $\psi \to U^\dagger\psi$ and $h^\dagger$ as $h^\dagger \to h^\dagger U$, for a finite gauge transformation U in SU(N), it is immediate to realize that ψ^h is gauge-invariant. Of course, the same procedure can be done for the Dirac adjoint field $\bar{\psi}$, giving rise to the Dirac adjoint invariant composite field $\bar{\psi}^h \equiv \bar{\psi}h$. This new composite invariant field can also be written as a series expansion of the Stückelberg-like field ξ^a,

$$\psi^{h,i}_\alpha = \psi^i_\alpha - ig\xi^a(T^a)^{ij}\psi^j_\alpha - \frac{g^2}{2}\xi^a\xi^b(T^a)^{ij}(T^b)^{jk}\psi^k_\alpha + \mathcal{O}(\xi^3)\,. \tag{239}$$

A complete study of this operator in the Euclidean Yang-Mills theory with matter fields was already done in (75), where its renormalizability was established to all orders in loop expansion.

A potential application of the operators $(\psi^h, \bar{\psi}^h)$ is that of allowing for a renormalizable non Abelian Landau-Khalatnikov-Fradkin (LKF) transformations which include spinor fields, within the setup worked out in (154) (see also references therein). In fact, due to the BRST invariance of $(A^h_\mu, \psi^h, \bar{\psi}^h)$, it follows that the correlation functions like

$$\langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_i}(x_i)\psi^h(y_1)\ldots\bar{\psi}^h(y_j)\rangle$$

are independent from the gauge parameter α, namely

$$\langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_i}(x_i)\psi^h(y_1)\ldots\bar{\psi}^h(y_j)\rangle_{\alpha\neq 0} = \langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_i}(x_i)\psi^h(y_1)\ldots\bar{\psi}^h(y_j)\rangle_{\alpha=0}\,. \tag{240}$$

Once expanded in powers of the Stückelberg field ξ, eq. (240) enables one to evaluate the Green function

$$\langle A^h_{\mu_1}(x_1)\ldots A^h_{\mu_i}(x_i)\psi^h(y_1)\ldots\bar{\psi}^h(y_j)\rangle_{\alpha\neq 0}$$

in a given α-gauge, with $\alpha \neq 0$, from the knowledge of the corresponding Green function evaluated in the Landau gauge, $\alpha = 0$, yielding thus the LKF transformations within a renormalizable environment. Also, eq. (240) might be employed to sheed some light on nonperturbative aspects of gauge theories, like the infrared behavior of the gauge and fermion propagators. As such, those equations could be exploited in order to show the gauge-independence of quantities like the chiral condensate.

Here in this thesis, once we have at our disposal the composite invariant field (239) (and its Dirac adjoint), we will be interested in construct and study the so-called horizon function for the matter sector. The immediate importance of this study relies on the behavior of the tree-level propagator of the Dirac fields. This will the subject of the next section.

Let us end this section by pointing out that it is also possible to construct an invariant composite field for scalar matter. In fact, considering a scalar field in the adjoint representation,

$$\phi = \phi^a T^a \,, \tag{241}$$

whose gauge transformation is

$$\phi \to U^\dagger \phi U \,, \tag{242}$$

with U in SU(N), a gauge-invariant composite scalar field is obtained by making use of the Stückelberg field, as

$$\phi^h \equiv h^\dagger \phi h \,. \tag{243}$$

The gauge invariance of ϕ^h is guaranteed by eqs. (242) and (213).

5.4 The horizon function for the matter

Let us start this section by reminding that the restriction to the Gribov region Ω_{LCG} is implemented in the extended GZ model by introducing the term

$$e^{-\gamma^4 H(A^h)} \,,$$

with γ being the Gribov mass parameter and $H(A^h)$ the gauge-invariant Zwanziger horizon function, given by

$$H(A^h) = g^2 \int d^4x d^4y \, f^{abc} A^{h,b}_\mu(x) \left[\mathcal{M}^{-1}(A^h)\right]^{ad} (x,y) \, f^{dec} A^{h,e}_\mu(y) \,. \tag{244}$$

Then, the horizon function above could be generalized in the following way

$$H_{\mathfrak{F}}(A^h, \mathfrak{F}) = -\int d^4x d^4y \; T^{a,ij} \, \bar{\mathfrak{F}}^i_N(x) \left[\mathcal{M}^{-1}(A^h)\right]^{ab} (x,y) \, T^{b,jk} \, \mathfrak{F}^k_N(y) \,. \tag{245}$$

Here, $\mathfrak{F}^i_N(x)$ is a generic local object that could be a fundamental field, or a local composite field. If $\mathfrak{F}$ is in the adjoint representation, one can replace $T^{a,ij}\mathfrak{F}^i$ by $-if^{abc}\mathfrak{F}^b$. The index N is a generic degree of freedom of $\mathfrak{F}$. Then, $\mathfrak{F}$ can be, for example, a Lorentz vector, or a tensor, or a Dirac spinor, or even a scalar. The bar in $\bar{\mathfrak{F}}$ represents a possible conjugation. For complex fields, the bar indicates the complex conjugation; for Dirac spinor fields, the bar indicates the Dirac adjoint; and for real fields, $\bar{\mathfrak{F}} = \mathfrak{F}$. The factor g^2 appearing in the Zwanziger horizon function (244) can be hidden in the definition of $\mathfrak{F}$.

In the original formulation of the GZ model, $\mathfrak{F}$ was taken as the gauge field $A^a_\mu(x)$. In (51), $\mathfrak{F}$ was chosen as a scalar field in the adjoint representation $\phi^a(x)$. In (155), it was chosen as a Dirac spinor field $\psi^i_\alpha(x)$. Recently, as presented in the last chapter, we have chosen $\mathfrak{F} \equiv A^h_\mu(x)$, giving rise to a gauge-invariant horizon function and a new formulation for the GZ model. Finally, we propose to choose $\mathfrak{F}$ as the gauge-invariant composite Dirac field $\psi^{h,i}_\alpha(x)$, generalizing the study previously done in (155). As we shall see later, the main reason for the introduction of the horizon term for the matter sector is the qualitative accordance with lattice results for the propagator of the fermionic fields.

Then, let us initially consider the following classical action

$$S = S^{LCG}_{RGZ} + S_{matter} + S_\sigma \,, \tag{246}$$

where S^{LCG}_{RGZ} is the RGZ action (233), while S_{matter} and S_σ are given by

$$S_{matter} = \int d^4x \left[i\bar{\psi}^{i\alpha}(\gamma_\mu)_{\alpha\beta} D^{ij}_\mu \psi^{j\beta} - m_\psi\, \bar{\psi}^{i\alpha}\psi^i_\alpha \right] , \tag{247}$$

$$\begin{aligned} S_\sigma &= -\sigma^3 H_\psi(A^h, \psi^h) \\ &= -\sigma^3 \int d^4x d^4y\, \bar{\psi}^{h,i}_\alpha(x) T^{a,ij} \left[\mathcal{M}^{-1}(A^h)\right]^{ab} (x,y) T^{b,jk} \psi^{h,k}_\alpha(y) \,. \end{aligned} \tag{248}$$

The term (247) is the usual fermionic matter sector coupled to pure Yang-Mills sector through the covariant derivative, $D^{ij}_\mu = \delta^{ij}\partial_\mu - igT^{a,ij}\, A^a_\mu$, in the fundamental representation. A mass parameter m_ψ is also introduced here in the case of massive fermions. The term (248) is the contribution of the horizon term for the matter. The parameter σ is analogous to the Gribov parameter for the matter sector. Notice that the horizon function (244) has mass dimension -4. Then it has to be introduced in the action together with a mass parameter of power four, i.e. γ^4. On the other hand, the horizon function (245) for $\mathfrak{F} \equiv \psi^{h,i}_\alpha(x)$ has mass dimension -3, as the composite fermionic fields $(\psi^h, \bar{\psi}^h)$ have dimension $3/2$, the later is introduced in the action with a mass of power three, justifying the σ^3 in (248).

The first issue that we have to face here is that the term (248) is nonlocal. Therefore, we need to develop a procedure, in a very similar way as the horizon function of the gauge-invariant bosonic composite field reviewed in chapter (4), in order to localize this term. As a result,

$$\begin{aligned} S^{local}_\sigma &= \int d^4x \Bigg[\bar{\lambda}^{ai}_\alpha \left(-\partial_\mu D^{ab}_\mu(A^h)\right) \lambda^{\alpha,bi} + \bar{\zeta}^{ai}_\alpha \left(-\partial_\mu D^{ab}_\mu(A^h)\right) \zeta^{\alpha,bi} \\ &+ \sigma^{\frac{3}{2}} \left(\bar{\lambda}^{ai}_\alpha T^{a,ij} \psi^{h,j\alpha} + \bar{\psi}^{h,i}_\alpha T^{a,ij} \lambda^{aj\alpha} \right) \Bigg] , \end{aligned} \tag{249}$$

where $(\lambda^{ai}_\alpha, \bar{\lambda}^{ai}_\alpha)$ are anticommuting spinor fields and $(\zeta^{ai}_\alpha, \bar{\zeta}^{ai}_\alpha)$ are the commuting ones. It is easy to prove that the integration over the auxiliary fields $(\lambda^{ai}_\alpha, \bar{\lambda}^{ai}_\alpha, \zeta^{ai}_\alpha, \bar{\zeta}^{ai}_\alpha)$ gives the original nonlocal expression (248). Then, action (246) gives place to an equivalent local version,

$$S^{local} = S^{LCG}_{RGZ} + S_{matter} + S^{local}_{\sigma}\,, \tag{250}$$

by replacing (248) by (249). The new local action (250) is left invariant by the BRST transformations (236) together with the BRST transformations of $(\psi, \bar{\psi}, \lambda, \bar{\lambda}, \zeta, \bar{\zeta})$,

$$\begin{aligned} & s\psi^i_\alpha = -i(T^a)^{ij}c^a\psi^j_\alpha\,, && s\bar{\psi}^i_\alpha = -i\bar{\psi}^j_\alpha(T^a)^{ji}c^a\,, \\ & s\bar{\zeta}^{ai}_\alpha = 0\,, && s\lambda^{ai}_\alpha = 0\,, \\ & s\bar{\lambda}^{ai}_\alpha = 0\,, && s\zeta^{ai}_\alpha = 0\,. \end{aligned} \tag{251}$$

5.5 Introduction of external sources

5.5.1 Embedding the theory into a more general one

Let us consider now the RGZ action (233) and take a look only in the term proportional to the Gribov parameter γ^2. Namely,

$$S_{\gamma^2} = \int d^4x\; \gamma^2 f^{abc}\, A^{h,a}_\mu\, (\varphi + \bar{\varphi})^{bc}_\mu\,. \tag{252}$$

Therefore, let us suppose that this term could be a particular case of a general term depending on a set of external sources:

$$\begin{aligned} S(M,N,U,V) &= \int d^4x \Big[M^{ac}_{\mu\nu} D^{ab}_\mu(A^h)\varphi^{bc}_\nu + V^{ac}_{\mu\nu} D^{ab}_\mu(A^h)\bar{\varphi}^{bc}_\nu - N^{ac}_{\mu\nu} D^{ab}_\mu(A^h)\omega^{bc}_\nu \\ &+ U^{ac}_{\mu\nu} D^{ab}_\mu(A^h)\bar{\omega}^{bc}_\nu - M^{ab}_{\mu\nu}V^{ab}_{\mu\nu} + N^{ab}_{\mu\nu}U^{ab}_{\mu\nu} \Big]\,, \end{aligned} \tag{253}$$

with (M, V) being commuting sources and (N, U) anticommuting ones. In fact, for the following particular values of physical interest

$$\begin{aligned} M^{ab}_{\mu\nu}\Big|_{phys} &= V^{ab}_{\mu\nu}\Big|_{phys} = \gamma^2\delta^{ab}\delta_{\mu\nu}\,, \\ N^{ab}_{\mu\nu}\Big|_{phys} &= U^{ab}_{\mu\nu}\Big|_{phys} = 0\,, \end{aligned} \tag{254}$$

we have

$$S(M,N,U,V)\Big|_{phys} = S_{\gamma^2}\,, \tag{255}$$

modulo a vacuum term coming from the MV product in (253), which is allowed by power-counting. The introduction of this set of external sources allow us to write the following symmetry transformations

$$
\begin{aligned}
\delta\varphi^{ab}_{\mu} &= \omega^{ab}_{\mu}\,, \qquad \delta\omega^{ab}_{\mu} \;=\; 0\,, \\
\delta\bar{\omega}^{ab}_{\mu} &= \bar{\varphi}^{ab}_{\mu}\,, \qquad \delta\bar{\varphi}^{ab}_{\mu} \;=\; 0\,, \\
\delta N^{ab}_{\mu\nu} &= M^{ab}_{\mu\nu}\,, \qquad \delta M^{ab}_{\mu\nu} \;=\; 0\,, \\
\delta V^{ab}_{\mu\nu} &= U^{ab}_{\mu\nu}\,, \qquad \delta U^{ab}_{\mu\nu} \;=\; 0\,.
\end{aligned}
\tag{256}
$$

The operator δ above is nilpotent and can be viewed as a kind of BRST acting only on the fields $(\varphi,\bar{\varphi},\omega,\bar{\omega},M,N,U,V)$. In fact, as we shall see later, operators s and δ can be linearly combined in order to write an extended BRST. Also, the general term (253) can be rewritten as a total δ-variation,

$$
S(M,N,U,V) \;=\; \delta\int d^4x\,\left[N^{ac}_{\mu\nu}D^{ab}_{\mu}(A^h)\varphi^{bc}_{\nu} + V^{ac}_{\mu\nu}D^{ab}_{\mu}(A^h)\bar{\omega}^{bc}_{\nu} - N^{ab}_{\mu\nu}V^{ab}_{\mu\nu}\right], \tag{257}
$$

as well as the RGZ dimension two operator

$$
\bar{\varphi}^{ab}_{\mu}\varphi^{ab}_{\mu} - \bar{\omega}^{ab}_{\mu}\omega^{ab}_{\mu} = \delta\left(\bar{\omega}^{ab}_{\mu}\varphi^{ab}_{\mu}\right)\,. \tag{258}
$$

Actually, the local version of the Zwanziger horizon function can be totally written as a δ-variation. Thus, once δ is a part of an extended BRST operator, the horizon function and the RGZ operator $(\bar{\varphi}^{ab}_{\mu}\varphi^{ab}_{\mu} - \bar{\omega}^{ab}_{\mu}\omega^{ab}_{\mu})$ belong to the trivial sector of the cohomology of such extended BRST operator. With respect to the s operator, the sources (M,N,U,V) transform as BRST singlets, i.e.

$$
sM^{ab}_{\mu\nu} = sN^{ab}_{\mu\nu} = sU^{ab}_{\mu\nu} = sV^{ab}_{\mu\nu} = 0\,. \tag{259}
$$

We can also define a $U(4(N^2-1))$ symmetry for $S(M,N,U,V)$,

$$
Q^{ab}_{\mu\nu}(S(M,N,U,V)) = 0\,, \tag{260}
$$

with

$$
\begin{aligned}
Q^{ab}_{\mu\nu} \;=\; & \int d^4x\,\Bigg(\varphi^{ca}_{\mu}\frac{\delta}{\delta\varphi^{cb}_{\nu}} - \bar{\varphi}^{cb}_{\nu}\frac{\delta}{\delta\bar{\varphi}^{ca}_{\mu}} + \omega^{ca}_{\mu}\frac{\delta}{\delta\omega^{cb}_{\nu}} - \bar{\omega}^{cb}_{\nu}\frac{\delta}{\delta\bar{\omega}^{ca}_{\mu}} \\
&+ V^{ca}_{\sigma\mu}\frac{\delta}{\delta V^{cb}_{\sigma\nu}} - M^{cb}_{\sigma\nu}\frac{\delta}{\delta M^{ca}_{\sigma\mu}} + U^{ca}_{\sigma\mu}\frac{\delta}{\delta U^{cb}_{\sigma\nu}} - N^{cb}_{\sigma\nu}\frac{\delta}{\delta N^{ca}_{\sigma\mu}}\Bigg)\,.
\end{aligned}
\tag{261}
$$

The trace of this operator defines a charge shared among the fields $(\varphi,\bar{\varphi},\omega,\bar{\omega},M,N,U,V)$. This symmetry also allows us to employ the so-called multi-index notation, as applied in (40, 114, 50, 155), where two more indices can be taken as only one. In this case, one index of the internal symmetry group in the adjoint representation and a vectorial index are composed as one index in the following way:

$$(\varphi^{ab}_{\nu}, \bar{\varphi}^{ab}_{\nu}, \omega^{ab}_{\nu}, \bar{\omega}^{ab}_{\nu}, M^{ab}_{\mu\nu}, N^{ab}_{\mu\nu}, U^{ab}_{\mu\nu}, V^{ab}_{\mu\nu}) \equiv (\varphi^{aI}, \bar{\varphi}^{aI}, \omega^{aI}, \bar{\omega}^{aI}, M^{aI}_{\mu}, N^{aI}_{\mu}, U^{aI}_{\mu}, V^{aI}_{\mu}) \,. \tag{262}$$

Here we have $I \equiv \{b, \nu\}$, i.e. index I is a combination of the indices b and ν. Then, we define a new set of indices $\{I, J, K, L, \dots\}$ in which each of the elements runs from 1 to $4 \times (N^2 - 1)$. In terms of the multi-index notation, the term (253) is written as

$$\begin{aligned} S(M,N,U,V) &= \delta \int d^4x \left[N^{aI}_{\mu} D^{ab}_{\mu}(A^h)\varphi^{bI} + V^{aI}_{\mu} D^{ab}_{\mu}(A^h)\bar{\omega}^{bI} - N^{aI}_{\mu} V^{aI}_{\mu} \right] \\ &= \int d^4x \Big[M^{aI}_{\mu} D^{ab}_{\mu}(A^h)\varphi^{bI} + V^{aI}_{\mu} D^{ab}_{\mu}(A^h)\bar{\varphi}^{bI} - N^{aI}_{\mu} D^{ab}_{\mu}(A^h)\omega^{bI} \\ &+ U^{aI}_{\mu} D^{ab}_{\mu}(A^h)\bar{\omega}^{bI} - M^{aI}_{\mu} V^{aI}_{\mu} + N^{aI}_{\mu} U^{aI}_{\mu} \Big] \,. \end{aligned} \tag{263}$$

Analogously, similar considerations can be done to the local version of the horizon function for fermionic matter. In this case, we will employ the set of external sources $(\Pi, \bar{\Pi}, \Lambda, \bar{\Lambda})^{ij}_{\alpha\beta}$, with $(\Lambda, \bar{\Lambda})$ being commuting and $(\Pi, \bar{\Pi})$ anticommuting ones and write the following expression:

$$\begin{aligned} S(\Pi, \bar{\Pi}, \Lambda, \bar{\Lambda}) &= \int d^4x \Big[\bar{\Lambda}^{jk}_{\alpha\beta} \bar{\psi}^{h,i\alpha} T^{a,ij} \lambda^{ak\beta} + \bar{\Pi}^{jk}_{\alpha\beta} \bar{\psi}^{h,i\alpha} T^{a,ij} \zeta^{ak\beta} \\ &+ \Pi^{ik\beta}_{\alpha} \bar{\zeta}^{ak}_{\beta} T^{a,ij} \psi^{h,j\alpha} + \Lambda^{ik\beta}_{\alpha} \bar{\lambda}^{ak}_{\beta} T^{a,ij} \psi^{h,j\alpha} \Big] \,. \end{aligned} \tag{264}$$

At the physical limit

$$\begin{aligned} \Lambda^{ij}_{\alpha\beta}\Big|_{phys} &= \bar{\Lambda}^{ij}_{\alpha\beta}\Big|_{phys} = \sigma^{\frac{3}{2}} \delta^{ij} \delta_{\alpha\beta} \,, \\ \Pi^{ij}_{\alpha\beta}\Big|_{phys} &= \bar{\Pi}^{ij}_{\alpha\beta}\Big|_{phys} = 0 \,, \end{aligned} \tag{265}$$

the term $S(\Pi, \bar{\Pi}, \Lambda, \bar{\Lambda})$ assumes the particular form

$$S(\Pi, \bar{\Pi}, \Lambda, \bar{\Lambda})\Big|_{phys} = \int d^4x \; \sigma^{\frac{3}{2}} \left(\bar{\lambda}^{ai}_{\alpha} T^{a,ij} \psi^{h,j\alpha} + \bar{\psi}^{h,i}_{\alpha} T^{a,ij} \lambda^{aj\alpha} \right) \,, \tag{266}$$

which is exactly the term proportional to $\sigma^{\frac{3}{2}}$ in (249). The sources are BRST singlets,

$$s\Lambda^{ij}_{\alpha\beta} = s\bar{\Lambda}^{ij}_{\alpha\beta} = s\Pi^{ij}_{\alpha\beta} = s\bar{\Pi}^{ij}_{\alpha\beta} = 0 \,. \tag{267}$$

Then, it is easy to check that $sS(\Pi, \bar{\Pi}, \Lambda, \bar{\Lambda}) = 0$. On the other hand, $S(\Pi, \bar{\Pi}, \Lambda, \bar{\Lambda})$ has "its own BRST" acting only on $(\lambda,\bar\lambda,\zeta,\bar\zeta,\Pi,\bar\Pi,\Lambda,\bar\Lambda)$,

$$\begin{aligned}
\hat\delta\lambda^{ai}_\alpha &= \zeta^{ai}_\alpha\,, & \hat\delta\zeta^{ai}_\alpha &= 0\,,\\
\hat\delta\bar\zeta^{ai}_\alpha &= \bar\lambda^{ai}_\alpha\,, & \hat\delta\bar\lambda^{ai}_\alpha &= 0\,,\\
\hat\delta\Lambda^{ij}_{\alpha\beta} &= \Pi^{ij}_{\alpha\beta}\,, & \hat\delta\Pi^{ij}_{\alpha\beta} &= 0\,,\\
\hat\delta\bar\Pi^{ij}_{\alpha\beta} &= \bar\Lambda^{ij}_{\alpha\beta}\,, & \hat\delta\bar\Lambda^{ij}_{\alpha\beta} &= 0\,.
\end{aligned} \tag{268}$$

In terms of this new BRST, $S(\Pi,\bar\Pi,\Lambda,\bar\Lambda)$ is written as an exact $\hat\delta$-variation,

$$S(\Pi,\bar\Pi,\Lambda,\bar\Lambda)=\hat\delta\int d^4x\,\left[\bar\Pi^{jk}_{\alpha\beta}\bar\psi^{h,i\alpha}T^{a,ij}\lambda^{ak\beta}+\Lambda^{ik\beta}_{\alpha}\bar\zeta^{ak}_{\beta}T^{a,ij}\psi^{h,j\alpha}\right]. \tag{269}$$

We can also define a new invariant dimension two operator

$$\bar\lambda^{ai}_\alpha\lambda^{ai\alpha}+\bar\zeta^{ai}_\alpha\zeta^{ai\alpha}=\hat\delta\left(\bar\zeta^{ai}_\alpha\lambda^{ai\alpha}\right)\,, \tag{270}$$

which will be considered later. Furthermore, there is a $U(4N)$ symmetry for $S(\Pi,\bar\Pi,\Lambda,\bar\Lambda)$,

$$\hat Q^{ij}_{\alpha\beta}(S(\Pi,\bar\Pi,\Lambda,\bar\Lambda))=0\,, \tag{271}$$

with

$$\begin{aligned}
\hat Q^{ij}_{\alpha\beta} &= \int d^4x\,\Bigg(\lambda^{ai}_\alpha\frac{\delta}{\delta\lambda^{aj\beta}}-\bar\lambda^{aj}_\beta\frac{\delta}{\delta\bar\lambda^{ai\alpha}}+\zeta^{ai}_\alpha\frac{\delta}{\delta\zeta^{aj\beta}}-\bar\zeta^{aj}_\beta\frac{\delta}{\delta\bar\zeta^{ai\alpha}}\\
&+\ \Lambda^{ki}_{\gamma\alpha}\frac{\delta}{\delta\Lambda^{kj\beta}_\gamma}-\bar\Lambda^{kj}_{\gamma\beta}\frac{\delta}{\delta\bar\Lambda^{ki\alpha}_\gamma}+\Pi^{ki}_{\gamma\alpha}\frac{\delta}{\delta\Pi^{kj\beta}_\gamma}-\bar\Pi^{kj}_{\gamma\beta}\frac{\delta}{\delta\bar\Pi^{ki\alpha}_\gamma}\Bigg)\,.
\end{aligned} \tag{272}$$

As in the case of (261), the trace of $\hat Q^{ij}_{\alpha\beta}$ defines a charge for $(\lambda,\bar\lambda,\zeta,\bar\zeta,\Pi,\bar\Pi,\Lambda,\bar\Lambda)$ and a new multi-index $\hat I\equiv\{j,\beta\}$ can be established:

$$(\lambda^{aj}_\beta,\bar\lambda^{aj}_\beta,\zeta^{aj}_\beta,\bar\zeta^{aj}_\beta,\Pi^{ij}_{\alpha\beta},\bar\Pi^{ij}_{\alpha\beta},\Lambda^{ij}_{\alpha\beta},\bar\Lambda^{ij}_{\alpha\beta})\equiv(\lambda^a_{\hat I},\bar\lambda^a_{\hat I},\zeta^a_{\hat I},\bar\zeta^a_{\hat I},\Pi^i_{\alpha\hat I},\bar\Pi^i_{\alpha\hat I},\Lambda^i_{\alpha\hat I},\bar\Lambda^i_{\alpha\hat I})\,. \tag{273}$$

Thus, we will use the indices $\{\hat I,\hat J,\hat K,\dots\}$, each of them varying from 1 to $4\times N$. Then, in terms of the new multi-index, expression (264) is written as

$$\begin{aligned}
S(\Pi,\bar\Pi,\Lambda,\bar\Lambda) &= \hat\delta\int d^4x\,\left[\bar\Pi^j_{\alpha\hat I}\,\bar\psi^{h,i\alpha}\,T^{a,ij}\,\lambda^{a\hat I}+\Lambda^{i\hat I}_\alpha\,\bar\zeta^a_{\hat I}\,T^{a,ij}\,\psi^{h,j\alpha}\right]\\
&= \int d^4x\,\Big[\bar\Lambda^j_{\alpha\hat I}\,\bar\psi^{h,i\alpha}\,T^{a,ij}\,\lambda^{a\hat I}+\bar\Pi^j_{\alpha\hat I}\,\bar\psi^{h,i\alpha}\,T^{a,ij}\,\zeta^{a\hat I}\\
&+\ \Pi^{i\hat I}_\alpha\,\bar\zeta^a_{\hat I}\,T^{a,ij}\,\psi^{h,j\alpha}+\Lambda^{i\hat I}_\alpha\,\bar\lambda^a_{\hat I}\,T^{a,ij}\,\psi^{h,j\alpha}\Big]\,.
\end{aligned} \tag{274}$$

Therefore, we can replace S^{local}, eq. (250), by a more general one, or, in other words, we can also say that action S^{local} is embedded in the following action

$$
\begin{aligned}
S_1 &= \int d^4x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{\alpha}{2} b^a b^a + i b^a \, \partial_\mu A^a_\mu + \bar{c}^a \, \partial_\mu D^{ab}_\mu(A) c^b + i \bar{\psi}^{i\alpha} (\gamma_\mu)_{\alpha\beta} D^{ij}_\mu \psi^{j\beta} \right] \\
&+ \int d^4x \left[\tau^a \, \partial_\mu A^{h,a}_\mu + \bar{\eta}^a \, \partial_\mu D^{ab}_\mu(A^h) \eta^b \right] \\
&+ \int d^4x \left[\bar{\varphi}^{aI} \, \partial_\mu D^{ab}_\mu(A^h) \varphi^{bI} - \bar{\omega}^{aI} \, \partial_\mu D^{ab}_\mu(A^h) \omega^{bI} + M^{aI}_\mu D^{ab}_\mu(A^h) \varphi^{bI} \right. \\
&+ \left. V^{aI}_\mu D^{ab}_\mu(A^h) \bar{\varphi}^{bI} - N^{aI}_\mu D^{ab}_\mu(A^h) \omega^{bI} + U^{aI}_\mu D^{ab}_\mu(A^h) \bar{\omega}^{bI} - M^{aI}_\mu V^{aI}_\mu + N^{aI}_\mu U^{aI}_\mu \right] \\
&+ \int d^4x \left[\bar{\lambda}^a_{\hat{I}} \left(-\partial_\mu D^{ab}_\mu(A^h) \right) \lambda^{b\hat{I}} + \bar{\zeta}^a_{\hat{I}} \left(-\partial_\mu D^{ab}_\mu(A^h) \right) \zeta^{b\hat{I}} \right. \\
&+ \left. \bar{\Lambda}^j_{\alpha\hat{I}} \, \bar{\psi}^{h,i\alpha} \, T^{a,ij} \, \lambda^{a\hat{I}} + \bar{\Pi}^j_{\alpha\hat{I}} \, \bar{\psi}^{h,i\alpha} \, T^{a,ij} \, \zeta^{a\hat{I}} + \Pi^{i\hat{I}}_\alpha \, \bar{\zeta}^a_{\hat{I}} \, T^{a,ij} \, \psi^{h,j\alpha} + \Lambda^{i\hat{I}}_\alpha \, \bar{\lambda}^a_{\hat{I}} \, T^{a,ij} \, \psi^{h,j\alpha} \right] \\
&+ \int d^4x \left[\frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu - M^2 \left(\bar{\varphi}^{aI} \varphi^{aI} - \bar{\omega}^{aI} \omega^{aI} \right) - m_\psi \, \bar{\psi}^{i\alpha} \psi^i_\alpha \right. \\
&+ \left. w^2 \left(\bar{\lambda}^{a\hat{I}} \lambda^a_{\hat{I}} + \bar{\zeta}^{a\hat{I}} \zeta^a_{\hat{I}} \right) \right] . \qquad (275)
\end{aligned}
$$

Here, the first integral is the Yang-Mills action in the LCG coupled to fermionic matter in the fundamental representation. The second integral is a constraint indicating the transversality of A^h_μ. The third integral is the local version of the invariant Zwanziger horizon function generalized by the set (M, N, U, V) of external sources. The fourth integral, analogous to the third one, is the local version of the horizon function for the matter generalized by the set of sources $(\Pi, \bar{\Pi}, \Lambda, \bar{\Lambda})$. Finally the fifth integral has all invariant lower-dimension scalar operators[29] that can be constructed in this theory. The parameter w^2 is a mass squared parameter as well as m^2 and M^2. Notice also that is immediate to verify that $\bar{\psi}\psi = \bar{\psi}^h \psi^h$. Then, it was not necessary to include an invariant lower -dimension operators with ψ^h.

The advantage of working with S_1 instead of S^{local} is that S_1 has a richer symmetry content and it will facilitate our task in the next chapter, which is the algebraic

[29] For lower-dimension operators we mean operators with dimension lower than the Euclidean space, in this case, lower than four. In action (275), the operator $\bar{\psi}\psi$ has dimension three and the remaining operators have dimension two.

renormalization proof of the theory. In fact, we can observe that

$$sS_1 = \delta S_1 = \hat{\delta} S_1 = Q_{IJ}(S_1) = \hat{Q}_{\hat{I}\hat{J}}(S_1) = 0\,, \tag{276}$$

reminding that, as said before and as will be clear later, operators s, δ and $\hat{\delta}$ can be linearly combined in order to obtain an extended BRST operator.

5.5.2 The BRST sources and the Slavnov-Taylor identity

There are several local composite operators that can be introduced in the theory. The requirement of introducing a specific local composite operator is sometimes related to the kind of Green functions desired and sometimes it is justified only *a posteriori* in the study of the renormalization. In general, when there are nonlinear symmetries involved, the renormalization of such symmetries requires that such nonlinear transformations be defined from the beginning in the starting point action. This is the case, for example, of the BRST symmetry. From eqs. (217), (218), (236) and (251) we realize that the transformations of the fields A^a_μ, c^a, ψ^i, $\bar{\psi}^i$ and ξ^a are nonlinear. These nonlinearity are taken into account in the theory by defining the following action

$$S_2 = S_1 + S_{BRST}\,, \tag{277}$$

with S_1 being given by (275) and S_{BRST} by

$$\begin{aligned} S_{BRST} &= \int d^4x \left[\Omega^a_\mu (sA^a_\mu) + L^a(sc^a) + K^a(s\xi^a) + (s\bar{\psi}^{i\alpha})\Upsilon^i_\alpha + \bar{\Upsilon}^i_\alpha (s\psi^{i\alpha})\right] \\ &= \int d^4x \left[\Omega^a_\mu D^{ab}_\mu(A)c^b + \frac{g}{2} f^{abc} L^a c^b c^c + K^a g^{ab}(\xi) c^b \right. \\ &- \left. i\bar{\psi}^{i\alpha}\, T^{a,ij}\, c^a\, \Upsilon^j_\alpha - i\bar{\Upsilon}^i_\alpha\, T^{a,ij}\, c^a\, \psi^{j\alpha}\right], \end{aligned} \tag{278}$$

where $(\Omega^a_\mu, L^a, K^a, \Upsilon^i_\alpha, \bar{\Upsilon}^i_\alpha)$ form a set of external sources invariant by the action of the BRST operator:

$$s\Omega^a_\mu = sL^a = sK^a = s\Upsilon^i_\alpha = \bar{\Upsilon}^i_\alpha = 0\,. \tag{279}$$

The main properties of these sources can be found in Tables (3) and (4). The BRST invariance can now be written as a functional identity known as Slavnov-Taylor identity:

$$\mathcal{S}(S_2) = 0\,, \tag{280}$$

where

$$\mathcal{S}(F) \;=\; \int d^4x \left(\frac{\delta F}{\delta \Omega^a_\mu}\frac{\delta F}{\delta A^a_\mu} + \frac{\delta F}{\delta L^a}\frac{\delta F}{\delta c^a} + \frac{\delta F}{\delta K^a}\frac{\delta F}{\delta \xi^a} + ib^a\frac{\delta F}{\delta \xi^a} \right), \tag{281}$$

with F being an arbitrary functional of the fields and sources.

5.5.3 Introducing additional relevant local composite operators

There are still some relevant composite operators that can be introduced in the theory. The relevance of such operators will be clear in the construction of the symmetry content of the theory and in the proof of its renormalization in the next chapter.

The gauge-invariant composite fields A^h_μ, ψ^h and $\bar{\psi}^h$ are examples of such operators. Therefore, let us define them in the theory as follows

$$S_3 = S_2 + \int d^4x \left(\mathcal{J}^a_\mu \, A^{h,a}_\mu + \bar{\psi}^{h,i}_\alpha \, \Theta^{i\alpha} + \bar{\Theta}^i_\alpha \, \psi^{h,i\alpha} \right), \tag{282}$$

where S_2 is given by (277). The local sources $(\mathcal{J}^a_\mu, \Theta^i_\alpha, \bar{\Theta}^i_\alpha)$ were employed in order to define the composite operators $(A^h_\mu, \psi^h, \bar{\psi}^h)$. Naturally, the BRST invariance is guaranteed by demanding simply that

$$s\mathcal{J}^a_\mu = s\Theta^i_\alpha = s\bar{\Theta}^i_\alpha = 0\,. \tag{283}$$

The gauge-invariant mass terms that can be constructed with $(A^h_\mu, \psi^h, \bar{\psi}^h)$ and with the two families of localizing auxiliary fields $(\varphi, \bar{\varphi}, \omega, \bar{\omega})$ and $(\lambda, \bar{\lambda}, \zeta, \bar{\zeta})$ are also composite local operators. Then, in the last integral of eq. (275), let us replace the mass parameters by local sources, obtaining the following term:

$$\begin{aligned} S(J, J_\psi, J_\varphi, J_\lambda) \;&=\; \int d^4x \left[J \, A^{h,a}_\mu A^{h,a}_\mu + J_\psi \, \bar{\psi}^i_\alpha \psi^{i\alpha} + J_\varphi \, (\bar{\varphi}^{aI}\varphi^{aI} - \bar{\omega}^{aI}\omega^{aI}) \right. \\ &+ \left. J_\lambda \, (\bar{\lambda}^{a\hat{I}}\lambda^a_{\hat{I}} + \bar{\zeta}^{a\hat{I}}\zeta^a_{\hat{I}}) \right]. \end{aligned} \tag{284}$$

Here, $(J, J_\psi, J_\varphi, J_\lambda)$ is a set of sources replacing the masses (m^2, m_ψ, M^2, w^2). Indeed, at the end, these sources are taken in their physical values

$$\begin{aligned} J(x)\Big|_{phys} \;&=\; \frac{m^2}{2}\,, \qquad J_\psi(x)\Big|_{phys} = -m_\psi\,, \\ J_\varphi(x)\Big|_{phys} \;&=\; -M^2\,, \qquad J_\lambda(x)\Big|_{phys} = w^2\,. \end{aligned} \tag{285}$$

For BRST invariance, we also have

$$sJ = sJ_\psi = sJ_\varphi = sJ_\lambda = 0\,. \tag{286}$$

At this point, the action we are dealing with is action S_3, given by eq. (282) but with the masses replaced by the local sources $(J, J_\psi, J_\varphi, J_\lambda)$, i.e. with the last integral of (275) replaced by the term (284). Of course, at the physical limits (285), action S_3 is recovered. In order to clarify the notation, let us define a new action

$$S_4 \equiv S_3\Big|_{masses\,\rightarrow\,sources}\,. \tag{287}$$

As will be clear in the next sections, this model has several symmetries and some of them are nonlinear as the BRST. Then, it is essential to introduce some extra local composite operators. Thus, we define

$$S_5 = S_4 + S_{extra}\,, \tag{288}$$

with S_{extra} being given by

$$\begin{aligned} S_{extra} &= \int d^4x\, \Big[\Xi^a_\mu\, D^{ab}_\mu(A^h)\eta^b + \Gamma^{ab}\eta^a\eta^b - X^I\,\eta^a\bar{\omega}^{aI} - Y^I\,\eta^a\bar{\varphi}^{aI} - \bar{X}^{abI}\,\eta^a\omega^{bI} \\ &- \bar{Y}^{abI}\,\eta^a\varphi^{bI} - Z^{\hat{I}}\,\eta^a\bar{\lambda}^a_{\hat{I}} - W^{\hat{I}}\,\eta^a\bar{\zeta}^a_{\hat{I}} - \bar{Z}^{ab\hat{I}}\,\eta^a\lambda^b_{\hat{I}} - \bar{W}^{ab\hat{I}}\,\eta^a\zeta^b_{\hat{I}} \\ &+ \Phi^i_\alpha\,\bar{\psi}^{h,j\alpha}\,T^{a,ji}\,\eta^a + \bar{\Phi}^{i\alpha}\,T^{a,ij}\,\eta^a\,\psi^{h,j}_\alpha + K_\varphi\,\bar{\omega}^{aI}\varphi^{aI} + K_\lambda\,\bar{\lambda}^{a\hat{I}}\zeta^a_{\hat{I}} \Big]\,. \end{aligned} \tag{289}$$

This extra term is invariant by the three modalities of BRST transformations $(s, \delta, \hat{\delta})$ if the new external local sources transform as

$$s(\Xi, \Gamma, \Phi, \bar{\Phi}, X, \bar{X}, Y, \bar{Y}, Z, \bar{Z}, W, \bar{W}, K_\varphi, K_\lambda) = 0\,; \tag{290}$$

$$\begin{aligned} &\delta(\Xi, \Gamma, \Phi, \bar{\Phi}, Z, \bar{Z}, W, \bar{W}, K_\lambda) = 0\,, \\ &\delta Y^I = X^I\,, \qquad \delta X^I = 0\,, \\ &\delta\bar{X}^{abI} = -\bar{Y}^{abI}\,, \qquad \delta\bar{Y}^{abI} = 0\,, \\ &\delta J_\varphi = K_\varphi\,, \qquad \delta K_\varphi = 0\,; \end{aligned} \tag{291}$$

$$\begin{aligned} &\hat{\delta}(\Xi, \Gamma, , \Phi, \bar{\Phi}X, \bar{X}, Y, \bar{Y}, K_\varphi) = 0\,, \\ &\hat{\delta}Z^{\hat{I}} = -W^{\hat{I}}\,, \qquad \hat{\delta}W^{\hat{I}} = 0\,, \\ &\hat{\delta}\bar{W}^{ab\hat{I}} = \bar{Z}^{ab\hat{I}}\,, \qquad \hat{\delta}\bar{Z}^{ab\hat{I}} = 0\,, \\ &\hat{\delta}J_\lambda = K_\lambda\,, \qquad \hat{\delta}K_\lambda = 0\,. \end{aligned} \tag{292}$$

Note that we have extended the δ and $\hat{\delta}$ transformations to the sources J_φ and J_λ, respectively.

Action S_5 can also be supplemented by terms allowed by power-counting (PC),

$$S_6 = S_5 + S_{PC} \,, \tag{293}$$

with

$$S_{PC} \;=\; \int d^4x \left[\frac{\kappa_1}{2}\, J^2 + \kappa_2\, J J_\psi^2 + \kappa_3\, J_\psi^4 + \kappa_4\, J_\psi (\bar{\Lambda}^{i\alpha\hat{I}}\Lambda^i_{\alpha\hat{I}} - \bar{\Pi}^{i\alpha\hat{I}}\Pi^i_{\alpha\hat{I}})\right], \tag{294}$$

where $(\kappa_1, \kappa_2, \kappa_3, \kappa_4)$ are coefficients necessary in order to reabsorb the UV divergences present in the vacuum correlation functions:

$$\Big\langle (A^h A^h)_x (A^h A^h)_y \Big\rangle \;,$$

$$\Big\langle (\bar{\psi}\psi)_x (\bar{\psi}\psi)_y (A^h A^h)_z \Big\rangle \;,$$

$$\Big\langle (\bar{\psi}\psi)_x (\bar{\psi}\psi)_y (\bar{\psi}\psi)_z (\bar{\psi}\psi)_w \Big\rangle \;,$$

$$\Big\langle (\bar{\psi}\psi)_x \Big((\bar{\psi}^h T\lambda)_y (\bar{\lambda} T\psi^h)_z - (\bar{\psi}^h T\zeta)_y (\bar{\zeta} T\psi^h)_z \Big) \Big\rangle \;,$$

respectively. Other combinations like J_φ^2, J_λ^2, $J_\psi^2 J_\varphi$, $J_\psi^2 J_\lambda$, $J J_\varphi$, $J J_\lambda$ and $J_\varphi J_\lambda$ are forbidden by the extended BRST transformations (291), (292) for J_φ and J_λ. This means that the correlation functions

$$\langle (\bar{\varphi}\varphi - \bar{\omega}\omega)_x (\bar{\varphi}\varphi - \bar{\omega}\omega)_y \rangle \;,$$

$$\Big\langle (\bar{\lambda}\lambda + \bar{\zeta}\zeta)_x (\bar{\lambda}\lambda + \bar{\zeta}\zeta)_y \Big\rangle \;,$$

$$\Big\langle (\bar{\psi}\psi)_x (\bar{\psi}\psi)_y (\bar{\varphi}\varphi - \bar{\omega}\omega)_z \Big\rangle \;,$$

$$\Big\langle (\bar{\psi}\psi)_x (\bar{\psi}\psi)_y (\bar{\lambda}\lambda + \bar{\zeta}\zeta)_z \Big\rangle \;,$$

$$\Big\langle (A^h A^h)_x (\bar{\varphi}\varphi - \bar{\omega}\omega)_y \Big\rangle \;,$$

$$\Big\langle (A^h A^h)_x (\bar{\lambda}\lambda + \bar{\zeta}\zeta)_y \Big\rangle \;,$$

$$\Big\langle (\bar{\varphi}\varphi - \bar{\omega}\omega)_x (\bar{\lambda}\lambda + \bar{\zeta}\zeta)_y \Big\rangle$$

do not present UV divergences.

5.6 **Extended BRST symmetry**

In this subsection, we will introduce a very useful trick, for more details see (156), which corresponds to extend the BRST transformations, (251), on the gauge parameter α. These extend transformations will give us the possibility of controlling the dependence of the Green functions from the gauge parameter α at quantum level, that is

$$s\alpha = \chi\,, \qquad s\chi = 0\,, \tag{295}$$

where χ is a Grassmann parameter with ghost number 1. Of course, this parameter can always be set to zero in order to restore the original action. In order to take into account this new BRST doublet we define the following action

$$\Sigma = S_6 - \frac{i}{2}\int d^4x\; \chi\,\bar{c}^a b^a\,, \tag{296}$$

where S_6 is given by (293). Action Σ above is invariant by the actions of the three BRST operators,

$$s\Sigma = \delta\Sigma = \hat{\delta}\Sigma = 0\,. \tag{297}$$

We can get them together in an unique BRST extended operator. Therefore, let us consider the operator

$$Q_{\epsilon,\hat{\epsilon}} = s + \epsilon\,\delta + \hat{\epsilon}\,\hat{\delta}\,, \tag{298}$$

which is a linear combination of the three BRST operators with ϵ and $\hat{\epsilon}$ being arbitrary coefficients. The nilpotency of $Q_{\epsilon,\hat{\epsilon}}$ is obtained from

$$s^2 = 0\,, \qquad \delta^2 = 0\,, \qquad \hat{\delta}^2 = 0\,, \qquad \{s,\delta\} = 0\,, \qquad \{s,\hat{\delta}\} = 0\,, \qquad \{\delta,\hat{\delta}\} = 0\,. \tag{299}$$

As the coefficients ϵ and $\hat{\epsilon}$ are completely arbitrary we will choose for simplicity $\epsilon = \hat{\epsilon} = 1$ and then our extended BRST will be given by

$$Q \equiv Q_{\epsilon=1,\hat{\epsilon}=1} = s + \delta + \hat{\delta}\,. \tag{300}$$

Thus, the extended BRST transformations are established as

- The nonlinear BRST transformations:

$$
\begin{aligned}
QA^a_\mu &= -D^{ab}_\mu(A)c^b\,,\\
Qc^a &= \frac{g}{2}f^{abc}c^b c^c\,,\\
Q\psi^i_\alpha &= -iT^{a,ij}c^a\psi^j_\alpha\,,\\
Q\bar{\psi}^i_\alpha &= -i\bar{\psi}^j_\alpha T^{a,ji}c^a\,,\\
Q\xi^a &= g^{ab}(\xi)c^b\,;
\end{aligned}
\tag{301}
$$

- The BRST doublets:

$$
\begin{aligned}
&Q\bar{c}^a = ib^a\,, && Qb^a = 0\,, && Q\alpha = \chi\,, && Q\chi = 0\,,\\
&Q\varphi^{aI} = \omega^{aI}\,, && Q\omega^{aI} = 0\,, && Q\bar{\omega}^{aI} = \bar{\varphi}^{aI}\,, && Q\bar{\varphi}^{aI} = 0\,,\\
&QN^{aI}_\mu = M^{aI}\,, && QM^{aI}_\mu = 0\,, && QV^{aI}_\mu = U^{aI}\,, && QU^{aI}_\mu = 0\,,\\
&Q\lambda^{a\hat{I}} = \zeta^{a\hat{I}}\,, && Q\zeta^{a\hat{I}} = 0\,, && Q\bar{\zeta}^{a\hat{I}} = \bar{\lambda}^{a\hat{I}}\,, && Q\bar{\lambda}^{a\hat{I}} = 0\,,\\
&Q\Lambda^{i\hat{I}}_\alpha = \Pi^{i\hat{I}}_\alpha\,, && Q\Pi^{i\hat{I}}_\alpha = 0\,, && Q\bar{\Pi}^{i\hat{I}}_\alpha = \bar{\Lambda}^{i\hat{I}}_\alpha\,, && Q\bar{\Lambda}^{i\hat{I}}_\alpha = 0\,,\\
&QJ_\varphi = K_\varphi\,, && QK_\varphi = 0\,, && QJ_\lambda = K_\lambda\,, && QK_\lambda = 0\,,\\
&QY^I = X^I\,, && QX^I = 0\,, && Q\bar{X}^{abI} = -\bar{Y}^{abI}\,, && Q\bar{Y}^{abI} = 0\,,\\
&QZ^{\hat{I}} = -W^{\hat{I}}\,, && QW^{\hat{I}} = 0\,, && Q\bar{W}^{ab\hat{I}} = \bar{Z}^{ab\hat{I}}\,, && Q\bar{Z}^{ab\hat{I}} = 0\,;
\end{aligned}
\tag{302}
$$

- The BRST singlets:

$$
Q\,\Big(\eta^a,\bar{\eta}^a,\tau^a,J,J_\psi,\Omega^a_\mu,L^a,K^a,\varUpsilon^i_\alpha,\bar{\varUpsilon}^i_\alpha,\mathcal{J}^a_\mu,\Theta^i_\alpha,\bar{\Theta}^i_\alpha,\Xi^a_\mu,\Gamma^{ab},\Phi^i_\alpha,\bar{\Phi}^i_\alpha\Big) = 0\,.
\tag{303}
$$

Once we have the complete BRST transformations we can write the action Σ explicitly as

$$
\begin{aligned}
\Sigma &= \int d^4x \left[\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + i\bar{\psi}^{i\alpha} (\gamma_\mu)_{\alpha\beta} D^{ij}_\mu \psi^{j\beta} + J\, A^{h,a}_\mu A^{h,a}_\mu + J_\psi\, \bar{\psi}^{i\alpha} \psi^i_\alpha \right. \\
&+ (\mathcal{J}^a_\mu - \partial_\mu \tau^a) A^{h,a}_\mu + (\Xi^a_\mu - \partial_\mu \bar{\eta}^a) D^{ab}_\mu (A^h) \eta^b + \bar{\psi}^{h,i}_\alpha \Theta^{i\alpha} + \bar{\Theta}^i_\alpha \psi^{h,i\alpha} \\
&+ \left. \Gamma^{ab} \eta^a \eta^b + \Phi^i_\alpha \bar{\psi}^{h,j\alpha}\, T^{a,ji} \eta^a + \bar{\Phi}^i_\alpha\, T^{a,ij} \eta^a \psi^{h,j\alpha} + \frac{\kappa_1}{2}\, J^2 + \kappa_2\, J J^2_\psi + \kappa_3\, J^4_\psi \right] \\
&+ Q \int d^4x \left[-\frac{i}{2}\, \alpha\, \bar{c}^a b^a - (\Omega^a_\mu + \partial_\mu \bar{c}^a) A^a_\mu + L^a c^a - K^a \xi^a + \bar{\psi}^{i\alpha} \Upsilon^i_\alpha + \bar{\Upsilon}^{i\alpha} \psi^i_\alpha \right. \\
&+ \bar{\omega}^{aI}\, \partial_\mu D^{ab}_\mu (A^h) \varphi^{bI} + N^{aI}_\mu\, D^{ab}_\mu (A^h) \varphi^{bI} + V^{aI}_\mu\, D^{ab}_\mu (A^h) \bar{\omega}^{bI} + J_\varphi\, \bar{\omega}^{aI} \varphi^{aI} - N^{aI}_\mu V^{aI}_\mu \\
&- \bar{\zeta}^{a\hat{I}}\, \partial_\mu D^{ab}_\mu (A^h) \lambda^b_{\hat{I}} + \bar{\Pi}^{j\hat{I}}_\alpha\, \bar{\psi}^{h,i\alpha}\, T^{a,ij}\, \lambda^a_{\hat{I}} + \Lambda^{i\hat{I}}_\alpha\, \bar{\zeta}^a_{\hat{I}}\, T^{a,ij}\, \psi^{h,j\alpha} + J_\lambda\, \bar{\zeta}^a_{\hat{I}} \lambda^{a\hat{I}} + \kappa_4\, J_\psi\, \bar{\Pi}^{i\alpha\hat{I}} \Lambda^i_{\alpha\hat{I}} \\
&- \left. Y^I \eta^a \bar{\omega}^{aI} + \bar{X}^{abI} \eta^a \varphi^{bI} - \bar{W}^{ab\hat{I}} \eta^a \lambda^b_{\hat{I}} + Z^{\hat{I}} \eta^a \bar{\zeta}^a_{\hat{I}} \right] . \qquad (304)
\end{aligned}
$$

$$
\begin{aligned}
\Sigma &= \int d^4x \left[\frac{1}{4} F^a_{\mu\nu}F^a_{\mu\nu} + i\bar{\psi}^{i\alpha}(\gamma_\mu)_{\alpha\beta}D^{ij}_\mu\psi^{j\beta} + J\, A^{h,a}_\mu A^{h,a}_\mu + J_\psi\, \bar{\psi}^{i\alpha}\psi^i_\alpha \right. \\
&+ (\mathcal{J}^a_\mu - \partial_\mu\tau^a)A^{h,a}_\mu + (\Xi^a_\mu - \partial_\mu\bar{\eta}^a)D^{ab}_\mu(A^h)\eta^b + \bar{\psi}^{h,i}_\alpha\Theta^{i\alpha} + \bar{\Theta}^i_\alpha\psi^{h,i\alpha} \\
&+ \Gamma^{ab}\eta^a\eta^b + \Phi^i_\alpha\bar{\psi}^{h,j\alpha}\,T^{a,ji}\eta^a + \bar{\Phi}^i_\alpha\,T^{a,ij}\eta^a\psi^{h,j\alpha} + \frac{\kappa_1}{2}\,J^2 + \kappa_2\,JJ^2_\psi + \kappa_3\,J^4_\psi \\
&- \frac{i}{2}\,\chi\,\bar{c}^a b^a + \frac{\alpha}{2}\,b^a b^a + ib^a\,\partial_\mu A^a_\mu - (\Omega^a_\mu + \partial_\mu\bar{c}^a)D^{ab}_\mu(A)c^b + \frac{g}{2}f^{abc}L^a c^b c^c \\
&+ K^a\,g^{ab}(\xi)c^b - i\bar{\psi}^{i\alpha}\,T^{a,ij}c^a\Upsilon^j_\alpha - i\bar{\Upsilon}^i_\alpha\,T^{a,ij}c^a\psi^{j\alpha} + \bar{\varphi}^{aI}\,\partial_\mu D^{ab}_\mu(A^h)\varphi^{bI} \\
&- \bar{\omega}^{aI}\,\partial_\mu D^{ab}_\mu(A^h)\omega^{bI} + M^{aI}_\mu\,D^{ab}_\mu(A^h)\varphi^{bI} - N^{aI}_\mu\,D^{ab}_\mu(A^h)\omega^{bI} + U^{aI}_\mu\,D^{ab}_\mu(A^h)\bar{\omega}^{bI} \\
&+ V^{aI}_\mu\,D^{ab}_\mu(A^h)\bar{\varphi}^{bI} + K_\varphi\,\bar{\omega}^{aI}\varphi^{aI} + J_\varphi(\bar{\varphi}^{aI}\varphi^{aI} - \bar{\omega}^{aI}\omega^{aI}) - M^{aI}_\mu V^{aI}_\mu + N^{aI}_\mu U^{aI}_\mu \\
&- \bar{\lambda}^{a\hat{I}}\,\partial_\mu D^{ab}_\mu(A^h)\lambda^b_{\hat{I}} - \bar{\zeta}^{a\hat{I}}\,\partial_\mu D^{ab}_\mu(A^h)\zeta^b_{\hat{I}} + \bar{\Lambda}^{j\hat{I}}_\alpha\,\bar{\psi}^{h,i\alpha}\,T^{a,ij}\lambda^a_{\hat{I}} - \bar{\Pi}^{j\hat{I}}_\alpha\,\bar{\psi}^{h,i\alpha}\,T^{a,ij}\zeta^a_{\hat{I}} \\
&+ \Pi^{i\hat{I}}_\alpha\bar{\zeta}^a_{\hat{I}}\,T^{a,ij}\psi^{h,j\alpha} + \Lambda^{i\hat{I}}_\alpha\bar{\lambda}^a_{\hat{I}}\,T^{a,ij}\psi^{h,j\alpha} + K_\lambda\,\bar{\zeta}^{a\hat{I}}\lambda^a_{\hat{I}} + J_\lambda\,(\bar{\lambda}^{a\hat{I}}\lambda^a_{\hat{I}} + \bar{\zeta}^{a\hat{I}}\zeta^a_{\hat{I}}) \\
&+ \kappa_4\,J_\psi(\bar{\Lambda}^{i\alpha\hat{I}}\Lambda^i_{\alpha\hat{I}} - \bar{\Pi}^{i\alpha\hat{I}}\Pi^i_{\alpha\hat{I}}) - X^I\eta^a\bar{\omega}^{aI} - Y^I\eta^a\bar{\varphi}^{aI} - \bar{Y}^{abI}\eta^a\varphi^{abI} - \bar{X}^{abI}\eta^a\omega^{abI} \\
&- \left. \bar{Z}^{ab\hat{I}}\eta^a\lambda^b_{\hat{I}} - \bar{W}^{ab\hat{I}}\eta^a\zeta^b_{\hat{I}} - W^{\hat{I}}\eta^a\bar{\zeta}^a_{\hat{I}} - Z^{\hat{I}}\eta^a\bar{\lambda}^a_{\hat{I}} \right] . \qquad (305)
\end{aligned}
$$

The action of physical interest is obtained from Σ by taken the particular values (254), (265) and (285) and setting the remaining sources and the Grassmannian parameter χ to zero. Such physical action is therefore a particular case of Σ, which enjoys a very rich symmetry content, as we shall see in the next section, being thus the suitable action to be considered in the study of renormalization.

It is also instructive to establish the mass dimension and the quantum numbers of fields and sources of the theory. These numbers are displayed in the Tables (1) – (5). In these tables is also displayed the so-called “nature” of each field and source. By nature we mean the commuting (C) or anticommuting (A) character of each field and source, which is determined by the parity of the summation of ghost numbers and the e-charge (or spinor number). If, for a certain field or source, the result of the summation is an even number, the considered field/source is commuting, otherwise it is anticommuting ones.

Table 1 - First set of quantum numbers of fields.

Fields	A	b	c	$\bar{c}$	$\bar{\psi}$	ψ	ξ	$\bar{\varphi}$	φ	$\bar{\omega}$	ω	α	χ	τ
Dimension	1	2	0	2	$\frac{3}{2}$	$\frac{3}{2}$	0	1	1	1	1	0	0	2
c-ghost number	0	0	1	-1	0	0	0	0	0	-1	1	0	1	0
η-ghost number	0	0	0	0	0	0	0	0	0	0	0	0	0	0
e-charge	0	0	0	0	-1	1	0	0	0	0	0	0	0	0
$U(4(N^2-1))$-charge	0	0	0	0	0	0	0	-1	1	-1	1	0	0	0
$U(4N)$-charge	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Nature	C	C	A	A	A	A	C	C	C	A	A	C	A	C

Source: The author, 2020.

Table 2 - Second set of quantum numbers of fields.

Fields	η	$\bar{\eta}$	$\bar{\lambda}$	λ	$\bar{\zeta}$	ζ
Dimension	0	2	1	1	1	1
c-ghost number	0	0	0	0	-1	1
η-ghost number	1	-1	0	0	0	0
e-charge	0	0	-1	1	-1	1
$U(4(N^2-1))$-charge	0	0	0	0	0	0
$U(4N)$-charge	0	0	-1	1	-1	1
Nature	A	A	A	A	C	C

Source: The author, 2020.

Table 3 - First set of quantum numbers of sources.

Sources	Ω	L	K	J	J_ψ	$\mathcal{J}$	M	N	U	V	J_φ	K_φ	Ξ	X	Y
Dimension	3	4	4	2	1	3	2	2	2	2	2	2	3	3	3
c-ghost number	-1	-2	-1	0	0	0	0	-1	1	0	0	1	0	1	0
η-ghost number	0	0	0	0	0	0	0	0	0	0	0	0	-1	-1	-1
e-charge	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$U(4(N^2-1))$-charge	0	0	0	0	0	0	-1	-1	1	1	0	0	0	1	1
$U(4N)$-charge	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Nature	A	C	A	C	C	C	C	A	A	C	C	A	A	C	A

Source: The author, 2020.

Table 4 - Second set of quantum numbers of sources.

Sources	$\bar{X}$	$\bar{Y}$	$\bar{\varUpsilon}$	$\varUpsilon$	$\bar{\Theta}$	Θ	$\bar{\Lambda}$	Λ	$\bar{\Pi}$	Π
Dimension	3	3	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
c-ghost number	-1	0	-1	-1	0	0	0	0	-1	1
η-ghost number	-1	-1	0	0	0	0	0	0	0	0
e-charge	0	0	-1	1	-1	1	0	0	0	0
$U(4(N^2-1))$-charge	-1	-1	0	0	0	0	0	0	0	0
$U(4N)$-charge	0	0	0	0	0	0	-1	1	-1	1
Nature	C	A	C	C	A	A	C	C	A	A

Source: The author, 2020.

Table 5 - Quantum numbers of the extra sources.

Extra Sources	J_λ	K_λ	$\bar{Z}$	$\bar{W}$	Z	W	$\bar{\Phi}$	Φ	Γ
Dimension	2	2	3	3	3	3	$\frac{5}{2}$	$\frac{5}{2}$	4
c-ghost number	0	1	0	-1	0	1	0	0	0
η-ghost number	0	0	-1	-1	-1	-1	-1	-1	-2
e-charge	0	0	-1	-1	1	1	-1	1	0
$U(4(N^2-1))$-charge	0	0	0	0	0	0	0	0	0
$U(4N)$-charge	0	0	-1	-1	1	1	0	0	0
Nature	C	A	C	A	C	A	A	A	C

Source: The author, 2020.

5.7 Ward identities

The classical extended action Σ defined by (305) enjoys a large set of symmetries specified by the following Ward identities,

- The Slavnov-Taylor identity:

$$\mathcal{B}(\Sigma) = 0\,, \tag{306}$$

with

$$\begin{aligned}
\mathcal{B}(\Sigma) &= \int d^4x \left(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta\Sigma}{\delta\Omega^a_\mu} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta\Sigma}{\delta L^a} + \frac{\delta\Sigma}{\delta\xi^a}\frac{\delta\Sigma}{\delta K^a} + \frac{\delta\Sigma}{\delta\Upsilon^i_\alpha}\frac{\delta\Sigma}{\delta\bar{\psi}^{i,\alpha}} + \frac{\delta\Sigma}{\delta\bar{\Upsilon}^i_\alpha}\frac{\delta\Sigma}{\delta\psi^{i,\alpha}} + ib^a\frac{\delta\Sigma}{\delta\bar{c}^a} \right.\\
&+ \omega^{aI}\frac{\delta\Sigma}{\delta\varphi^{aI}} + \bar{\varphi}^{aI}\frac{\delta\Sigma}{\delta\bar{\omega}^{aI}} + M^{aI}_\mu\frac{\delta\Sigma}{\delta N^{aI}_\mu} + U^{aI}_\mu\frac{\delta\Sigma}{\delta V^{aI}_\mu} + K_\varphi\frac{\delta\Sigma}{\delta J_\varphi} + X^I\frac{\delta\Sigma}{\delta Y^I}\\
&- \bar{Y}^{abI}\frac{\delta\Sigma}{\delta\bar{X}^{abI}} + \bar{\lambda}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} + \zeta^{a\hat{I}}\frac{\delta\Sigma}{\delta\lambda^a_{\hat{I}}} + \bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} + \Pi^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Lambda^i_{\alpha\hat{I}}} + K_\lambda\frac{\delta\Sigma}{\delta G_\lambda}\\
&\left. - W^{\hat{I}}\frac{\delta\Sigma}{\delta Z_{\hat{I}}} + \bar{Z}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}} \right) + \chi\frac{\partial\Sigma}{\partial\alpha}\,.
\end{aligned} \tag{307}$$

From the Slavnov-Taylor identity (307), we can define the well-known linearized Slavnov-Taylor operator $\mathcal{B}_\Sigma$ (76),

$$\begin{aligned}
\mathcal{B}_\Sigma &= \int d^4x \left(\frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta\Omega^a_\mu} + \frac{\delta\Sigma}{\delta\Omega^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta\xi^a}\frac{\delta}{\delta K^a} + \frac{\delta\Sigma}{\delta K^a}\frac{\delta}{\delta\xi^a} \right.\\
&+ \frac{\delta\Sigma}{\delta\Upsilon^i_\alpha}\frac{\delta}{\delta\bar{\psi}^{i,\alpha}} + \frac{\delta\Sigma}{\delta\bar{\psi}^{i,\alpha}}\frac{\delta}{\delta\Upsilon^i_\alpha} + \frac{\delta\Sigma}{\delta\bar{\Upsilon}^i_\alpha}\frac{\delta}{\delta\psi^{i,\alpha}} + \frac{\delta\Sigma}{\delta\psi^{i,\alpha}}\frac{\delta}{\delta\bar{\Upsilon}^i_\alpha} + ib^a\frac{\delta}{\delta\bar{c}^a} + \omega^{aI}\frac{\delta}{\delta\varphi^{aI}}\\
&+ \bar{\varphi}^{aI}\frac{\delta}{\delta\bar{\omega}^{aI}} + M^{aI}_\mu\frac{\delta}{\delta N^{aI}_\mu} + U^{aI}_\mu\frac{\delta}{\delta V^{aI}_\mu} + K_\varphi\frac{\delta}{\delta J_\varphi} + X^I\frac{\delta}{\delta Y^I} - \bar{Y}^{abI}\frac{\delta}{\delta\bar{X}^{abI}}\\
&+ \bar{\lambda}^{a\hat{I}}\frac{\delta}{\delta\bar{\zeta}^a_{\hat{I}}} + \zeta^{a\hat{I}}\frac{\delta}{\delta\lambda^a_{\hat{I}}} + \bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} + \Pi^{i\alpha\hat{I}}\frac{\delta}{\delta\Lambda^i_{\alpha\hat{I}}} + K_\lambda\frac{\delta}{\delta J_\lambda} - W^{\hat{I}}\frac{\delta}{\delta Z_{\hat{I}}}\\
&\left. + \bar{Z}^{ab\hat{I}}\frac{\delta}{\delta\bar{W}^{ab}_{\hat{I}}} \right) + \chi\frac{\partial}{\partial\alpha}\,,
\end{aligned} \tag{308}$$

with

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0\,. \tag{309}$$

In the next chapter we will solve the cohomology problem of the operator $\mathcal{B}_\Sigma$ in order to obtain the most general counterterm that can be added to any order of loop correction.

- The antighost equation:

$$\frac{\delta\Sigma}{\delta\bar{c}^a} + \partial_\mu \frac{\delta\Sigma}{\delta\Omega^a_\mu} = \frac{i}{2}\chi b^a \,. \tag{310}$$

- The equation of motion of the Lagrange multiplier b^a:

$$\frac{\delta\Sigma}{\delta b^a} = i\partial_\mu A^a_\mu + \alpha b^a - \frac{i}{2}\chi\bar{c}^a \,, \tag{311}$$

corresponding to the linear covariant gauge-fixing adopted here, has the meaning of a Ward identity. This follows from the fact that the right-hand side of (311) is linear in the fields. As such, it represents a linearly breaking term which is not affected by quantum corrections (76).

- The equation of τ^a:

Analogously to the antighost equation, the equation of motion of the τ^a field and the variation of the action with respect to the source $\mathcal{J}^a_\mu$, yields the following identity, i.e.

$$\frac{\delta\Sigma}{\delta\tau^a} - \partial_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu} = 0 \,. \tag{312}$$

- The equation of the antighost $\bar{\eta}^a$:

$$\frac{\delta\Sigma}{\delta\bar{\eta}^a} + \partial_\mu \frac{\delta\Sigma}{\delta\Xi^a_\mu} = 0 \,. \tag{313}$$

Note that the presence of the composite field operator $D^{ab}_\mu(A^h)\eta^b$, coupled to the source Ξ^a_μ, is needed in order to establish this identity.

- The integrated equation of the ghost η^a:

$$\int d^4x \left(\frac{\delta\Sigma}{\delta\eta^a} + gf^{abc}\bar{\eta}^b \frac{\delta\Sigma}{\delta\tau^c} - gf^{abc}\Xi^b_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^c_\mu} \right) = \int d^4x \Big(\bar{X}^{abI}\omega^{bI} - \bar{Y}^{abI}\varphi^{bI} + X\bar{\omega}^{aI}$$
$$- Y^I\bar{\varphi}^{aI} + Z^{\hat{I}}\bar{\lambda}^a_{\hat{I}} - W^{\hat{I}}\bar{\zeta}^a_{\hat{I}} + \bar{Z}^{ab\hat{I}}\lambda^b_{\hat{I}}$$
$$- \bar{W}^{ab\hat{I}}\zeta^b_{\hat{I}} + \Gamma^{ab}\eta^b + \Phi^i_\alpha\bar{\psi}^{h,j\alpha}T^{a,ji}$$
$$- \bar{\Phi}^{i\alpha}T^{a,ij}\psi^{h,j}_\alpha \Big) . \tag{314}$$

- The global $U(4(N^2-1))$ symmetry:

$$U_{IJ}(\Sigma) = 0 \,, \tag{315}$$

with

$$U_{IJ}(\Sigma) = \int d^4x \Big(\varphi^{aI}\frac{\delta\Sigma}{\delta\varphi^{aJ}} - \bar{\varphi}^{aJ}\frac{\delta\Sigma}{\delta\bar{\varphi}^{aI}} + \omega^{aI}\frac{\delta\Sigma}{\delta\omega^{aJ}} - \bar{\omega}^{aJ}\frac{\delta\Sigma}{\delta\bar{\omega}^{aI}}$$
$$- M^{aJ}_\mu\frac{\delta\Sigma}{\delta M^{aI}_\mu} + V^{aI}_\mu\frac{\delta\Sigma}{\delta V^{aJ}_\mu} - N^{aJ}_\mu\frac{\delta\Sigma}{\delta N^{aI}_\mu} + U^{aJ}_\mu\frac{\delta\Sigma}{\delta U^{aI}_\mu}$$
$$+ X^J\frac{\delta\Sigma}{\delta X^I} + Y^I\frac{\delta\Sigma}{\delta Y^J} - \bar{X}^{abJ}\frac{\delta\Sigma}{\delta\bar{X}^{abI}} - \bar{Y}^{abJ}\frac{\delta\Sigma}{\delta\bar{Y}^{abI}} \Big) . \tag{316}$$

- The $U(4N)$ symmetry:

$$\hat{U}^{\hat{I}\hat{J}}(\Sigma) = 0 \,, \tag{317}$$

where

$$\hat{U}^{\hat{I}\hat{J}}(\Sigma) = \int d^4x \Big(\lambda^{a\hat{I}}\frac{\delta\Sigma}{\delta\lambda^a_{\hat{J}}} - \bar{\lambda}^{a\hat{J}}\frac{\delta\Sigma}{\delta\bar{\lambda}^a_{\hat{I}}} + \zeta^{a\hat{I}}\frac{\delta\Sigma}{\delta\zeta^a_{\hat{J}}} - \bar{\zeta}^{a\hat{J}}\frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} + \Lambda^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Lambda^i_{\alpha\hat{J}}}$$
$$- \bar{\Lambda}^{i\alpha\hat{J}}\frac{\delta\Sigma}{\delta\bar{\Lambda}^i_{\alpha\hat{I}}} + \Pi^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Pi^i_{\alpha\hat{J}}} - \bar{\Pi}^{i\alpha\hat{J}}\frac{\delta\Sigma}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} + Z^{\hat{J}}\frac{\delta\Sigma}{\delta Z_{\hat{I}}} + W^{\hat{I}}\frac{\delta\Sigma}{\delta W_{\hat{J}}}$$
$$- \bar{Z}^{ab\hat{J}}\frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}} - \bar{W}^{ab\hat{J}}\frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}} \Big) . \tag{318}$$

- The e-charge, or spinor number:

$$\mathcal{N}_e(\Sigma) = 0 \,,$$

$$
\begin{aligned}
\mathcal{N}_e(\Sigma) &= \int d^4x \left(\psi^{i\alpha} \frac{\delta\Sigma}{\delta\psi^i_\alpha} - \bar{\psi}^{i\alpha} \frac{\delta\Sigma}{\delta\bar{\psi}^i_\alpha} + \Upsilon^{i\alpha} \frac{\delta\Sigma}{\delta\Upsilon^i_\alpha} - \bar{\Upsilon}^{i\alpha} \frac{\delta\Sigma}{\delta\bar{\Upsilon}^i_\alpha} + \Theta^{i\alpha} \frac{\delta\Sigma}{\delta\Theta^i_\alpha} - \bar{\Theta}^{i\alpha} \frac{\delta\Sigma}{\delta\bar{\Theta}^i_\alpha} \right. \\
&+ \lambda^{a\hat{I}} \frac{\delta\Sigma}{\delta\lambda^a_{\hat{I}}} - \bar{\lambda}^{a\hat{I}} \frac{\delta\Sigma}{\delta\bar{\lambda}^a_{\hat{I}}} + \zeta^{a\hat{I}} \frac{\delta\Sigma}{\delta\zeta^a_{\hat{I}}} - \bar{\zeta}^{a\hat{I}} \frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} + \Phi^{i\alpha} \frac{\delta\Sigma}{\delta\Phi^i_\alpha} - \bar{\Phi}^{i\alpha} \frac{\delta\Sigma}{\delta\bar{\Phi}^i_\alpha} + Z^{\hat{I}} \frac{\delta\Sigma}{\delta Z_{\hat{I}}} \\
&+ \left. W^{\hat{I}} \frac{\delta\Sigma}{\delta W_{\hat{I}}} - \bar{Z}^{ab\hat{I}} \frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}} - \bar{W}^{ab\hat{I}} \frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}} \right) . \qquad (319)
\end{aligned}
$$

- The linearly broken constraints:

$$
\frac{\delta\Sigma}{\delta\bar{\varphi}^{aI}} + \partial_\mu \frac{\delta\Sigma}{\delta M^{aI}_\mu} + g f^{abc} V^{bI}_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^c_\mu} = -J_\varphi \varphi^{aI} + Y^I \eta^a , \qquad (320)
$$

$$
\frac{\delta\Sigma}{\delta\varphi^{aI}} + \partial_\mu \frac{\delta\Sigma}{\delta V^{aI}_\mu} - g f^{abc} \bar{\varphi}^{bI} \frac{\delta\Sigma}{\delta\tau^c} + g f^{abc} M^{bI}_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^c_\mu} = -J_\varphi \bar{\varphi}^{aI} - K_\varphi \bar{\omega}^{aI} + \bar{Y}^{baI} \eta^b , \qquad (321)
$$

$$
\frac{\delta\Sigma}{\delta\bar{\omega}^{aI}} + \partial_\mu \frac{\delta\Sigma}{\delta N^{aI}_\mu} - g f^{abc} U^{bI}_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^c_\mu} = J_\varphi \omega^{aI} - K_\varphi \varphi^{aI} - X^I \eta^a , \qquad (322)
$$

$$
\frac{\delta\Sigma}{\delta\omega^{aI}} + \partial_\mu \frac{\delta\Sigma}{\delta U^{aI}_\mu} - g f^{abc} \bar{\omega}^{bI} \frac{\delta\Sigma}{\delta\tau^c} + g f^{abc} N^{bI}_\mu \frac{\delta\Sigma}{\delta\mathcal{J}^c_\mu} = -J_\varphi \bar{\omega}^{aI} - \bar{X}^{baI} \eta^a . \qquad (323)
$$

- The linearly broken integrated Ward Identities for the matter sector:

$$
\int d^4x \left(\frac{\delta\Sigma}{\delta\zeta^{a\hat{I}}} + g f^{abc} \bar{\zeta}^b_{\hat{I}} \frac{\delta\Sigma}{\delta\tau^c} - \bar{\Pi}^{i\alpha}_{\hat{I}} T^{a,ij} \frac{\delta\Sigma}{\delta\Theta^j_\alpha} \right) = \int d^4x \left(J_\lambda \bar{\zeta}^a_{\hat{I}} - \bar{W}^{ba}_{\hat{I}} \eta^b \right) , \qquad (324)
$$

$$
\int d^4x \left(\frac{\delta\Sigma}{\delta\lambda^{a\hat{I}}} + g f^{abc} \frac{\delta\Sigma}{\delta\tau^c} \bar{\lambda}^b_{\hat{I}} + T^{a,ij} \bar{\Lambda}^{i\alpha}_{\hat{I}} \frac{\delta\Sigma}{\delta\Theta^j_\alpha} \right) = \int d^4x \left(J_\lambda \bar{\lambda}^a_{\hat{I}} + K_\lambda \bar{\zeta}^a_{\hat{I}} + \bar{Z}^{ba}_{\hat{I}} \eta^b \right) , \qquad (325)
$$

$$
\int d^4x \left(\frac{\delta\Sigma}{\delta\bar{\zeta}^{a\hat{I}}} + \Pi^{i\alpha}_{\hat{I}} T^{a,ij} \frac{\delta\Sigma}{\delta\bar{\Theta}^j_\alpha} \right) = \int d^4x \left(J_\lambda \zeta^a_{\hat{I}} - W_{\hat{I}} \eta^a - K_\lambda \lambda^a_{\hat{I}} \right) , \qquad (326)
$$

$$\int d^4x \left(\frac{\delta\Sigma}{\delta\bar{\lambda}^{a\hat{I}}} + \Lambda^{i\alpha}_{\hat{I}}\, T^{a,ij}\, \frac{\delta\Sigma}{\delta\bar{\Theta}^j_\alpha} \right) \;=\; \int d^4x \left(J_\lambda \lambda^a_{\hat{I}} + Z_{\hat{I}}\, \eta^a \right). \tag{327}$$

- The η-ghost number:

  A ghost number can be assigned to the anticommuting fields $(\bar{\eta}, \eta)$ and to the source Ξ_μ, resulting in the following η-ghost number Ward identity

$$\begin{aligned}
\mathcal{N}_{\eta-ghost}(\Sigma) \;&=\; \int d^4x \left(\eta^a \frac{\delta\Sigma}{\delta\eta^a} + -\bar{\eta}^a \frac{\delta\Sigma}{\delta\bar{\eta}^a} - \Xi^a_\mu \frac{\delta\Sigma}{\delta\Xi^a_\mu} - X^I \frac{\delta\Sigma}{\delta X^I} - Y^I \frac{\delta\Sigma}{\delta Y^I} \right. \\
&-\; \bar{X}^{abI} \frac{\delta\Sigma}{\delta\bar{X}^{abI}} - \bar{Y}^{abI} \frac{\delta\Sigma}{\delta\bar{Y}^{abI}} - Z^{\hat{I}} \frac{\delta\Sigma}{\delta Z_{\hat{I}}} - W^{\hat{I}} \frac{\delta\Sigma}{\delta W_{\hat{I}}} - \bar{Z}^{ab\hat{I}} \frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}} \\
&-\; \left. \bar{W}^{ab\hat{I}} \frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}} - 2\Gamma^{ab} \frac{\delta\Sigma}{\delta\Gamma^{ab}} \right) = 0\,.
\end{aligned} \tag{328}$$

- The c-ghost number:

  Analogously, we have also the usual c-ghost number in the Faddeev-Popov sector, expressed by

$$\begin{aligned}
\mathcal{N}_{c-ghost}(\Sigma) \;&=\; \int d^4x \left(c^a \frac{\delta\Sigma}{\delta c^a} - \bar{c}^a \frac{\delta\Sigma}{\delta\bar{c}^a} + \omega^{aI} \frac{\delta\Sigma}{\delta\omega^{aI}} - \bar{\omega}^{aI} \frac{\delta\Sigma}{\delta\bar{\omega}^{aI}} - \Upsilon^{i\alpha} \frac{\delta\Sigma}{\delta\Upsilon^i_\alpha} - \bar{\Upsilon}^{i\alpha} \frac{\delta\Sigma}{\delta\bar{\Upsilon}^i_\alpha} \right. \\
&-\; \Omega^a_\mu \frac{\delta\Sigma}{\delta\Omega^a_\mu} - 2L^a \frac{\delta\Sigma}{\delta L^a} - K^a \frac{\delta\Sigma}{\delta K^a} + U^{aI}_\mu \frac{\delta\Sigma}{\delta U^{aI}_\mu} - N^{aI}_\mu \frac{\delta\Sigma}{\delta N^{aI}_\mu} + K_\varphi \frac{\partial\Sigma}{\delta K_\varphi} \\
&+\; X^I \frac{\delta\Sigma}{\delta X^I} - \bar{X}^{abI} \frac{\delta\Sigma}{\delta\bar{X}^{abI}} + \zeta^{a\hat{I}} \frac{\delta\Sigma}{\delta\zeta^a_{\hat{I}}} - \bar{\zeta}^{a\hat{I}} \frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} + \Pi^{i\alpha\hat{I}} \frac{\delta\Sigma}{\delta\Pi^i_{\alpha\hat{I}}} - \bar{\Pi}^{i\alpha\hat{I}} \frac{\delta\Sigma}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} \\
&+\; \left. K_\lambda \frac{\delta\Sigma}{\delta K_\lambda} + B^{\hat{I}} \frac{\delta\Sigma}{\delta B_{\hat{I}}} - \bar{B}^{ab\hat{I}} \frac{\delta\Sigma}{\delta\bar{B}^{ab}_{\hat{I}}} \right) + \chi \frac{\partial\Sigma}{\partial\chi} = 0\,.
\end{aligned} \tag{329}$$

- The exactly $\mathcal{R}_{IJ}$ symmetry;

$$\mathcal{R}_{IJ}(\Sigma) = 0\,, \tag{330}$$

  with

$$\mathcal{R}_{IJ}(\Sigma) = \int d^4x \left(\varphi^{aI} \frac{\delta\Sigma}{\delta\omega^{aJ}} - \bar{\omega}^{aJ} \frac{\delta\Sigma}{\delta\bar{\varphi}^{aI}} + V^{aI}_\mu \frac{\delta\Sigma}{\delta U^{aJ}_\mu} - N^{aI}_\mu \frac{\delta\Sigma}{\delta M^{aJ}_\mu} + \bar{X}^{abJ} \frac{\delta\Sigma}{\delta\bar{Y}^{abI}} + Y^I \frac{\delta\Sigma}{\delta X^J} \right). \tag{331}$$

- Identities that mix the Zwanziger ghosts with $(\eta,\bar{\eta})$ ghosts

$$W^{I}_{(1)}(\Sigma) = \int d^4x \left(\bar{\omega}^{aI}\frac{\delta\Sigma}{\delta\bar{\eta}^a} + \eta^a\frac{\delta\Sigma}{\delta\omega^{aI}} + N^{aI}_{\mu}\frac{\delta\Sigma}{\delta\Xi^a_{\mu}} + J_{\varphi}\frac{\delta\Sigma}{\delta X^I} + \bar{X}^{abI}\frac{\delta\Sigma}{\delta\Gamma^{ab}}\right) = 0\,, \tag{332}$$

$$\begin{aligned} W^{I}_{(2)}(\Sigma) &= \int d^4x \left(\bar{\varphi}^{aI}\frac{\delta\Sigma}{\delta\bar{\eta}^a} - \eta^a\frac{\delta\Sigma}{\delta\varphi^{aI}} + M^{aI}_{\mu}\frac{\delta\Sigma}{\delta\Xi^a_{\mu}} - J_{\varphi}\frac{\delta\Sigma}{\delta Y^I} + K_{\varphi}\frac{\delta\Sigma}{\delta X^I}\right. \\ &- \left.\bar{Y}^{abI}\frac{\delta\Sigma}{\delta\Gamma^{ab}}\right) = 0\,, \end{aligned} \tag{333}$$

$$\begin{aligned} W^{I}_{(3)}(\Sigma) &= \int d^4x \left(\varphi^{aI}\frac{\delta\Sigma}{\delta\bar{\eta}^a} - \eta^a\frac{\delta\Sigma}{\delta\bar{\varphi}^{aI}} - gf^{abc}\frac{\delta\Sigma}{\delta\bar{Y}^{abI}}\frac{\delta\Sigma}{\delta\tau^c} - V^{aI}_{\mu}\frac{\delta\Sigma}{\delta\Xi^a_{\mu}} + J_{\varphi}\frac{\delta\Sigma}{\delta\bar{Y}^{aaI}}\right. \\ &+ \left.Y^I\frac{\delta\Sigma}{\delta\Gamma^{aa}}\right) = 0\,, \end{aligned} \tag{334}$$

$$\begin{aligned} W^{I}_{(4)}(\Sigma) &= \int d^4x \left(\omega^{aI}\frac{\delta\Sigma}{\delta\bar{\eta}^a} - \eta^a\frac{\delta\Sigma}{\delta\bar{\omega}^{aI}} + gf^{abc}\frac{\delta\Sigma}{\delta\bar{X}^{abI}}\frac{\delta\Sigma}{\delta\tau^c} + U^{aI}_{\mu}\frac{\delta\Sigma}{\delta\Xi^a_{\mu}} + J_{\varphi}\frac{\delta\Sigma}{\delta\bar{X}^{aaI}}\right. \\ &+ \left.K_{\varphi}\frac{\delta\Sigma}{\delta\bar{Y}^{aaI}} + X^I\frac{\delta\Sigma}{\delta\Gamma^{aa}}\right) = 0\,. \end{aligned} \tag{335}$$

- Identities that mix the ghosts related to fermionic matter fields with the $(\eta,\bar{\eta})$ ghosts:

$$W^{\hat{I}}_{(1)}(\Sigma) = \int d^4x \left(\eta^a\frac{\delta\Sigma}{\delta\lambda^a_{\hat{I}}} + \bar{\lambda}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\eta}^a} + \bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Phi^i_{\alpha}} + J_{\lambda}\frac{\delta\Sigma}{\delta Z_{\hat{I}}} - \bar{Z}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\Gamma^{ab}}\right) = 0\,, \tag{336}$$

$$\begin{aligned} W^{\hat{I}}_{(2)}(\Sigma) &= \int d^4x \left(\bar{\zeta}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\eta}^a} - \eta^a\frac{\delta\Sigma}{\delta\zeta^a_{\hat{I}}} + \bar{\Pi}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Phi^i_{\alpha}} - J_{\lambda}\frac{\delta\Sigma}{\delta W_{\hat{I}}} + K_{\lambda}\frac{\delta\Sigma}{\delta Z_{\hat{I}}} - \bar{W}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\Gamma^{ab}}\right) \\ &= 0\,, \end{aligned} \tag{337}$$

$$\begin{aligned} W^{\hat{I}}_{(3)}(\Sigma) &= \int d^4x \left(\eta^a\frac{\delta\Sigma}{\delta\bar{\lambda}^a_{\hat{I}}} + \lambda^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\eta}^a} - \Lambda^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Phi}^i_{\alpha}} - J_{\lambda}\frac{\delta\Sigma}{\delta\bar{Z}^{aa}_{\hat{I}}} - K_{\lambda}\frac{\delta\Sigma}{\delta\bar{W}^{aa}_{\hat{I}}} - Z^{\hat{I}}\frac{\delta\Sigma}{\delta\Gamma^{aa}}\right) \\ &= 0\,, \end{aligned} \tag{338}$$

$$W^{\hat{I}}_{(4)}(\Sigma) = \int d^4x \left(\zeta^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\eta}^a} + \eta^a\frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} + \Pi^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Phi}^i_{\alpha}} + J_{\lambda}\frac{\delta\Sigma}{\delta\bar{W}^{aa}_{\hat{I}}} - W^{\hat{I}}\frac{\delta\Sigma}{\delta\Gamma^{aa}}\right) = 0\,. \tag{339}$$

5.8 Some remarks

Here we would like to perform some comments on the model we have determined in this chapter, expressed by action Σ, eq. (305). The introduction of the horizon term for the matter sector has an important meaning. The Gribov problem is a partial answer to the question of how to correctly quantize a non-Abelian gauge field theory and apparently this answer has something to do with the problem of confinement of gauge bosons. Then, when we reproduce the same procedure of GZ (or RGZ) model for the matter sector, by introducing the fermionic horizon function (248), we are implicitly assuming that the mechanism of fermion confinement, which corresponds to the quarks of QCD in the case of $SU(3)$ group, is similar to the mechanism of bosons (the gluons in the case of QCD). This is a strong assumption and it is based on the fact that action (305) is local, BRST invariant and, as we shall see in the next chapter, renormalizable. It is also justified by the form of the fermion propagator. Taking the physical limits of the sources, in particular (254), (265) and (285), we have for the tree level fermion propagator in the Landau gauge ($\alpha = 0$) the following result

$$\langle \bar{\psi}^i_\alpha(p)\psi^j_\beta(-p)\rangle = \frac{-ip_\mu(\gamma_\mu)_{\alpha\beta} + \mathcal{A}(p^2)\,\delta_{\alpha\beta}}{p^2 + \mathcal{A}^2(p^2)}\,\delta^{ij}\,, \tag{340}$$

where $\mathcal{A}(p^2)$ is a "mass function" given by

$$\mathcal{A}(p^2) = m_\psi + g^2\,\frac{N^2-1}{2N}\,\frac{\sigma^3}{p^2+w^2}\,. \tag{341}$$

This kind of propagator is in qualitative agreement with some lattice results as (157).

However, we do not have at the present moment a clear interpretation for the fermionic horizon function as occurs in the case of the original Zwanziger horizon function of GZ (or RGZ) model. It seems that somehow the horizon function has an universal character giving rise to the generalization (245). Nevertheless, there is an intriguing result obtained in (158) where a dimensional reduction (from five to four) is studied in the RGZ model in five dimensional Euclidean space and the scalar horizon function naturally shows up[30]. This was a first evidence that the generalization of the horizon function could have a geometrical origin. In the case of the fermionic horizon function, we can only speculate that in a supersymmetric scenario in higher dimensions a similar result could happen and a dimensional reduction could provide the fermionic horizon function.

[30] In the dimensional reduction the scalar field $\phi^a(x)$ is identified with the fifth component of the gauge field $A^a_5(x)$. Of course the interaction term $\lambda(\phi^a\phi^a)^2$ can not be reproduced from the dimensional reduction, then the resulting action in four dimensions is not renormalizable.

6 ALGEBRAIC RENORMALIZATION ANALYSIS

In section (5.7), we displayed all Ward Identities that action (305) obeys. Now we will focus on search for the most general counterterm, which gives us the possibility to study the renormalizability of this model.

An useful convention that we will adopt in this chapter is the following reparametrization

$$\Big(A^a_\mu\,,\ b^a\,,\ \xi^a\,,\ \alpha\,,\ \tau^a\,,\ \mathcal{J}^a_\mu\,,\ J\Big) \to \left(\frac{1}{g}A^a_\mu\,,\ gb^a\,,\ \frac{1}{g}\xi^a\,,\ \frac{1}{g^2}\alpha\,,\ g\tau^a\,,\ g\mathcal{J}^a_\mu\,,\ g^2 J\right)\,. \tag{342}$$

6.1 Algebraic characterization of the most general counterterm

In order to characterize the most general invariant counterterm, which can be freely added to all orders in perturbation theory we follow the setup of the algebraic renormalization (76) and perturb the classical action Σ by adding an integrated local polynomial in the fields and sources, Σ_{CT}, with dimension bounded by four and vanishing c-ghost number, η-ghost number, e-charge, $U(4(N^2-1))$-charge and $U(4N)$-charge. We demand thus that the perturbed action, $(\Sigma+\epsilon\Sigma_{\mathrm{CT}})$, with ϵ being an expansion parameter, fulfills, to the first order in ϵ, the same Ward identities obeyed by the classical action, i.e.

$$\begin{aligned}
\mathcal{B}_\Sigma(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) &= \mathcal{O}(\epsilon^2)\,,\\
\left(\frac{\delta}{\delta\bar{c}^a}+\partial_\mu\frac{\delta}{\delta\Omega^a_\mu}\right)(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) - \frac{i}{2}\chi b^a &= \mathcal{O}(\epsilon^2)\,,\\
\frac{\delta}{\delta b^a}(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) = i\partial_\mu A^a_\mu + \alpha b^a - \frac{i}{2}\chi\bar{c}^a &+ \mathcal{O}(\epsilon^2)\,,\\
\left(\frac{\delta}{\delta\tau^a}-\partial_\mu\frac{\delta}{\delta\mathcal{J}^a_\mu}\right)(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) &= \mathcal{O}(\epsilon^2)\,,\\
U_{IJ}(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) &= \mathcal{O}(\epsilon^2)\,,\\
\hat{U}^{\hat{I}\hat{J}}(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) &= \mathcal{O}(\epsilon^2)\,,\\
\mathcal{N}_e(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) &= \mathcal{O}(\epsilon^2)\,,\\
\mathcal{N}_{c-ghost}(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) &= \mathcal{O}(\epsilon^2)\,,\\
\mathcal{N}_{\eta-ghost}(\Sigma+\epsilon\Sigma_{\mathrm{CT}}) &= \mathcal{O}(\epsilon^2)\,,
\end{aligned}$$

$$
\begin{aligned}
\left(\frac{\delta}{\delta\bar{\varphi}^{aI}}+\partial_\mu\frac{\delta}{\delta M^{ai}_\mu}+f^{abc}V^{bI}_\mu\frac{\delta}{\delta\mathcal{J}^c_\mu}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= -J_\varphi\varphi^{aI}+Y^I\eta^a \\
&+\ \mathcal{O}(\epsilon^2)\,, \\
\left(\frac{\delta}{\delta\varphi^{aI}}+\partial_\mu\frac{\delta}{\delta V^{aI}_\mu}-f^{abc}\bar{\varphi}^{bI}\frac{\delta}{\delta\tau^c}+f^{abc}M^{bI}_\mu\frac{\delta}{\delta\mathcal{J}^c_\mu}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= -J_\varphi\bar{\varphi}^{aI}-K_\varphi\bar{\omega}^{aI} \\
&+\ \bar{Y}^{baI}\eta^b+\mathcal{O}(\epsilon^2) \\
\left(\frac{\delta}{\delta\bar{\omega}^{aI}}+\partial_\mu\frac{\delta}{\delta N^{aI}_\mu}-f^{abc}U^{bI}_\mu\frac{\delta}{\delta\mathcal{J}^c_\mu}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= J_\varphi\omega^{aI}-K_\varphi\varphi^{aI} \\
&-\ X^I\eta^a+\mathcal{O}(\epsilon^2)\,, \\
\left(\frac{\delta}{\delta\omega^{aI}}+\partial_\mu\frac{\delta}{\delta U^{aI}_\mu}-f^{abc}\bar{\omega}^{bI}\frac{\delta}{\delta\tau^c}+f^{abc}N^{bI}_\mu\frac{\delta}{\delta\mathcal{J}^c_\mu}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= -J_\varphi\bar{\omega}^{aI}-\bar{X}^{baI}\eta^b \\
&+\ \mathcal{O}(\epsilon^2)\,, \\
\int d^4x\left(\frac{\delta}{\delta\zeta^{a\hat{I}}}+f^{abc}\bar{\zeta}^b_{\hat{I}}\frac{\delta}{\delta\tau^c}-\bar{\Pi}^{i\alpha}_{\hat{I}}\,T^{a,ij}\,\frac{\delta}{\delta\Theta^j_\alpha}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= \int d^4x\left(J_\lambda\bar{\zeta}^a_{\hat{I}}\right. \\
&-\ \left.\bar{W}^{ba}_{\hat{I}}\,\eta^b\right)+\mathcal{O}(\epsilon^2)\,, \\
\int d^4x\left(\frac{\delta}{\delta\lambda^{a\hat{I}}}+f^{abc}\frac{\delta}{\delta\tau^c}\bar{\lambda}^b_{\hat{I}}+T^{a,ij}\,\bar{\Lambda}^{i\alpha}_{\hat{I}}\,\frac{\delta}{\delta\Theta^j_\alpha}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= \int d^4x\left(J_\lambda\bar{\lambda}^a_{\hat{I}}+\bar{Z}^{ba}_{\hat{I}}\,\eta^b\right) \\
&+\ K_\lambda\bar{\zeta}^a_{\hat{I}}+\mathcal{O}(\epsilon^2)\,, \\
\int d^4x\left(\frac{\delta}{\delta\bar{\zeta}^{a\hat{I}}}+\Pi^{i\alpha}_{\hat{I}}\,T^{a,ij}\,\frac{\delta}{\delta\bar{\Theta}^j_\alpha}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= \int d^4x\left(J_\lambda\zeta^a_{\hat{I}}-W_{\hat{I}}\,\eta^a\right. \\
&-\ \left.K_\lambda\lambda^a_{\hat{I}}\right)+\mathcal{O}(\epsilon^2)\,, \\
\int d^4x\left(\frac{\delta}{\delta\bar{\lambda}^{a\hat{I}}}+\Lambda^{i\alpha}_{\hat{I}}(T^a)^{ij}\frac{\delta}{\delta\bar{\Theta}^j_\alpha}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= \int d^4x\left(J_\lambda\lambda^a_{\hat{I}}\right. \\
&+\ \left.Z_{\hat{I}}\,\eta^a\right)+\mathcal{O}(\epsilon^2)\,, \\
\mathcal{R}_{ij}(\Sigma+\epsilon\Sigma_{\rm CT}) &= \mathcal{O}(\epsilon^2)\,, \\
\left(\frac{\delta}{\delta\bar{\eta}^a}+\partial_\mu\frac{\delta}{\delta\Xi^a_\mu}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= \mathcal{O}(\epsilon^2)\,, \\
\int d^4x\left(\frac{\delta}{\delta\eta^a}+f^{abc}\bar{\eta}^b\frac{\delta}{\delta\tau^c}-f^{abc}\Xi^b_\mu\frac{\delta}{\delta\mathcal{J}^c_\mu}\right)(\Sigma+\epsilon\Sigma_{\rm CT}) &= \int d^4x\left(-\bar{Y}^{abI}\varphi^{bI}+\bar{X}^{abI}\omega^{bI}\right. \\
&+\ X\bar{\omega}^{aI}-Y^I\bar{\varphi}^{aI}+Z^{\hat{I}}\bar{\lambda}^a_{\hat{I}} \\
&-\ W^{\hat{I}}\bar{\zeta}^a_{\hat{I}}+\bar{Z}^{ab\hat{I}}\lambda^b_{\hat{I}}-\bar{W}^{ab\hat{I}}\zeta^b_{\hat{I}}+\Gamma^{ab}\eta^b \\
&+\ \left.\Phi^i_\alpha\bar{\psi}^{h,j\alpha}\,T^{a,ji}-\bar{\Phi}^{i\alpha}\,T^{a,ij}\,\psi^{h,j}_\alpha\right) \\
&+\ \mathcal{O}(\epsilon^2)\,, \\
W^I_{(1,2,3,4)}(\Sigma+\epsilon\Sigma_{\rm CT}) &= \mathcal{O}(\epsilon^2)\,.
\end{aligned}
$$

$$W^{\hat{I}}_{(1,2,3,4)}(\Sigma + \epsilon\Sigma_{\mathrm{CT}}) \;=\; \mathcal{O}(\epsilon^2)\,. \tag{343}$$

Looking to the first condition of eqs.(343), one gets

$$\mathcal{B}_\Sigma \Sigma_{\mathrm{CT}} = 0\,, \tag{344}$$

where $\mathcal{B}_\Sigma$ was defined in (308). The equation (344) reveals that the invariant counterterm Σ_{CT} belongs to the cohomolgy of $\mathcal{B}_\Sigma$ in the space of the integrated local polynomials in the fields and sources. From the results of Yang-Mills theories cohomology, for instance see (76), the most general solution for Σ_{CT} is given by

$$\Sigma_{\mathrm{CT}} = \Delta + a_1 \int d^4x\, J_\psi\, \bar{\psi}^i_\alpha \psi^{i,\alpha} + \mathcal{B}_\Sigma \Delta^{(-1)}\,, \tag{345}$$

with Δ and $\Delta^{(-1)}$ being the nontrivial and trivial solutions of the first equation of the set (343), respectively. Furthermore, for the nontrivial part, one has $\Delta \neq \mathcal{B}_\Sigma T$ for some local integrated T. Also notice that, according to the quantum numbers of the fields, $\Delta^{(-1)}$ is an integrated polynomial of dimension four, c-ghost number -1 and η-ghost number equals to zero. At this moment, it is possible to visualize the convenience of the extended operator Q. The auxiliary fields and sources introduced due to the restriction of the functional measure to the Gribov region are doublets with respect to Q, which indicate that they belong to the exact part of the cohomology of $\mathcal{B}_\Sigma$ (76), that is they come into view only in $\Delta^{(-1)}$. Enjoying this characteristic, the most general expression allowed for Δ can be written as[31]

$$\begin{aligned}
\Delta &= \int d^4x \Big[\frac{a_0}{4g^2} F^a_{\mu\nu}F^a_{\mu\nu} + a_2(\partial_\mu A^{h,a}_\mu)(\partial_\nu A^{h,a}_\nu) + a_3(\partial_\mu A^{h,a}_\nu)(\partial_\mu A^{h,a}_\nu) \\
&+ a_4 f^{abc} A^{h,a}_\mu A^{h,b}_\nu \partial_\mu A^{h,c}_\nu + \pi^{abcd} A^{h,a}_\mu A^{h,b}_\mu A^{h,c}_\nu A^{h,d}_\nu + \hat{\mathcal{J}}^a_\mu \mathcal{O}^a_\mu(A,\xi) \\
&+ J\mathcal{O}(A,\xi) + J^2_\psi \mathcal{O}'(A,\xi) + a_5(\partial_\mu \bar{\eta}^a + \Xi^a_\mu)(\partial_\mu \eta^a) + f^{abc}(\partial_\mu \bar{\eta}^a + \Xi^a_\mu)\mathcal{P}^b_\mu(A,\xi)\eta^c \\
&+ \bar{\Theta}^{i\alpha}\mathcal{F}^i_\alpha(\psi,\xi) + \Theta^{i\alpha}\bar{\mathcal{F}}^i_\alpha(\bar{\psi},\xi) + \bar{\Phi}^{i\alpha}\, T^{a,ij}\, \eta^a \mathcal{W}^j_\alpha(\psi,\xi) \\
&+ \Phi^{i\alpha}\, T^{a,ij}\, \eta^a \bar{\mathcal{W}}^j_\alpha(\bar{\psi},\xi)\Big]\,,
\end{aligned} \tag{346}$$

where $(a_0, a_1, \ldots, a_5, \pi^{abcd})$ are arbitrary dimensionless coefficients, while $\mathcal{O}^a_\mu(A,\xi)$, $\mathcal{O}(A,\xi)$, $\mathcal{O}'(A,\xi)$ and $\mathcal{P}^a_\mu(A,\xi)$ are local expressions in terms of the fields A^a_μ and ξ^a with c-ghost number zero and dimensions 1, 2, 2 and 1, respectively. Moreover, $\mathcal{W}^i_\alpha(\psi,\xi)$ and $\bar{\mathcal{W}}^i_\alpha(\psi,\xi)$ are local functional of ψ^i_α and ξ^a and for the last, $\bar{\mathcal{F}}^i_\alpha(\bar{\psi},\xi)$ and $\bar{\mathcal{W}}^i_\alpha(\bar{\psi},\xi)$ are local expressions of $\bar{\psi}^i_\alpha$ and ξ^a.

---

[31] For simplicity we have omitted the vacuum terms. These terms can be considered in a future calculation but the general results obtained here remain unchanged.

It is important to mention that in the equation (346) we have already taken into account the fact that the variables $(\tau^a, \mathcal{J}^a_\mu)$ can enter in the counterterm only through the combination,

$$\hat{\mathcal{J}}^a_\mu = \mathcal{J}^a_\mu - \partial_\mu \tau^a \ . \tag{347}$$

This occurs because of the Ward identity (312). Furthermore, from (345), one gets

$$\mathcal{B}_\Sigma \mathcal{O}^a_\mu(A,\xi) \ = \ Q\mathcal{O}^a_\mu(A,\xi) = s\mathcal{O}^a_\mu(A,\xi) = 0\,, \tag{348}$$

$$\mathcal{B}_\Sigma \mathcal{O}(A,\xi) \ = \ Q\mathcal{O}(A,\xi) = s\mathcal{O}(A,\xi) = 0\,, \tag{349}$$

$$\mathcal{B}_\Sigma \mathcal{P}^a_\mu(A,\xi) \ = \ Q\mathcal{P}^a_\mu(A,\xi) = s\mathcal{P}^a_\mu(A,\xi) = 0\,, \tag{350}$$

$$\mathcal{B}_\Sigma \mathcal{F}^i_\alpha(\psi,\xi) \ = \ Q\mathcal{F}^i_\alpha(\psi,\xi) = s\mathcal{F}^i_\alpha(\psi,\xi) = 0\,, \tag{351}$$

$$\mathcal{B}_\Sigma \bar{\mathcal{F}}^i_\alpha(\bar\psi,\xi) \ = \ Q\bar{\mathcal{F}}^i_\alpha(\bar\psi,\xi) = s\bar{\mathcal{F}}^i_\alpha(\bar\psi,\xi) = 0\,, \tag{352}$$

$$\mathcal{B}_\Sigma \mathcal{W}^i_\alpha(\psi,\xi) \ = \ Q\mathcal{W}^i_\alpha(\psi,\xi) = s\mathcal{W}^i_\alpha(\psi,\xi) = 0\,, \tag{353}$$

$$\mathcal{B}_\Sigma \bar{\mathcal{W}}^i_\alpha(\bar\psi,\xi) \ = \ Q\bar{\mathcal{W}}^i_\alpha(\bar\psi,\xi) = s\bar{\mathcal{W}}^i_\alpha(\bar\psi,\xi) = 0\,, \tag{354}$$

which implies in the BRST-invariance of $\mathcal{O}^a_\mu(A,\xi)$, $\mathcal{O}(A,\xi)$, $\mathcal{P}^b_\mu(A,\xi)$, $\mathcal{F}^i_\alpha(\psi,\xi)$, $\bar{\mathcal{F}}^i_\alpha(\bar\psi,\xi)$, $\mathcal{W}^i_\alpha(\psi,\xi)$ and $\bar{\mathcal{W}}^i_\alpha(\bar\psi,\xi)$. In (75) and (87), the general solution of the eqs. (350)-(352) were obtained, yielding

$$\mathcal{O}^a_\mu(A,\xi) \ = \ b_1 (A^h)^a_\mu\,, \tag{355}$$

$$\mathcal{O}(A,\xi) \ = \ \frac{b_2}{2}(A^h)^a_\mu (A^h)^a_\mu\,, \tag{356}$$

$$\mathcal{O}'(A,\xi) \ = \ \frac{b_2'}{2}(A^h)^a_\mu (A^h)^a_\mu\,, \tag{357}$$

$$\mathcal{P}^a_\mu(A,\xi) \ = \ b_3 (A^h)^a_\mu\,, \tag{358}$$

$$\mathcal{F}^i_\alpha(\psi,\xi) \ = \ b_4 \psi^{(h)i}_\alpha\,, \tag{359}$$

$$\bar{\mathcal{F}}^{i}_{\alpha}(\bar{\psi},\xi) \;=\; b_5\bar{\psi}^{(h)i}_{\alpha}\,, \tag{360}$$

where $(b_1, b_2, b_2', b_3, b_4, b_5)$ are free dimensionless parameters. Now, let us show with details the procedure to determine the results for the coefficients of (353) and (354), that is

$$\mathcal{B}_{\Sigma}\mathcal{W}^{i}_{\alpha}(\psi,\xi) \;=\; Q\mathcal{W}^{i}_{\alpha}(\psi,\xi) = s\mathcal{W}^{i}_{\alpha}(\psi,\xi) = 0\,, \tag{361}$$

$$\mathcal{B}_{\Sigma}\bar{\mathcal{W}}^{i}_{\alpha}(\bar{\psi},\xi) \;=\; Q\bar{\mathcal{W}}^{i}_{\alpha}(\bar{\psi},\xi) = s\bar{\mathcal{W}}^{i}_{\alpha}(\bar{\psi},\xi) = 0\,, \tag{362}$$

with $\mathcal{W}^{i}_{\alpha}(\psi,\xi)$ and $\bar{\mathcal{W}}^{i}_{\alpha}(\bar{\psi},\xi)$ being BRST invariant, as mentioned before. First, we will analyze the equation (361). The operator presented in this case has dimension $(\frac{3}{2})$, null ghosts number and has two indices: one is related to the internal symmetry group in the fundamental representation and the other one is associated with the fermionic nature of this operator. Thus, we consider the following parametrization

$$\mathcal{W}^{i}_{\alpha}(\psi,\xi) \;=\; \sigma^{ij}_{\alpha\beta}(\xi)\psi^{j\beta}\,, \tag{363}$$

with $\sigma^{ij}(\xi)$ being a quantity that does not has mass dimension in the Stückelberg field ξ^a, and given by

$$\begin{aligned}
\sigma^{ij}(\xi) \;&=\; \sigma_1(\xi)\delta^{ij} + \sigma_2^{ij}(\xi)(\gamma^0) + \sigma_3^{ij}(\xi)(\gamma^1) + \sigma_4^{ij}(\xi)(\gamma^2)\\
&+\; \sigma_5^{ij}(\xi)(\gamma^3) + \sigma_6^{ij}(\xi)(\gamma^5) + \sigma_7^{ij}(\xi)(\gamma^0\gamma^5) + \sigma_8^{ij}(\xi)(\gamma^1\gamma^5)\\
&+\; \sigma_9^{ij}(\xi)(\gamma^2\gamma^5) + \sigma_{10}^{ij}(\xi)(\gamma^3\gamma^5) + \sigma_{11}^{ij}(\xi)\left[\gamma^0,\gamma^1\right]\\
&+\; \sigma_{12}^{ij}(\xi)\left[\gamma^0,\gamma^2\right] + \sigma_{13}^{ij}(\xi)\left[\gamma^0,\gamma^3\right] + \sigma_{14}^{ij}(\xi)\left[\gamma^1,\gamma^2\right]\\
&+\; \sigma_{15}^{ij}(\xi)\left[\gamma^1,\gamma^3\right] + \sigma_{16}^{ij}(\xi)\left[\gamma^2,\gamma^3\right]\,.
\end{aligned} \tag{364}$$

where the γ's are known as the Dirac matrices. In the next, we explicitly characterize the expression of operator $\mathcal{W}^{i}_{\alpha}(\psi,\xi)$ as

$$\begin{aligned}
\mathcal{W}^{i}(\psi,\xi) \;&=\; \sigma_1(\xi)\delta^{ij}\psi^j + \sigma_2^{ij}(\xi)(\gamma^0)\psi^j + \sigma_3^{ij}(\xi)(\gamma^1)\psi^j + \sigma_4^{ij}(\xi)(\gamma^2)\psi^j\\
&+\; \sigma_5^{ij}(\xi)(\gamma^3)\psi^j + \sigma_6^{ij}(\xi)(\gamma^5)\psi^j + \sigma_7^{ij}(\xi)(\gamma^0\gamma^5)\psi^j + \sigma_8^{ij}(\xi)(\gamma^1\gamma^5)\psi^j\\
&+\; \sigma_9^{ij}(\xi)(\gamma^2\gamma^5)\psi^j + \sigma_{10}^{ij}(\xi)(\gamma^3\gamma^5)\psi^j + \sigma_{11}^{ij}(\xi)\left[\gamma^0,\gamma^1\right]\psi^j\\
&+\; \sigma_{12}^{ij}(\xi)\left[\gamma^0,\gamma^2\right]\psi^j + \sigma_{13}^{ij}(\xi)\left[\gamma^0,\gamma^3\right]\psi^j + \sigma_{14}^{ij}(\xi)\left[\gamma^1,\gamma^2\right]\psi^j\\
&+\; \sigma_{15}^{ij}(\xi)\left[\gamma^1,\gamma^3\right]\psi^j + \sigma_{16}^{ij}(\xi)\left[\gamma^2,\gamma^3\right]\psi^j\,.
\end{aligned} \tag{365}$$

Enjoying the equation (238), we make a convenient replacement in equation (365), which is the substitution of ψ^{i}_{α} by the gauge-invariant matter field $\psi^{h,i}_{\alpha}$ and, consequently, we redefine the quantities, previously mentioned, for a new one $(\hat{\sigma}^{ij}(\xi))$. Thus, (365) is redefined as

$$\mathcal{W}^i_\alpha(\psi,\xi) = \hat{\sigma}^{ij}_{\alpha\beta}(\xi)\psi^{h,j\beta}\,, \tag{366}$$

with

$$\begin{aligned}\mathcal{W}^i(\psi,\xi) &= \hat{\sigma}_1\delta^{ij}\psi^{h,j} + \hat{\sigma}^{ij}_2(\gamma^0)\psi^{h,j} + \hat{\sigma}^{ij}_3(\gamma^1)\psi^{(h)j}_\alpha + \hat{\sigma}^{ij}_4(\gamma^2)\psi^{h,j}\\ &+ \hat{\sigma}^{ij}_5(\gamma^3)\psi^{h,j} + \hat{\sigma}^{ij}_6(\gamma^5)\psi^{h,j} + \hat{\sigma}^{ij}_7(\gamma^0\gamma^5)\psi^{h,j} + \hat{\sigma}^{ij}_8(\gamma^1\gamma^5)\psi^{h,j}\\ &+ \hat{\sigma}^{ij}_9(\gamma^2\gamma^5)\psi^{h,j} + \hat{\sigma}^{ij}_{10}(\gamma^3\gamma^5)\psi^{h,j} + \hat{\sigma}^{ij}_{11}\left[\gamma^0,\gamma^1\right]\psi^{h,j}\\ &+ \hat{\sigma}^{ij}_{12}\left[\gamma^0,\gamma^2\right]\psi^{h,j} + \hat{\sigma}^{ij}_{13}\left[\gamma^0,\gamma^3\right]\psi^{h,j} + \hat{\sigma}^{ij}_{14}\left[\gamma^1,\gamma^2\right]\psi^{h,j}\\ &+ \hat{\sigma}^{ij}_{15}\left[\gamma^1,\gamma^3\right]\psi^{h,j} + \hat{\sigma}^{ij}_{16}\left[\gamma^2,\gamma^3\right]\psi^{h,j}\,.\end{aligned} \tag{367}$$

Now, let us explore with more details the equation (361). Thus,

$$\mathcal{B}_\Sigma\mathcal{F}^i_\alpha(\psi,\xi) = s\mathcal{F}^i_\alpha(\psi,\xi) = 0\,, \tag{368}$$

$$\begin{aligned}\mathcal{B}_\Sigma\mathcal{W}^i_\alpha(\psi,\xi) &= \int d^4x\left\{\frac{\delta\Sigma}{\delta\bar{U}^i_\alpha}\frac{\delta\mathcal{W}^i_\alpha(\psi,\xi)}{\delta\psi^{i,\alpha}} + \frac{\delta\Sigma}{\delta K^k}\frac{\delta\mathcal{W}^i_\alpha(\psi,\xi)}{\delta\xi^k}\right\}\\ &= \int d^4x\left\{\frac{\delta\Sigma}{\delta K^a}\frac{\partial\hat{\sigma}^{ij}(\xi)}{\partial\xi^a}\psi^{(h)j}_\alpha\right\}\\ &= \int d^4x\left\{g^{cb}(\xi)\,c^b\frac{\partial\hat{\sigma}^{ij}(\xi)}{\partial\xi^c}\psi^{(h)j}_\alpha\right\}\\ &= 0,\end{aligned} \tag{369}$$

which immediately gives

$$\begin{aligned}\frac{\partial\hat{\sigma}^{ij}(\xi)}{\partial\xi^c} &= \frac{\partial\hat{\sigma}_1(\xi)}{\partial\xi^c}\delta^{ij} + \frac{\partial\hat{\sigma}^{ij}_2(\xi)}{\partial\xi^c}(\gamma^0) + \frac{\partial\hat{\sigma}^{ij}_3(\xi)}{\partial\xi^c}(\gamma^1) + \frac{\partial\hat{\sigma}^{ij}_4(\xi)}{\partial\xi^c}(\gamma^2) + \frac{\partial\hat{\sigma}^{ij}_5(\xi)}{\partial\xi^c}(\gamma^3)\\ &+ \frac{\partial\hat{\sigma}^{ij}_6(\xi)}{\partial\xi^c}(\gamma^5) + \frac{\partial\hat{\sigma}^{ij}_7(\xi)}{\partial\xi^c}(\gamma^0\gamma^5) + \frac{\partial\hat{\sigma}^{ij}_8(\xi)}{\partial\xi^c}(\gamma^1\gamma^5) + \frac{\partial\hat{\sigma}^{ij}_9(\xi)}{\partial\xi^c}(\gamma^2\gamma^5)\\ &+ \frac{\partial\hat{\sigma}^{ij}_{10}(\xi)}{\partial\xi^c}(\gamma^3\gamma^5) + \frac{\partial\hat{\sigma}^{ij}_{11}(\xi)}{\partial\xi^c}\left[\gamma^0,\gamma^1\right] + \frac{\partial\hat{\sigma}^{ij}_{12}(\xi)}{\partial\xi^c}\left[\gamma^0,\gamma^2\right] + \frac{\partial\hat{\sigma}^{ij}_{13}(\xi)}{\partial\xi^c}\left[\gamma^0,\gamma^3\right]\\ &+ \frac{\partial\hat{\sigma}^{ij}_{14}(\xi)}{\partial\xi^c}\left[\gamma^1,\gamma^2\right] + \frac{\partial\hat{\sigma}^{ij}_{15}(\xi)}{\partial\xi^c}\left[\gamma^1,\gamma^3\right] + \frac{\partial\hat{\sigma}^{ij}_{16}(\xi)}{\partial\xi^c}\left[\gamma^2,\gamma^3\right]\\ &= 0\,.\end{aligned} \tag{370}$$

Moreover, from the so-called discrete symmetries, that is, parity, time-reversal, charge conjugation and chirality conditions, see appendix (C), one has the following result,

$$\hat{\sigma}^{ij}_{\alpha\beta} = b_6\delta^{ij}\delta_{\alpha\beta}\,, \tag{371}$$

where b_6 is a constant. The other 15 parameters are null because of the discrete symmetries previously mentioned. Therefore, we conclude that the most general expression for $\mathcal{W}^i_\alpha(\psi,\xi)$ is given as follows

$$\mathcal{W}^i_\alpha(\psi,\xi) \;=\; b_6\,\psi^{h,i}_\alpha\,. \tag{372}$$

The same procedure can be done for $\bar{\mathcal{W}}^i_\alpha(\bar{\psi},\xi)$ and we will not repeat the development established before, however we will show in the sequence the results for this functional

$$\bar{\mathcal{W}}^i_\alpha(\bar{\psi},\xi) \;=\; \varsigma^{ij}_{\alpha\beta}(\xi)\bar{\psi}^{j\beta}\,, \tag{373}$$

Making the same redefinition as done in equation (366), one gets

$$\bar{\mathcal{W}}^i_\alpha(\bar{\psi},\xi) \;=\; \hat{\varsigma}^{ij}_{\alpha\beta}(\xi)\bar{\psi}^{h,j\beta}\,. \tag{374}$$

After using the equation (374) and the discrete symmetries,

$$\hat{\varsigma}^{ij}_{\alpha\beta} \;=\; b_7\delta^{ij}\delta_{\alpha\beta}\,, \tag{375}$$

Thus, the functional $\bar{\mathcal{W}}^i_\alpha(\bar{\psi},\xi)$ is specified by

$$\bar{\mathcal{W}}^i_\alpha(\bar{\psi},\xi) \;=\; b_7\,\bar{\psi}^{h,i}_\alpha\,. \tag{376}$$

Finally, the most general Δ after imposing the constraints (350)-(353) is given by

$$\begin{aligned}\Delta \;=\; & \int d^4x\bigg[\frac{a_0}{4g^2}F^a_{\mu\nu}F^a_{\mu\nu} + a_2(\partial_\mu A^{h,a}_\mu)(\partial_\nu A^{h,a}_\nu) + a_3(\partial_\mu A^{h,a}_\nu)(\partial_\mu A^{h,a}_\nu)\\ + \;& a_4 f^{abc}A^{h,a}_\mu A^{h,b}_\nu \partial_\mu A^{h,c}_\nu + \pi^{abcd}A^{h,a}_\mu A^{h,b}_\mu A^{h,c}_\nu A^{h,d}_\nu + b_1\,\hat{\mathcal{J}}^a_\mu A^{h,a}_\mu\\ + \;& \left(b_2\frac{J}{2} + b_2'\frac{J^2_\psi}{2}\right)A^{h,a}_\mu A^{h,a}_\mu + a_5(\partial_\mu\bar{\eta}^a + \Xi^a_\mu)(\partial_\mu\eta^a) + b_3 f^{abc}(\partial_\mu\bar{\eta}^a + \Xi^a_\mu)A^{h,b}_\nu\eta^c\\ + \;& b_4\bar{\Theta}^{i\alpha}\psi^{h,i}_\alpha + b_5\Theta^{i\alpha}\bar{\psi}^{h,i}_\alpha + b_6\bar{\Phi}^{i\alpha}\,T^{a,ij}\,\eta^a\psi^{h,i}_\alpha + b_7\Phi^{i\alpha}\,T^{a,ij}\,\eta^a\bar{\psi}^{h,i}_\alpha\bigg]\,.\end{aligned} \tag{377}$$

It is important to remember that the parameters (α,χ) were inserted as a Q-doublet, thereby they cannot appear in the nontrivial sector of the Q cohomology. Which means, these parameters are not present in Δ. In expectation of determining the trivial sector of the cohomology, $\Delta^{(-1)}$, first we desire to show if the values of the extra sources are set to zero as

$$J = J_\varphi = M = N = V = U = K_\varphi = \chi = K = \mathcal{J} = \Xi = \Gamma = X = Y = \bar{X} = \bar{Y} \;=\; 0\,, \tag{378}$$

$$\Lambda = \bar{\Lambda} = \Pi = \bar{\Pi} = 0\,, \tag{379}$$

and

$$\Upsilon = \bar{\Upsilon} = \Lambda = \bar{\Lambda} = \Theta = \bar{\Theta} = \Phi = \bar{\Phi} \;\; = \;\; 0\,, \tag{380}$$

in (305), thus the remaining action with $\Omega = L = 0$ and $J_\psi = -m_\psi$ is

$$\begin{aligned}
\Sigma_{\mathrm{LCG}M} \;\; = \;\; & \int d^4x \,\Bigg[\frac{1}{4g^2}F^a_{\mu\nu}F^a_{\mu\nu} + i\bar{\psi}^{i\alpha}(\gamma_\mu)_{\alpha\beta}D^{ij}_\mu\psi^{j\beta} - m_\psi\bar{\psi}^i_\alpha\psi^{i\alpha} + ib^a\partial_\mu A^a_\mu \\
+ \;\; & \frac{\alpha}{2}b^ab^a + \bar{c}^a\partial_\mu D^{ab}_\mu(A)c^b - \bar{\varphi}^{ac}_\mu\mathcal{M}^{ab}(A^h)\varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu\mathcal{M}^{ab}(A^h)\omega^{bc}_\mu \\
+ \;\; & \tau^a\partial_\mu A^{h,a}_\mu - \bar{\eta}^a\mathcal{M}^{ab}(A^h)\eta^b - \bar{\lambda}^{ai}_\alpha\partial_\mu D^{ab}_\mu(A^h)\lambda^{bi\alpha} \\
- \;\; & \bar{\zeta}^{ai\alpha}\partial_\mu D^{ab}_\mu(A^h)\zeta^{bi}_\alpha\Bigg]\,,
\end{aligned} \tag{381}$$

which is the YM action carrying the Dirac fermionic matter fields, in the fundamental representation, gauge fixed in linear covariant gauges with the addition of the following terms:

$$-\int d^4x\left(\bar{\varphi}^{ac}_\mu\mathcal{M}^{ab}(A^h)\varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu\mathcal{M}^{ab}(A^h)\omega^{bc}_\mu + \tau^a\partial_\mu A^{h,a}_\mu - \bar{\eta}^a\mathcal{M}^{ab}(A^h)\eta^b\right), \tag{382}$$

$$-\int d^4x\left(\bar{\lambda}^{ai}_\alpha\partial_\mu D^{ab}_\mu(A^h)\lambda^{\alpha,bi} + \bar{\zeta}^{ai\alpha}\partial_\mu D^{ab}_\mu(A^h)\zeta^{bi}_\alpha\right). \tag{383}$$

Nevertheless, upon integration over $(\bar{\varphi}, \varphi, \bar{\omega}, \omega, \tau, \bar{\eta}, \eta, \lambda, \bar{\lambda}, \zeta, \bar{\zeta})$, the terms (382) and (383) are equal to a unity. Therefore, correlation functions of the original fields of the FP quantization, i.e. $(A, \bar{c}, c, b)$, are the same as those computed with the standard YM action in the linear covariant gauges (202). From this observation, it follows that, in the limits (378)–(380), the counterterm (377) should reduce to the standard one in the YMFP action in linear covariant gauges, see also (87, 83). This gives

$$a_2 = a_3 = a_4 = 0\,, \qquad a_5 = b_3\,, \qquad \pi^{abcd} = 0\,, \tag{384}$$

yielding

$$
\begin{aligned}
\Delta &= \int d^4x \Bigg[\frac{a_0}{4g^2} F^a_{\mu\nu} F^a_{\mu\nu} + b_1 \hat{\mathcal{J}}^a_\mu A^{h,a}_\mu + \left(b_2 \frac{J}{2} + b_2' \frac{J^2_\psi}{2} \right) A^{h,a}_\mu A^{h,a}_\mu + b_3 (\partial_\mu \bar{\eta}^a + \Xi^a_\mu) D^{ab}_\mu (A^h) \eta^b \\
&+ b_4 \bar{\Theta}^{i\alpha} \psi^{h,i}_\alpha + b_5 \Theta^{i\alpha} \bar{\psi}^{h,i}_\alpha + b_6 \bar{\Phi}^{i\alpha} T^{a,ij} \eta^a \psi^{h,i}_\alpha + b_7 \Phi^{i\alpha} T^{a,ij} \eta^a \bar{\psi}^{h,i}_\alpha \Bigg] . \qquad (385)
\end{aligned}
$$

Finally, the Ward identity (314) imposes the following constraint

$$
a_5 = -b_1 . \qquad (386)
$$

Now, let us turn our attention to the trivial part of the cohomology of $\mathcal{B}_\Sigma$ which has dimension four, it is a local expression in the fields and sources and it has c-ghost number (-1). Moreover, considering the quantum numbers of all the fields and sources given by the Tables (1)–(5) and the set of constraints (343), we have the following expression for $\Delta^{(-1)}$,

$$
\begin{aligned}
\Delta^{(-1)} &= \int d^4x \Bigg[f^{ab}_1(\xi,\alpha)(\Omega^a_\mu + \partial_\mu \bar{c}^a) A^b_\mu + f^{ab}_2(\xi,\alpha) c^a L^b + K^a f^{ab}(\xi,\alpha) \xi^b \\
&- b_1 \left(V^{aI}_\mu D^{ab}_\mu (A^h) \bar{\omega}^{bI} + N^{aI}_\mu D^{ab}_\mu (A^h) \varphi^{bI} + (\partial_\mu \bar{\omega}^{aI}) D^{ab}_\mu (A^h) \varphi^{bI} \right) \\
&+ f_3(\xi,\alpha) \left(\bar{\psi}^{i,\alpha} \Upsilon^i_\alpha + \bar{\Upsilon}^{i,\alpha} \psi^i_\alpha \right) + b_4 \left(T^{a,ij}\, \psi^{h,i\alpha} \bar{\zeta}^{a\hat{I}} \Lambda^j_{\alpha\hat{I}} + T^{aij}\, \bar{\psi}^{h,i\alpha} \lambda^{a\hat{I}} \bar{\Pi}^j_{\alpha\hat{I}} \right) \\
&+ f_4(\xi,\alpha)(\partial^2 \lambda^{a\hat{I}}) \bar{\zeta}^a_{\hat{I}} \Bigg] , \qquad (387)
\end{aligned}
$$

with $f^{ab}_1(\xi,\alpha)$, $f^{ab}_2(\xi,\alpha)$, $f_3(\xi,\alpha)\,, f_4(\xi,\alpha)$ and $f^{ab}(\xi,\alpha)$ arbitrary functions of ξ^a and α. Invoking again the limits (378) and (380), one is able to conclude that

$$
\begin{aligned}
f^{ab}_1(\xi,\alpha) &= \delta^{ab} d_1 , \\
f^{ab}_2(\xi,\alpha) &= \delta^{ab} d_2 , \\
f_3(\xi,\alpha) &= d_3 , \\
f_4(\xi,\alpha) &= d_4 , \qquad (388)
\end{aligned}
$$

where (d_1, d_2, d_3, d_4) are free parameters which has the possibility of being dependent of the gauge parameter α. Applying $\mathcal{B}_\Sigma$ on $\Delta^{(-1)}$, one has

$$
\begin{aligned}
\mathcal{B}_\Sigma\Delta^{(-1)} &= \int d^4x \Bigg[d_1\left(\frac{\delta\Sigma}{\delta A^a_\mu} + i\partial_\mu b^a A^a_\mu\right) - d_1(\Omega^a_\mu + \partial_\mu\bar{c}^a)\frac{\delta\Sigma}{\delta\Omega^a_\mu} + d_2\left(\frac{\delta\Sigma}{\delta L^a}L^a + \frac{\delta\Sigma}{\delta c^a}c^a\right) \\
&+ \frac{\delta\Sigma}{\delta\xi^a}f^{ab}(\xi)\xi^b - K^b\frac{\delta\Sigma}{\delta K^a}\left(\frac{\partial f^{bc}}{\partial\xi^a}\xi^c + f^{ba}(\xi)\right) + d_3\left(\bar{\psi}^{i\alpha}\Upsilon^i_\alpha + \bar{\Upsilon}^{i\alpha}\psi^i_\alpha\right) \\
&+ b_1 f^{abc}(A^h)^c_\mu\Big(U^{aI}_\mu\bar{\omega}^{bI} + V^{aI}_\mu\bar{\varphi}^{bI} + M^{aI}_\mu\varphi^{bI} - N^{aI}_\mu\omega^{bI} \\
&- \omega^{aI}\partial_\mu\bar{\omega}^{bI} - \varphi^{aI}\partial_\mu\bar{\varphi}^{bI}\Big) - b_1\Big(U^{aI}_\mu\partial_\mu\bar{\omega}^{aI} \\
&+ V^{aI}_\mu\partial_\mu\bar{\varphi}^{aI} + M^{aI}_\mu\partial_\mu\varphi^{aI} - N^{aI}_\mu\partial_\mu\omega^{aI} + (\partial_\mu\bar{\varphi}^{aI})\partial_\mu\varphi^{aI} \\
&- (\partial_\mu\bar{\omega}^{aI})\partial_\mu\omega^{aI} + \bar{\lambda}^{a\hat{I}}\partial^2\lambda^a_{\hat{I}} + \zeta^{a\hat{I}}\partial^2\bar{\zeta}^a_{\hat{I}}\Big) + b_4\Big(\bar{\lambda}^{a\hat{I}}\,T^{a,ij}\,\psi^{h,i\alpha}\Lambda^j_{\alpha\hat{I}} \\
&- T^{a,ij}\,\Pi^{j\alpha\hat{I}}\psi^{h,i}_\alpha\bar{\zeta}^a_{\hat{I}} - \zeta^{a\hat{I}}\,T^{a,ij}\,\bar{\psi}^{h,i\alpha}\bar{\Pi}^j_{\alpha\hat{I}} + T^{a,ij}\,\bar{\Lambda}^{j\alpha\hat{I}}\bar{\psi}^{h,i}_\alpha\lambda^a_{\hat{I}}\Big) \\
&+ \chi\frac{\partial d_1}{\partial\alpha}(\Omega^a_\mu + \partial_\mu\bar{c}^a)A^a_\mu + \chi\frac{\partial d_2}{\partial\alpha}L^a c^a \\
&+ \chi K^a\frac{\partial f^{ab}}{\partial\alpha}\xi^b + \chi\bar{\zeta}^a_{\hat{I}}\frac{\partial d_4}{\partial\alpha}(\partial^2\lambda^{a\hat{I}})\Bigg]. \qquad (389)
\end{aligned}
$$

Thus, substituting the expressions (385) and (389) in (345), the most general, invariant and local counterterm will be given by

$$
\begin{aligned}
\Sigma_{\mathrm{CT}} &= \Delta + a_1\int d^4x\, J_\psi\,\bar{\psi}^i_\alpha\psi^{i\alpha} + \mathcal{B}_\Sigma\Delta^{(-1)} \\
&= \int d^4x\,\Bigg[\frac{a_0}{4g^2}F^a_{\mu\nu}F^a_{\mu\nu} + a_1\,J_\psi\,\bar{\psi}^{i\alpha}\psi^i_\alpha + b_1\left(\mathcal{J}^a_\mu A^{h,a}_\mu - (\partial_\mu\bar{\eta}^a + \Xi^a_\mu)D^{ab}_\mu(A^h)\eta^b\right) \\
&+ \left(b_2\frac{J}{2} + b_2'\frac{J^2_\psi}{2}\right)A^{h,a}_\mu A^{h,a}_\mu + \frac{\delta\Sigma}{\delta\xi^a}f^{ab}(\xi)\xi^b \\
&+ d_1\left(\frac{\delta\Sigma}{\delta A^a_\mu} + i\partial_\mu b^a A^a_\mu - (\Omega^a_\mu + \partial_\mu\bar{c}^a)\frac{\delta\Sigma}{\delta\Omega^a_\mu}\right) + d_2\left(\frac{\delta\Sigma}{\delta L^a}L^a + \frac{\delta\Sigma}{\delta c^a}c^a\right) \\
&- K^b\frac{\delta\Sigma}{\delta K^a}\left(\frac{\partial f^{bc}}{\partial\xi^a}\xi^c + f^{ba}(\xi)\right) + d_3\left(\bar{\psi}^{i\alpha}\Upsilon^i_\alpha + \bar{\Upsilon}^{i\alpha}\psi^i_\alpha\right) + b_1 f^{abc}A^{h,c}_\mu\Big(U^{aI}_\mu\bar{\omega}^{bI} + \\
&+ V^{aI}_\mu\bar{\varphi}^{bI} + M^{aI}_\mu\varphi^{bI} - N^{aI}_\mu\omega^{bI} - \omega^{aI}\partial_\mu\bar{\omega}^{bI} - \varphi^{aI}\partial_\mu\bar{\varphi}^{bI}\Big) \\
&- b_1\Big(U^{aI}_\mu\partial_\mu\bar{\omega}^{aI} + V^{aI}_\mu\partial_\mu\bar{\varphi}^{aI} + M^{aI}_\mu\partial_\mu\varphi^{aI} - N^{aI}_\mu\partial_\mu\omega^{aI} \\
&+ (\partial_\mu\bar{\varphi}^{aI})\partial_\mu\varphi^{aI} - (\partial_\mu\bar{\omega}^{aI})\partial_\mu\omega^{aI} + \bar{\lambda}^{a\hat{I}}\partial^2\lambda^a_{\hat{I}} + \zeta^{a\hat{I}}\partial^2\bar{\zeta}^a_{\hat{I}}\Big) + b_4\Big(\bar{\Theta}^{i\alpha}\psi^{h,i}_\alpha + \Theta^{i\alpha}\bar{\psi}^{h,i}_\alpha \\
&+ \bar{\lambda}^{a\hat{I}}\,T^{a,ij}\,\psi^{h,i\alpha}\Lambda^j_{\alpha\hat{I}} - T^{a,ji}\,\Pi^{i\alpha\hat{I}}\psi^{h,j}_\alpha\bar{\zeta}^a_{\hat{I}} - \zeta^{a\hat{I}}\,T^{a,ij}\,\bar{\psi}^{h,i\alpha}\bar{\Pi}^j_{\alpha\hat{I}} + T^{a,ji}\,\bar{\Lambda}^{i\alpha\hat{I}}\bar{\psi}^{h,j}_\alpha\lambda^a_{\hat{I}}
\end{aligned}
$$

$$+ \quad \bar{\Phi}^{i\alpha}\, T^{a,ij}\, \eta^a \psi^{h,j}_\alpha + \Phi^{i\alpha}\, T^{a,ij}\, \eta^a \bar{\psi}^{h,j}_\alpha \Big) + \chi \frac{\partial d_1}{\partial \alpha}(\Omega^a_\mu + \partial_\mu \bar{c}^a) A^a_\mu + \chi \frac{\partial d_2}{\partial \alpha} L^a c^a$$
$$+ \quad \chi K^a \frac{\partial f^{ab}}{\partial \alpha} \xi^b + \chi \bar{\zeta}^a_{\hat{I}} \frac{\partial d_4}{\partial \alpha} (\partial^2 \lambda^{a\hat{I}}) \Big] . \tag{390}$$

Therefore, eq. (390) has been obtained and it is consistent in relation to the set of Ward identities, in the sequence we have to verify the stability of the theory, i.e. if (390) can be reabsorbed in the original full action, (305), via redefining the fields, parameters and sources.

6.2 **Parametric form of the counterterm**

In order to study the stability of (390), it seens very appropriate to treat this case by transforming the counterterm in the parametric form, see (87, 75, 83, 85). With this purpose, we consider the counterterm (390) as

$$\Sigma_{\mathrm{CT}} = \sum_{n=1}^{11} \Sigma^{\mathrm{CT}}_n \tag{391}$$

where

$$\begin{aligned}
\Sigma^{\mathrm{CT}}_1 &= \frac{a_0}{4g^2} \int d^4x\ F^a_{\mu\nu} F^a_{\mu\nu}\,, \\
\Sigma^{\mathrm{CT}}_2 &= a_1 \int d^4x\ J_\psi\, \bar{\psi}^{i\alpha} \psi^i_\alpha\,, \\
\Sigma^{\mathrm{CT}}_3 &= d_3 \int d^4x \left(\bar{\psi}^{i\alpha} \Upsilon^i_\alpha + \bar{\Upsilon}^{i\alpha} \psi^i_\alpha \right), \\
\Sigma^{\mathrm{CT}}_4 &= b_1 \int d^4x\ \mathcal{J}^a_\mu A^{h,a}_\mu\,, \\
\Sigma^{\mathrm{CT}}_5 &= b_1 \int d^4x \left(\tau^a \partial_\mu A^{h,a}_\mu - (\partial_\mu \bar{\eta}^a + \Xi^a_\mu) D^{ab}_\mu (A^h) \eta^b \right), \\
\Sigma^{\mathrm{CT}}_6 &= \int d^4x \left(b_2 \frac{J}{2} + b_2' \frac{J^2_\psi}{2} \right) A^{h,a}_\mu A^{h,a}_\mu\,, \\
\Sigma^{\mathrm{CT}}_7 &= d_1 \int d^4x \Big(- i b^a \partial_\mu A^a_\mu \Big), \\
\Sigma^{\mathrm{CT}}_8 &= d_1 \int d^4x\ \bar{c}^a \partial_\mu \frac{\delta \Sigma}{\delta \Omega^a_\mu}\,,
\end{aligned}$$

$$
\begin{aligned}
\Sigma_9^{\rm CT} &= \int d^4x \left(d_1 A^a_\mu \frac{\delta\Sigma}{\delta A^a_\mu} - d_1 \Omega^a_\mu \frac{\delta\Sigma}{\delta\Omega^a_\mu} + d_2 L^a \frac{\delta\Sigma}{\delta L^a} + d_2 c^a \frac{\delta\Sigma}{\delta c^a} + f^{ab}(\xi)\xi^b \frac{\delta\Sigma}{\delta\xi^a} \right. \\
&- \left. K^a \frac{\delta\Sigma}{\delta K^a}\frac{\partial f^{bc}}{\partial\xi^a}\xi^c - K^b \frac{\delta\Sigma}{\delta K^a} f^{ba}(\xi) \right) , \\
\Sigma_{10}^{\rm CT} &= b_1 \int d^4x \left(\bar{\varphi}^{ai}\mathcal{M}^{ab}(A^h)\varphi^{bi} - \bar{\omega}^{ai}\mathcal{M}^{ab}(A^h)\omega^{bi} + U^{ai}_\mu D^{ab}_\mu(A^h)\bar{\omega}^{bi} + V^{ai} D^{ab}_\mu(A^h)\bar{\varphi}^{bi} \right. \\
&+ \left. M^{ai}_\mu D^{ab}_\mu(A^h)\varphi^{bi} - N^{ai}_\mu D^{ab}_\mu(A^h)\omega^{bi} - \bar{\lambda}^{a\hat{I}}\partial^2\lambda^a_{\hat{I}} - \zeta^{a\hat{I}}\partial^2\bar{\zeta}^a_{\hat{I}} \right) , \\
\Sigma_{11}^{\rm CT} &= b_4 \int d^4x \left(\bar{\Theta}^{i\alpha}\psi^{h,i}_\alpha + \Theta^{i\alpha}\bar{\psi}^{h,i}_\alpha + \bar{\lambda}^{a\hat{I}}\, T^{a,ij}\, \psi^{h,i\alpha}\Lambda^j_{\alpha\hat{I}} - T^{a,ji}\,\Pi^{i\alpha\hat{I}}\psi^{h,j}_\alpha\bar{\zeta}^a_{\hat{I}} \right. \\
&- \left. \zeta^{a\hat{I}}\, T^{a,ij}\, \bar{\psi}^{h,i\alpha}\bar{\Pi}^j_{\alpha\hat{I}} + T^{a,ji}\bar{\Lambda}^{i\alpha\hat{I}}\bar{\psi}^{h,j}_\alpha\lambda^a_{\hat{I}} + \bar{\Phi}^{i\alpha}T^{a,ij}\,\eta^a\psi^{h,j}_\alpha + \Phi^{i\alpha}\,T^{a,ij}\,\eta^a\bar{\psi}^{h,j}_\alpha \right) .
\end{aligned}
\tag{392}
$$

It is easy to notice that is possible to write[32]

$$
\Sigma_1^{\rm CT} = -a_0 g^2 \frac{\partial\Sigma}{\partial g^2} \tag{393}
$$

and

$$
\Sigma_2^{\rm CT} = a_1\, J_\psi \frac{\delta\Sigma}{\delta J_\psi} \,. \tag{394}
$$

Looking the functional equation for the sources $(\Upsilon, \bar{\Upsilon})$, we have

$$
\begin{aligned}
\frac{\delta\Sigma}{\delta\Upsilon^i_\alpha} &= -i\bar{\psi}^{j\alpha}\,T^{a,ji}\,c^a \,, \\
\frac{\delta\Sigma}{\delta\bar{\Upsilon}^i_\alpha} &= -iT^{a,ij}\,c^a\psi^{j\alpha} \,,
\end{aligned}
$$

which give us

$$
\Sigma_3^{\rm CT} = d_3\left(\frac{\delta\Sigma}{\delta\Upsilon^i_\alpha}\Upsilon^i_\alpha + \bar{\Upsilon}^i_\alpha\frac{\delta\Sigma}{\delta\bar{\Upsilon}^i_\alpha} - \frac{\delta\Sigma}{\delta\psi^i_\alpha}\psi^i_\alpha - \bar{\psi}^i_\alpha\frac{\delta\Sigma}{\delta\bar{\psi}^i_\alpha} \right) . \tag{395}
$$

---

[32] Modulo vacuum terms that we are neglecting.

Additionally, it is able to be done

$$
\begin{aligned}
\frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu} &= A^{h,a}_\mu \,, \\
\frac{\delta\Sigma}{\delta\tau^a} &= \partial_\mu A^{h,a}_\mu \,, \\
\frac{\delta\Sigma}{\delta\bar{\eta}^a} &= \partial_\mu D^{ab}_\mu(A^h)\eta^b \,, \\
\frac{\delta\Sigma}{\delta\eta^a} &= \Bigg(- D^{ab}_\mu(A^h)\partial_\mu\bar{\eta}^b - \bar{Y}^{abI}\varphi^{bI} + \bar{X}^{abI}\omega^{bI} + X^I\bar{\omega}^{aI} - Y^I\bar{\varphi}^{bI} - D^{ab}_\mu(A^h)\Xi^b_\mu \\
&- Z^{\hat{I}}\bar{\lambda}^a_{\hat{I}} + W^{\hat{I}}\bar{\zeta}^a_{\hat{I}} - \bar{Z}^{ab\hat{I}}\lambda^b_{\hat{I}} + \bar{W}^{ab\hat{I}}\zeta^b_{\hat{I}} + 2\Gamma^{ab}\eta^b\Bigg) \,, \\
\frac{\delta\Sigma}{\delta X^I} &= \eta^a\bar{\omega}^{aI} \,, \qquad \frac{\delta\Sigma}{\delta Y^I} = \eta^a\bar{\varphi}^{aI} \,, \qquad \frac{\delta\Sigma}{\delta\bar{X}^{abI}} = \eta^a\omega^{bI} \,, \qquad \frac{\delta\Sigma}{\delta\bar{Y}^{abI}} = \eta^a\varphi^{bI} \,, \\
\frac{\delta\Sigma}{\delta Z_{\hat{I}}} &= -\eta^a\bar{\lambda}^a_{\hat{I}} \,, \qquad \frac{\delta\Sigma}{\delta W_{\hat{I}}} = -\eta^a\bar{\zeta}^a_{\hat{I}} \,, \qquad \frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}} = -\eta^a\lambda^b_{\hat{I}} \,, \qquad \frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}} = -\eta^a\zeta^b_{\hat{I}} \,, \\
\frac{\delta\Sigma}{\delta\Gamma^{ab}} &= \eta^a\eta^b \,, \\
\frac{\delta\Sigma}{\delta J} &= A^{h,a}_\mu A^{h,a}_\mu \,,
\end{aligned} \tag{396}
$$

with the set (396), we can write the following expressions

$$
\begin{aligned}
\Sigma^{\mathrm{CT}}_4 &= b_1\int d^4x\; \mathcal{J}^a_\mu\frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu} \,, \\
\Sigma^{\mathrm{CT}}_5 &= b_1\int d^4x\Bigg[\tau^a\frac{\delta\Sigma}{\delta\tau^a} + \frac{1}{2}\Bigg(\bar{\eta}^a\frac{\delta\Sigma}{\delta\bar{\eta}^a} + \eta^a\frac{\delta\Sigma}{\delta\eta^a} + \Xi^a_\mu\frac{\delta\Sigma}{\delta\Xi^a_\mu} - X^I\frac{\delta\Sigma}{\delta X^I} - Y^I\frac{\delta\Sigma}{\delta Y^I} \\
&- \bar{X}^{abI}\frac{\delta\Sigma}{\delta\bar{X}^{abI}} - \bar{Y}^{abI}\frac{\delta\Sigma}{\delta\bar{Y}^{abI}} - Z^{\hat{I}}\frac{\delta\Sigma}{\delta Z_{\hat{I}}} - W^{\hat{I}}\frac{\delta\Sigma}{\delta W_{\hat{I}}} - \bar{Z}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}} - \bar{W}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}} \\
&- \Gamma^{ab}\frac{\delta\Sigma}{\delta\Gamma^{ab}}\Bigg)\Bigg] \,, \\
\Sigma^{\mathrm{CT}}_6 &= \int d^4x\;\frac{1}{2}\,(b_2J + b'_2J^2_\psi)\frac{\delta\Sigma}{\delta J} \,.
\end{aligned} \tag{397}
$$

Regarding the term Σ^{CT}_7, the parametric form is obtained by considering the following equations

$$
\frac{\delta\Sigma}{\delta b^a} = i\partial_\mu A^a_\mu + \alpha b^a \,, \qquad \frac{\partial\Sigma}{\partial\alpha} = \frac{1}{2}\int d^4x\, b^ab^a \,. \tag{398}
$$

Thus,

$$
\Sigma^{\mathrm{CT}}_7 = -d_1\int d^4x\; b^a\frac{\delta\Sigma}{\delta b^a} + 2d_1\alpha\frac{\partial\Sigma}{\partial\alpha} \,. \tag{399}
$$

Currently, to establish the parametric form of the term $\Sigma_8^{\rm CT}$ one needs

$$\frac{\delta\Sigma}{\delta\bar{c}^a} = -\partial_\mu \frac{\delta\Sigma}{\delta\Omega^a_\mu}\,, \tag{400}$$

which allows

$$\Sigma_8^{\rm CT} = -d_1 \int d^4x\ \bar{c}^a \frac{\delta\Sigma}{\delta\bar{c}^a}\,. \tag{401}$$

To find the expression of $\Sigma_{10}^{\rm CT}$ in parametric form, we need to employ the following set of relations

$$\begin{aligned}
\int d^4x\ \bar{\varphi}^{aI}\frac{\delta\Sigma}{\delta\bar{\varphi}^{aI}} &= \int d^4x\Big[\bar{\varphi}^{aI}\partial^2\varphi^{aI} - f^{abc}A^{h,c}_\mu\varphi^{aI}\partial_\mu\bar{\varphi}^{bI} - V^{aI}_\mu\partial_\mu\bar{\varphi}^{aI} \\
&+ f^{abc}A^{h,c}_\mu V^{aI}_\mu\bar{\varphi}^{bI} - J_\varphi\,\bar{\varphi}^{aI}\varphi^{aI} + Y^I\eta^a\bar{\varphi}^{aI}\Big]\,, \\
\int d^4x\ \bar{\omega}^{aI}\frac{\delta\Sigma}{\delta\bar{\omega}^{aI}} &= \int d^4x\Big[-\bar{\omega}^{aI}\partial^2\omega^{aI} + f^{abc}\bar{\omega}^{aI}\partial_\mu(A^{h,c}_\mu\omega^{bI}) - \bar{\omega}^{aI}\partial_\mu U^{aI}_\mu \\
&- f^{bac}\bar{\omega}^{aI}U^{bI}_\mu A^{h,c}_\mu + J_\varphi\,\bar{\omega}^{aI}\omega^{aI} + K_\varphi\,\bar{\omega}^{aI}\varphi^{aI} + X^I\eta^a\bar{\omega}^{aI}\Big]\,, \\
\int d^4x\ \varphi^{aI}\frac{\delta\Sigma}{\delta\varphi^{aI}} &= \int d^4x\Big[\varphi^{aI}\partial^2\bar{\varphi}^{aI} + f^{abc}\varphi^{bI}(\partial_\mu\bar{\varphi}^{aI})A^{h,c}_\mu + \varphi^{aI}\partial_\mu M^{aI}_\mu \\
&+ f^{abc}\varphi^{bI}M^{aI}_\mu A^{h,c}_\mu - J_\varphi\,\bar{\varphi}^{aI}\varphi^{aI} + K_\varphi\,\bar{\omega}^{aI}\varphi^{aI} + \bar{Y}^{abI}\eta^a\varphi^{bI}\Big]\,, \\
\int d^4x\ \omega^{aI}\frac{\delta\Sigma}{\delta\omega^{aI}} &= \int d^4x\Big[\omega^{aI}\partial^2\bar{\omega}^{aI} + f^{abc}\omega^{bI}(\partial_\mu\bar{\omega}^{aI})A^{h,c}_\mu + \omega^{aI}\partial_\mu N^{aI}_\mu \\
&+ f^{abc}\omega^{bI}N^{aI}_\mu A^{h,c}_\mu - J_\varphi\,\omega^{aI}\bar{\omega}^{aI} + \bar{X}^{abI}\eta^a\omega^{bI}\Big]\,, \\
\int d^4x\ M^{aI}_\mu\frac{\delta\Sigma}{\delta M^{aI}_\mu} &= -\int d^4x\, M^{aI}_\mu D^{ab}_\mu(A^h)\varphi^{bI}\,, \\
\int d^4x\ V^{aI}_\mu\frac{\delta\Sigma}{\delta V^{aI}_\mu} &= -\int d^4x\, V^{aI}_\mu D^{ab}_\mu(A^h)\bar{\varphi}^{bI}\,, \\
\int d^4x\ N^{aI}_\mu\frac{\delta\Sigma}{\delta N^{aI}_\mu} &= \int d^4x\, N^{aI}_\mu D^{ab}_\mu(A^h)\omega^{bI}\,, \\
\int d^4x\ U^{aI}_\mu\frac{\delta\Sigma}{\delta U^{aI}_\mu} &= -\int d^4x\, U^{aI}_\mu D^{ab}_\mu(A^h)\bar{\omega}^{bI}\,,
\end{aligned}$$

$$
\begin{aligned}
\int d^4x\ \bar{\lambda}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\lambda}^a_{\hat{I}}} &= \int d^4x\Big[-\bar{\lambda}^{a\hat{I}}\partial_\mu D^{ab}_\mu(A^h)\lambda^b_{\hat{I}} + \bar{\lambda}^{a\hat{I}}\Lambda^{i\alpha}_{\hat{I}}\,T^{a,ij}\,\psi^{h,j}_\alpha - J_\lambda\,\bar{\lambda}^{a\hat{I}}\lambda^a_{\hat{I}} \\
&+ K_\lambda\,\bar{\lambda}^{a\hat{I}}\zeta^a_{\hat{I}} + \bar{\lambda}^{a\hat{I}}Z_{\hat{I}}\,\eta^a\Big], \\
\int d^4x\ \lambda^{a\hat{I}}\frac{\delta\Sigma}{\delta\lambda^a_{\hat{I}}} &= \int d^4x\Big[\lambda^{a\hat{I}}D^{ba}_\mu(A^h)\partial_\mu\bar{\lambda}^b_{\hat{I}} - \lambda^{a\hat{I}}\bar{\Lambda}^j_{\alpha\hat{I}}\bar{\psi}^{h,i\alpha}\,T^{a,ij} - J_\lambda\,\lambda^{a\hat{I}}\bar{\lambda}^a_{\hat{I}} \\
&+ \lambda^{a\hat{I}}\bar{Z}^{ba}_{\hat{I}}\,\eta^b\Big], \\
\int d^4x\ \bar{\zeta}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} &= \int d^4x\Big[-\bar{\zeta}^{a\hat{I}}\partial_\mu D^{ab}_\mu(A^h)\zeta^b_{\hat{I}} + \bar{\zeta}^{a\hat{I}}\Pi^{i\alpha}_{\hat{I}}\,T^{a,ij}\,\psi^{h,j}_\alpha + J_\lambda\bar{\zeta}^{a\hat{I}}\zeta^a_{\hat{I}} \\
&- \bar{\zeta}^{a\hat{I}}W_{\hat{I}}\,\eta^a\Big], \\
\int d^4x\ \zeta^{a\hat{I}}\frac{\delta\Sigma}{\delta\zeta^a_{\hat{I}}} &= \int d^4x\Big[-\bar{\zeta}^{a\hat{I}}\partial_\mu D^{ab}_\mu(A^h)\zeta^b_{\hat{I}} - \bar{\Pi}^{j\alpha\hat{I}}\bar{\psi}^{h,i\alpha}\,T^{a,ij}\,\zeta^a_{\hat{I}} + J_\lambda\,\bar{\zeta}^{a\hat{I}}\zeta^a_{\hat{I}} \\
&- \bar{W}^{ab\hat{I}}\eta^a\zeta^b_{\hat{I}}\Big]
\end{aligned}
\tag{402}
$$

and

$$
\frac{\delta\Sigma}{\delta J_\varphi} = \bar{\omega}^{aI}\omega^{aI} - \bar{\varphi}^{aI}\varphi^{aI}\,, \qquad \frac{\delta\Sigma}{\delta K_\varphi} = \bar{\omega}^{aI}\varphi^{aI}, \qquad \frac{\delta\Sigma}{\delta J_\lambda} = \bar{\lambda}^{a\hat{I}}\lambda^a_{\hat{I}} + \bar{\zeta}^{a\hat{I}}\zeta^a_{\hat{I}}\,,
$$
$$
\frac{\delta\Sigma}{\delta K_\lambda} = \bar{\lambda}^{a\hat{I}}\zeta^a_{\hat{I}}\,. \tag{403}
$$

Therefore, we have

$$
\begin{aligned}
\Sigma^{\rm CT}_{10} &= -\frac{b_1}{2}\int d^4x\Bigg(\bar{\varphi}^{aI}\frac{\delta\Sigma}{\delta\bar{\varphi}^{aI}} + \varphi^{aI}\frac{\delta\Sigma}{\delta\varphi^{aI}} + \bar{\omega}^{aI}\frac{\delta\Sigma}{\delta\bar{\omega}^{aI}} + \omega^{aI}\frac{\delta\Sigma}{\delta\omega^{aI}} + M^{aI}_\mu\frac{\delta\Sigma}{\delta M^{aI}_\mu} \\
&+ V^{aI}_\mu\frac{\delta\Sigma}{\delta V^{ai}_\mu} + N^{aI}_\mu\frac{\delta\Sigma}{\delta N^{aI}_\mu} + U^{aI}\frac{\delta\Sigma}{\delta U^{aI}_\mu} + 2J_\varphi\frac{\delta\Sigma}{\delta J_\varphi} + 2K_\varphi\frac{\delta\Sigma}{\delta K_\varphi} + 2J_\lambda\frac{\delta\Sigma}{\delta J_\lambda} + 2K_\lambda\frac{\delta\Sigma}{\delta K_\lambda} \\
&+ \bar{\lambda}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\lambda}^a_{\hat{I}}} + \lambda^{a\hat{I}}\frac{\delta\Sigma}{\delta\lambda^a_{\hat{I}}} - \bar{\zeta}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} - \zeta^{a\hat{I}}\frac{\delta\Sigma}{\delta\zeta^a_{\hat{I}}} - X^I\frac{\delta\Sigma}{\delta X^I} - Y^I\frac{\delta\Sigma}{\delta Y^I} - \bar{X}^{abI}\frac{\delta\Sigma}{\delta\bar{X}^{abI}} \\
&- \bar{Y}^{abI}\frac{\delta\Sigma}{\delta\bar{Y}^{abI}} - Z^{\hat{I}}\frac{\delta\Sigma}{\delta Z_{\hat{I}}} - W^{\hat{I}}\frac{\delta\Sigma}{\delta W_{\hat{I}}} - \bar{Z}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}} - \bar{W}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}} - \Pi^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Pi^i_{\alpha\hat{I}}} \\
&- \bar{\Pi}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} - \bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Lambda}^i_{\alpha\hat{I}}} - \Lambda^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Lambda^i_{\alpha\hat{I}}}\Bigg)\,.
\end{aligned}
\tag{404}
$$

In order to obtain the counterterm $\Sigma_{11}^{\rm CT}$ in the parametric form, one should employ the equations

$$
\begin{aligned}
\int d^4x\ \bar{\Pi}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} &= \int d^4x\,\bar{\Pi}^{i\alpha\hat{I}}\bar{\psi}^{h,j}_\alpha\,T^{a,ji}\,\zeta^a_{\hat{I}}\,,\\
\int d^4x\ \Pi^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Pi^i_{\alpha\hat{I}}} &= \int d^4x\,\Pi^{i\alpha\hat{I}}\bar{\zeta}^a_{\hat{I}}\,T^{a,ij}\,\psi^{h,j}_\alpha\,,\\
\int d^4x\ \bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Lambda}^i_{\alpha\hat{I}}} &= \int d^4x\,\bar{\Lambda}^{i\alpha\hat{I}}\bar{\psi}^{h,j}_\alpha\,T^{a,ji}\,\lambda^a_{\hat{I}}\,,\\
\int d^4x\ \Lambda^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Lambda^i_{\alpha\hat{I}}} &= \int d^4x\,\Lambda^{i\alpha\hat{I}}\bar{\lambda}^a_{\hat{I}}\,T^{a,ij}\,\psi^{h,j}_\alpha\,,
\end{aligned}
\tag{405}
$$

Thus, $\Sigma_{11}^{\rm CT}$ is written as

$$
\begin{aligned}
\Sigma_{11}^{\rm CT} &= b_4\int d^4x\Bigg[\Theta^{i\alpha}\frac{\delta\Sigma}{\delta\Theta^{i\alpha}}+\bar{\Theta}^{i\alpha}\frac{\delta\Sigma}{\delta\bar{\Theta}^{i\alpha}}-\frac{1}{2}\Bigg(\lambda^{a\hat{I}}\frac{\delta\Sigma}{\delta\lambda^{a\hat{I}}}-\bar{\lambda}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\lambda}^{a\hat{I}}}+\zeta^{a\hat{I}}\frac{\delta\Sigma}{\delta\zeta^{a\hat{I}}}-\bar{\zeta}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\zeta}^{a\hat{I}}}\\
&+ \Pi^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Pi^i_{\alpha\hat{I}}}-\bar{\Pi}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Pi}^i_{\alpha\hat{I}}}+\bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Lambda}^i_{\alpha\hat{I}}}-\Lambda^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Lambda^i_{\alpha\hat{I}}}+Z^{\hat{I}}\frac{\delta\Sigma}{\delta Z_{\hat{I}}}-\bar{Z}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}}\\
&+ W^{\hat{I}}\frac{\delta\Sigma}{\delta W_{\hat{I}}}-\bar{W}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}}\Bigg)+\Phi^{i\alpha}\frac{\delta\Sigma}{\delta\Phi^i_\alpha}+\bar{\Phi}^{i\alpha}\frac{\delta\Sigma}{\delta\bar{\Phi}^i_\alpha}\Bigg]\,.
\end{aligned}
\tag{406}
$$

Finally, the most general counterterm $\Sigma^{\rm CT}$ in the parametric form is

$$
\begin{aligned}
\Sigma^{\rm CT} &= -a_0g^2\frac{\partial\Sigma}{\partial g^2}+a_1\int d^4x\,J_\psi\frac{\delta\Sigma}{\delta J_\psi}+b_1\int d^4x\Bigg[\mathcal{J}^a_\mu\frac{\delta\Sigma}{\delta\mathcal{J}^a_\mu}+\tau^a\frac{\delta\Sigma}{\delta\tau^a}+\frac{1}{2}\Bigg(\bar{\eta}^a\frac{\delta\Sigma}{\delta\bar{\eta}^a}+\eta^a\frac{\delta\Sigma}{\delta\eta^a}\\
&+ \Xi^a_\mu\frac{\delta\Sigma}{\delta\Xi^a_\mu}-\Gamma^{ab}\frac{\delta\Sigma}{\delta\Gamma^{ab}}\Bigg)\Bigg]+d_3\Bigg(\frac{\delta\Sigma}{\delta\varUpsilon_\alpha{}^i}\varUpsilon_\alpha{}^i+\bar{\varUpsilon}^i_\alpha\frac{\delta\Sigma}{\delta\bar{\varUpsilon}^i_\alpha}-\frac{\delta\Sigma}{\delta\psi_\alpha{}^i}\psi_\alpha{}^i-\bar{\psi}^i_\alpha\frac{\delta\Sigma}{\delta\bar{\psi}^i_\alpha}\Bigg)\\
&+ \int d^4x\ \frac{1}{2}\left(b_2J+b_2'J_\psi^2\right)\frac{\delta\Sigma}{\delta J}-d_1\int d^4x\ b^a\frac{\delta\Sigma}{\delta b^a}+2d_1\alpha\frac{\partial\Sigma}{\partial\alpha}\\
&- d_1\int d^4x\ \bar{c}^a\frac{\delta\Sigma}{\delta\bar{c}^a}+\int d^4x\Bigg[d_1A^a_\mu\frac{\delta\Sigma}{\delta A^a_\mu}-d_1\Omega^a_\mu\frac{\delta\Sigma}{\delta\Omega^a_\mu}+d_2L^a\frac{\delta\Sigma}{\delta L^a}+d_2c^a\frac{\delta\Sigma}{\delta c^a}\\
&+ f^{ab}(\xi)\xi^b\frac{\delta\Sigma}{\delta\xi^a}-K^a\frac{\delta\Sigma}{\delta K^a}\left(\frac{\partial f^{bc}}{\partial\xi^a}\xi^c+f^{ba}(\xi)\right)\Bigg]-\frac{b_1}{2}\int d^4x\Bigg[\bar{\varphi}^{aI}\frac{\delta\Sigma}{\delta\bar{\varphi}^{aI}}+\varphi^{aI}\frac{\delta\Sigma}{\delta\varphi^{aI}}\\
&+ \bar{\omega}^{aI}\frac{\delta\Sigma}{\delta\bar{\omega}^{aI}}+\omega^{aI}\frac{\delta\Sigma}{\delta\omega^{aI}}+M^{aI}_\mu\frac{\delta\Sigma}{\delta M^{aI}_\mu}+V^{aI}_\mu\frac{\delta\Sigma}{\delta V^{aI}_\mu}+N^{aI}_\mu\frac{\delta\Sigma}{\delta N^{aI}_\mu}+U^{aI}_\mu\frac{\delta\Sigma}{\delta U^{aI}_\mu}\\
&+ 2J_\varphi\frac{\delta\Sigma}{\delta J_\varphi}+2K_\varphi\frac{\delta\Sigma}{\delta K_\varphi}+2J_\lambda\frac{\delta\Sigma}{\delta J_\lambda}+2K_\lambda\frac{\delta\Sigma}{\delta K_\lambda}\Bigg]+\int d^4x\Bigg[\left(\frac{b_4-b_1}{2}\right)\bar{\lambda}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\lambda}^a_{\hat{I}}}
\end{aligned}
$$

$$
\begin{aligned}
- & \left(\frac{b_1+b_4}{2}\right)\lambda^{a\hat{I}}\frac{\delta\Sigma}{\delta\lambda^a_{\hat{I}}} + \frac{b_4}{2}\left(\bar{Z}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{Z}^{ab}_{\hat{I}}} + \bar{W}^{ab\hat{I}}\frac{\delta\Sigma}{\delta\bar{W}^{ab}_{\hat{I}}}\right) \\
+ & \left(\frac{b_1+b_4}{2}\right)\left(\bar{\zeta}^{a\hat{I}}\frac{\delta\Sigma}{\delta\bar{\zeta}^a_{\hat{I}}} + \bar{\Pi}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} + \Lambda^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Lambda^i_{\alpha\hat{I}}} + \bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\bar{\Lambda}^i_{\alpha\hat{I}}}\right) \\
+ & \left(\frac{b_1-b_4}{2}\right)\left(\zeta^{a\hat{I}}\frac{\delta\Sigma}{\delta\zeta^a_{\hat{I}}} + \Pi^{i\alpha\hat{I}}\frac{\delta\Sigma}{\delta\Pi^i_{\alpha\hat{I}}}\right) - \frac{b_4}{2}\left(Z^{\hat{I}}\frac{\delta\Sigma}{\delta Z_{\hat{I}}} + W^{\hat{I}}\frac{\delta\Sigma}{\delta W_{\hat{I}}}\right)\Bigg] \\
+ & \; b_4\int d^4x\left[\Theta^{i\alpha}\frac{\delta\Sigma}{\delta\Theta^i_\alpha} + \bar{\Theta}^{i\alpha}\frac{\delta\Sigma}{\delta\bar{\Theta}^i_\alpha} + \Phi^{i\alpha}\frac{\delta\Sigma}{\delta\Phi^i_\alpha} + \bar{\Phi}^{i\alpha}\frac{\delta\Sigma}{\delta\bar{\Phi}^i_\alpha}\right],
\end{aligned}
\tag{407}
$$

which is possible to rewrite as

$$
\Sigma^{\mathrm{CT}} = \mathcal{R}\Sigma\,,
\tag{408}
$$

where $\mathcal{R}$ is the operator

$$
\begin{aligned}
\mathcal{R} \;=\; & -a_0 g^2\frac{\partial}{\partial g^2} + a_1\int d^4x\, J_\psi\frac{\delta}{\delta J_\psi} + b_1\int d^4x\Bigg[\mathcal{J}^a_\mu\frac{\delta}{\delta\mathcal{J}^a_\mu} + \tau^a\frac{\delta}{\delta\tau^a} + \frac{1}{2}\Bigg(\bar{\eta}^a\frac{\delta}{\delta\bar{\eta}^a} + \eta^a\frac{\delta}{\delta\eta^a} \\
+ & \;\Xi^a_\mu\frac{\delta}{\delta\Xi^a_\mu} - \Gamma^{ab}\frac{\delta}{\delta\Gamma^{ab}}\Bigg)\Bigg] + d_3\left(\frac{\delta}{\delta\Upsilon^i_\alpha}\Upsilon^i_\alpha + \bar{\Upsilon}^i_\alpha\frac{\delta}{\delta\bar{\Upsilon}^i_\alpha} - \frac{\delta}{\delta\psi^i_\alpha}\psi^i_\alpha - \bar{\psi}^i_\alpha\frac{\delta}{\delta\bar{\psi}^i_\alpha}\right) \\
+ & \int d^4x\;\frac{1}{2}\left(b_2 J + b_2' J_\psi^2\right)\frac{\delta}{\delta J} - d_1\int d^4x\; b^a\frac{\delta}{\delta b^a} + 2d_1\alpha\frac{\partial}{\partial\alpha} \\
- & \; d_1\int d^4x\;\bar{c}^a\frac{\delta}{\delta\bar{c}^a} + \int d^4x\Bigg[d_1 A^a_\mu\frac{\delta}{\delta A^a_\mu} - d_1\Omega^a_\mu\frac{\delta}{\delta\Omega^a_\mu} + d_2 L^a\frac{\delta}{\delta L^a} + d_2 c^a\frac{\delta}{\delta c^a} \\
+ & \; f^{ab}(\xi)\xi^b\frac{\delta}{\delta\xi^a} - K^a\frac{\delta}{\delta K^a}\left(\frac{\partial f^{bc}}{\partial\xi^a}\xi^c + f^{ba}(\xi)\right)\Bigg] - \frac{b_1}{2}\int d^4x\Bigg[\bar{\varphi}^{aI}\frac{\delta}{\delta\bar{\varphi}^{aI}} + \varphi^{aI}\frac{\delta}{\delta\varphi^{aI}} \\
+ & \;\bar{\omega}^{aI}\frac{\delta}{\delta\bar{\omega}^{aI}} + \omega^{aI}\frac{\delta}{\delta\omega^{aI}} + M^{aI}_\mu\frac{\delta}{\delta M^{aI}_\mu} + V^{aI}_\mu\frac{\delta}{\delta V^{aI}_\mu} + N^{aI}_\mu\frac{\delta}{\delta N^{aI}_\mu} + U^{aI}_\mu\frac{\delta}{\delta U^{aI}_\mu} \\
+ & \; 2J_\varphi\frac{\delta}{\delta J_\varphi} + 2K_\varphi\frac{\delta}{\delta K_\varphi} + 2J_\lambda\frac{\delta}{\delta J_\lambda} + 2K_\lambda\frac{\delta}{\delta K_\lambda}\Bigg] + \int d^4x\Bigg[\left(\frac{b_4-b_1}{2}\right)\bar{\lambda}^{a\hat{I}}\frac{\delta}{\delta\bar{\lambda}^a_{\hat{I}}} \\
- & \left(\frac{b_1+b_4}{2}\right)\lambda^{a\hat{I}}\frac{\delta}{\delta\lambda^a_{\hat{I}}} + \frac{b_4}{2}\left(\bar{Z}^{ab\hat{I}}\frac{\delta}{\delta\bar{Z}^{ab}_{\hat{I}}} + \bar{W}^{ab\hat{I}}\frac{\delta}{\delta\bar{W}^{ab}_{\hat{I}}}\right) \\
+ & \left(\frac{b_1+b_4}{2}\right)\left(\bar{\zeta}^{a\hat{I}}\frac{\delta}{\delta\bar{\zeta}^a_{\hat{I}}} + \bar{\Pi}^{i\alpha\hat{I}}\frac{\delta}{\delta\bar{\Pi}^i_{\alpha\hat{I}}} + \Lambda^{i\alpha\hat{I}}\frac{\delta}{\delta\Lambda^i_{\alpha\hat{I}}} + \bar{\Lambda}^{i\alpha\hat{I}}\frac{\delta}{\delta\bar{\Lambda}^i_{\alpha\hat{I}}}\right) \\
+ & \left(\frac{b_1-b_4}{2}\right)\left(\zeta^{a\hat{I}}\frac{\delta}{\delta\zeta^a_{\hat{I}}} + \Pi^{i\alpha\hat{I}}\frac{\delta}{\delta\Pi^i_{\alpha\hat{I}}}\right) - \frac{b_4}{2}\left(Z^{\hat{I}}\frac{\delta}{\delta Z_{\hat{I}}} + W^{\hat{I}}\frac{\delta}{\delta W_{\hat{I}}}\right)\Bigg] \\
+ & \; b_4\int d^4x\left[\Theta^{i\alpha}\frac{\delta}{\delta\Theta^i_\alpha} + \bar{\Theta}^{i\alpha}\frac{\delta}{\delta\bar{\Theta}^i_\alpha} + \Phi^{i\alpha}\frac{\delta}{\delta\Phi^i_\alpha} + \bar{\Phi}^{i\alpha}\frac{\delta}{\delta\bar{\Phi}^i_\alpha}\right],
\end{aligned}
\tag{409}
$$

The parametric form of the counterterm (408) is very practical to study the stability of the full action action Σ, a subject which will be set about in the next subsection.

6.3 Stability of the action and the renormalization factors

In order to complete the algebraic renormalization analysis of the model, we have to show that the counterterm (407) can be reabsorbed into the starting action (305) through a redefinition of fields, sources and parameters. These redefinitions are made in a simplest way using the parametric form (407). If it is possible to reabsorb this counterterm in the starting action, subsequently, for the first order in the parameter expansion ϵ, the following definition can be written (76),

$$\Sigma[\Phi_0] = \Sigma[\Phi] + \epsilon\Sigma^{\mathrm{CT}}[\Phi] + O(\epsilon)\,, \tag{410}$$

where Φ represents all fields, sources and parameters of the theory and the "0" subscribed represents the bare (nonrenormalized) quantities. From (408), we have

$$\Sigma[\Phi_0] = \Sigma[\Phi] + \epsilon\mathcal{R}\Sigma[\Phi] + O(\epsilon)\,, \tag{411}$$

and due to the form of $\mathcal{R}$, one can observes

$$\Phi_0 = (1+\epsilon\mathcal{R})\Phi\,. \tag{412}$$

Thereby, the fields, sources and parameters are redefined as

$$\begin{aligned}
A_0 &= Z_A^{1/2}A\,, \qquad b_0 = Z_b^{1/2}b\,, \qquad c_0 = Z_c^{1/2}c\,, \qquad \bar{c}_0 = Z_{\bar{c}}^{1/2}\bar{c}\,,\\
\xi_0^a &= Z_\xi^{ab}(\xi)\xi^b\,, \qquad \tau_0 = Z_\tau^{1/2}\tau\,, \qquad \eta_0 = Z_\eta^{1/2}\eta\,, \qquad \bar{\eta}_0 = Z_{\bar{\eta}}^{1/2}\bar{\eta}\,,\\
\bar{\psi}_0 &= Z_{\bar{\psi}}^{1/2}\bar{\psi}\,, \qquad \psi_0 = Z_\psi^{1/2}\psi\,, \qquad \bar{\Theta}_0 = Z_{\bar{\Theta}}\bar{\Theta}\,, \qquad \Theta_0 = Z_\Theta\Theta\,,\\
\bar{\varphi}_0 &= Z_{\bar{\varphi}}^{1/2}\bar{\varphi}\,, \qquad \varphi_0 = Z_\varphi^{1/2}\varphi\,, \qquad \bar{\omega}_0 = Z_{\bar{\omega}}^{1/2}\bar{\omega}\,, \qquad \omega_0 = Z_\omega^{1/2}\omega\,,\\
\Omega_0 &= Z_\Omega\Omega\,, \qquad L_0 = Z_L L\,, \qquad K_0^a = Z_K^{ab}(\xi)K^b\,, \qquad \mathcal{J}_0 = Z_{\mathcal{J}}\mathcal{J}\,,\\
J_{\varphi,0} &= Z_{J_\varphi}J_\varphi\,, \qquad K_{\varphi,0} = Z_{K_\varphi}K_\varphi\,, \qquad g_0 = Z_g g\,, \qquad J_{\psi,0} = Z_{J_\psi}J_\psi\,,\\
\alpha_0 &= Z_\alpha\alpha\,, \qquad M_0 = Z_M M\,, \qquad V_0 = Z_V V\,,\\
N_0 &= Z_N N\,, \qquad U_0 = Z_U U\,, \qquad \Xi_0 = Z_\Xi\,\Xi\,, \qquad X_0 = Z_X X\,,\\
Y_0 &= Z_Y Y\,, \qquad \bar{X}_0 = Z_{\bar{X}}\bar{X}\,, \qquad \bar{Y}_0 = Z_{\bar{Y}}\bar{Y}\,, \qquad \Gamma_0 = Z_\Gamma\Gamma\,,\\
\bar{\lambda}_0 &= Z_{\bar{\lambda}}^{1/2}\bar{\lambda}\,, \qquad \lambda_0 = Z_\lambda^{1/2}\lambda\,, \qquad \bar{\zeta}_0 = Z_{\bar{\zeta}}^{1/2}\bar{\zeta}\,, \qquad \zeta_0 = Z_\zeta^{1/2}\zeta\,,\\
J_{\lambda,0} &= Z_{J_\lambda}J_\lambda\,, \qquad K_{\lambda,0} = Z_{K_\lambda}K_\lambda\,,\\
\Pi_0 &= Z_\Pi\Pi\,, \qquad \bar{\Pi}_0 = Z_{\bar{\Pi}}\bar{\Pi}\,, \qquad \bar{\Lambda}_0 = Z_{\bar{\Lambda}}\bar{\Lambda}\,, \qquad \Lambda_0 = Z_\Lambda\Lambda\,.,\\
Z_0 &= Z_Z Z\,, \qquad \bar{Z}_0 = Z_{\bar{Z}}\bar{Z}\,, \qquad \bar{W}_0 = Z_{\bar{W}}\bar{W}\,, \qquad W_0 = Z_W W\,,\\
\Phi_0 &= Z_\Phi\Phi\,, \qquad \bar{\Phi}_0 = Z_{\bar{\Phi}}\bar{\Phi}\,, \qquad \bar{\Upsilon}_0 = Z_{\bar{\Upsilon}}\bar{\Upsilon}\,, \qquad \Upsilon_0 = Z_\Upsilon\Upsilon\,,
\end{aligned} \tag{413}$$

and

$$J_0 = J + \frac{\epsilon}{2}(b_2\, J + b_2'\, J_\psi^2)\,, \tag{414}$$

with

$$\begin{aligned}
Z_A^{1/2} &= 1+\epsilon d_1\,, \qquad Z_c^{1/2} = 1+\epsilon d_2\,, \qquad Z_g = 1-\epsilon\frac{a_0}{2}\,, \qquad Z_\tau^{1/2} = 1+\epsilon b_1\,,\\
Z_{\bar\Theta}^{1/2} &= Z_\Theta^{1/2} = Z_\Phi = Z_{\bar\Phi} = 1+\epsilon b_4\,,\\
Z_\Upsilon &= Z_{\bar\Upsilon} = Z_\psi^{-\frac{1}{2}} = Z_{\bar\psi}^{-\frac{1}{2}} = 1+\epsilon d_3\,, \qquad Z_{J_\psi} = 1+\epsilon a_1\,,\\
Z_\xi^{ab}(\xi) &= \delta^{ab}+\epsilon f^{ab}(\xi)\,,\\
Z_K^{ab}(\xi) &= \delta^{ab}-\epsilon\left(f^{ba}(\xi)+\frac{\partial f^{bc}}{\partial \xi^a}\xi^c\right)\,.
\end{aligned} \tag{415}$$

For the other fields, sources and parameters, the following relations hold

$$\begin{aligned}
Z_A^{1/2} &= Z_\Omega^{-1} = Z_{\bar c}^{-1/2} = Z_b^{-1/2} = Z_\alpha^{1/2}\,,\\
Z_\tau^{1/2} &= Z_{\bar\eta} = Z_\eta = Z_\Xi^2 = Z_\Gamma^2 = Z_{\mathcal{J}}\,,\\
Z_c^{1/2} &= Z_L\,, \qquad Z_X = Z_Y = Z_{\bar X} = Z_{\bar Y} = 1\,.
\end{aligned} \tag{416}$$

Looking at this moment for the renormalization factors of fields and sources introduced to implement the Gribov horizon related to the gauge-invariant field sector, we have

$$\begin{aligned}
Z_\tau^{-1/4} &= Z_{\bar\varphi}^{1/2} = Z_\varphi^{1/2} = Z_{\bar\omega}^{1/2} = Z_\omega^{1/2} = Z_M = Z_V = Z_N = Z_U\\
&= Z_{J_\varphi}^{1/2} = Z_{K_\varphi}^{1/2} = Z_{\tilde J_\lambda}^{1/2} = Z_{K_\lambda}^{1/2}\,,
\end{aligned} \tag{417}$$

while the renormalization factors of fields and sources associated to the horizon-like function of the fermionic gauge-invariant composite fields are given by

$$\begin{aligned}
Z_\lambda^{1/2} &= 1-\epsilon\left(\frac{b_1+b_4}{2}\right)\,,\\
Z_{\bar\lambda}^{1/2} &= 1+\epsilon\left(\frac{b_4-b_1}{2}\right)\,,\\
Z_{\bar\zeta} &= Z_{\bar\Pi} = Z_{\bar\Lambda} = Z_\Lambda = 1+\epsilon\left(\frac{b_1+b_4}{2}\right)\,,\\
Z_\zeta &= Z_\Pi = 1+\epsilon\left(\frac{b_1-b_4}{2}\right)\,,\\
Z_Z &= Z_W = 1-\epsilon\,\frac{b_4}{2}\,,\\
Z_{\bar Z} &= Z_{\bar W} = 1+\epsilon\,\frac{b_4}{2}\,.
\end{aligned} \tag{418}$$

To end up, we also have

$$Z_{\mathcal{J}} Z_{\varphi}^{1/2} Z_{\bar{\varphi}}^{1/2} = 1 \,, \qquad\qquad Z_{\mathcal{J}} Z_{\omega}^{1/2} Z_{\bar{\omega}}^{1/2} = 1 \,. \tag{419}$$

After doing a proper redefinition of fields, sources and parameters as reported in (415), (416), (417) and (418) the most general, local and invariant counterterm consistent with the Ward identities can be reabsorbed in the classical action (305). Therefore, by using the algebraic renormalization procedure (76) the theory is renormalizable at all orders in a loop expansion. Since the Stückelberg field has null dimension, the renormalization factors $(Z^{ab}_{\xi}(\xi), Z^{ab}_{K}(\xi))$ are nonlinear in ξ^a, which is a well-known behavior of dimensionless fields, for this we refer to (87, 75, 83, 85). It is important to point out that the renormalization factors related to the Gribov parameters γ^2 and σ^3 are not independent quantities of the model, *i.e.* they are expressed in terms of other renormalization factors. Finally, equations (419) tells us that the vertices $(A, \varphi, \bar{\varphi})$ and $(A, \omega, \bar{\omega})$ are nonrenormalizable as already observed in (63, 74).

CONCLUSIONS AND PERSPECTIVES

In the first part of this thesis, we have proposed by using the Serreau-Tissier gauge-fixing framework a good explanation for the origin of the gluon mass in the particular CF action in Landau gauge. This issue has been finally well-explained by the analogy between NLσ and Yang-Mills theories through the phenomenon of symmetry restoration, which cames originally from the NLσ model. This was possible since the Serreau-Tissier action has been written in an elegant supersymmetric manner, *i.e.*, in terms of a $SU(2)$ superfield V or analogously, by a unit norm vector n^A. Therefore, we have added a term which breaks explicitly the supersymmetry, all in all with this breaking event, a mass for the ghost condensate has been generated and a constraint term directly related with the NLσ model has been added. Moreover, the actions S_{gf} and $\widetilde{S}_{gf}$ have been responsible to give an equally contribution for the mass squared of the gluons which is equal to the parameter β_0. The contribution of $\widetilde{S}_{gf}$ to the square mass of the gluon was described in Eq. (158) by the third term with $(n^A)^2 = 1$. As the action $\widetilde{S}_{gf}$ came into view $(p-1)$ times because of the replica trick in the action Eq (155), the total gluon square mass has been determined as $\beta_0 + (p-1)\beta_0$, which in previous works tended to zero in the limit of vanishing number of replicas. Thus, in this thesis we have employed another viewpoint. We have used the tree-level equation of motion for the n which has admitted as a solution $n = 0$ if $\varsigma \neq 0$ and this matches with the symmetry restoration phenomenon. Thus, the third term in Eq. (158) have not contributed to the tree-level square mass of the gluons representing in this case a 4-point vertex. Taking into account this argument, the total gluon square mass has been obtained as β_0. This observation turned out to be fundamental to us in order to understand the possibility of the average value of the field n vanishes or not. In order to accomplish a better comprehension, we have studied the symmetry restoration phenomenon following the strategy developed by n (147, 146) which has consisted in studying the equation of motion for the ς field. After that, we have obtained the behavior of $\hat{\varsigma}$ as a function of the ghost condensate mass ϱ_0. Finally, we have concluded that the sign of the renormalizable gluon mass parameter β_r has offered different types of solutions.

In the second part, we have studied the algebraic renormalization procedure for a model which has the inverse of the Faddeev-Popov operator also coupled with the fermionic gauge-invariant local composite fields. The linear covariant gauges has been used as a gauge-fixing and this class of gauges has been chosen, thankfully, to the introduction of the bosonic gauge-invariant field. Thus, the Faddeev-Popov action has been extended using the Gribov-Zwanziger framework with the inclusion of two horizon functions, one coupled to the A^h and another one with the fields $(\bar{\psi}^h, \psi^h)$. The expression (248) was nonlocal and could be localized following two steps, the first step was adding a set of local fields to localize this horizon-like function and the second one, we have introduced the auxiliary dimensionless Stückelberg field, which has permitted the localization of $(\bar{\psi}^h, \psi^h)$ themselves. The method has been used here was the same as done in (73). Thereby, the resulting action has obeyed a large quantity of Ward identities, which gave us the possibility of proving that this model is renormalizable to all orders in perturbation theory. Moreover, the idea of the Faddeev-Popov operator being coupled with the bosonic and fermionic gauge-invariant fields in a universal way has allowed us to reproduce in a very good agreement the results from lattice simulations about the gluon and quark propagators. This nontrivial result is an important argument in favor of the inverse Faddeev-Popov to play a pivotal role in the study of confining Yang-Mills theories.

One possible subject to investigate in the future is the study of the analytical structure of the correlation functions in both Serreau-Tissier and refined Gribov-Zwanziger frameworks. Moreover the same analysis can be realized in a simple phenomenological model, e.g., the CF in Landau gauge, now having not only the massive gauge field term but also the massive ghost ones discovered in this manuscript in the Serreau-Tissier approach. Furthermore, in the bosonic and fermionic gauge-invariant composite fields formalism we can also extend the CF model for the linear covariant gauges and develop the same idea for this case. To do so, one has to make computations at 1-loop for the propagators of gauge field (gluon), ghost and fermions (quarks) to each model presented here and compare those results with lattice simulations. After that, one can study, e.g., the consequences of massive ghost in CF model in the positivity violation and the spectral density of the gluon.

Furthermore, using the idea of making computations of correlation functions in YM models without considering Gribov ambiguities (to make life more easily), we can study the structure of the quark-gluon vertex in 1-loop and see how the gauge parameter present in linear covariant gauges or the massive ghost term in Landau gauge can influence such vertex construction. Therefore, one can analyse the structure of the Slavnov-Taylor identity, which relates the longitudinal form factors and the quark-ghost kernel. Thus, the analytical structure of the form factors of quark-ghost kernel can be acquired and compared with results recently obtained numerically from the inverse of the Dyson-Schwinger equations, which enjoy as known quantities the quark and gluon propagators achieved from lattice simulations. Thereby, it is expected some novel effects in the infrared sector. This study can be realized through the traditional integral representations of Källén-Lehman (KL) and Nakanishi (NIR). After that, another subject to investigate is to achieve transverse components of the quark-gluon vertex in those formulations at 1-loop.

Then, turning back our attention to the first principles models (which take into account the Gribov copies), e.g., with all the propagators of the fundamental fields acquired in the RGZ and Serreau-Tissier frameworks we desire to construct a novel approach for the nonperturbative structure of quark-gluon vertex in Landau gauge. In particular, it is going to be possible to obtain the form factors through the Slavnov-Taylor identity and all the steps presented before will be done for both cases in a, obviously, more complicated way.

Finally, having all the ingredients achieved previously, our dream will be defining the Bethe-Salpeter equation (BSE) for the pion. As a first case, we intend to solve the BSE in Euclidean space taking into account only longitudinal components of the quark-gluon vertex. Thereby, using the analytical forms for the vertex and propagators obtained previously, we intend to access the Bethe-Salpeter amplitude in Minkowski space via NIR to achieve the quark momenta distribution.

REFERENCES

1 CHATRCHYAN, S. et al. Observation of a new boson at a mass of 125 gev with the cms experiment at the lhc. *Physics Letters B*, [S.l.], v. 716, n. 1, p. 30 – 61, 2012. Disponível em: http://www.sciencedirect.com/science/article/pii/S0370269312008581. Acesso em: 27 mai. 2019.

2 GELL-MANN, M. A schematic model of baryons and mesons. *Physics Letters*, [S.l.], v. 8, n. 3, p. 214 – 215, 1964. Disponível em: https://doi.org/10.1016/S0031-9163(64)92001-3. Acesso em: 18 jun. 2019.

3 ZWEIG, G. An SU(3) model for strong interaction symmetry and its breaking. Version 2. In: LICHTENBERG, D.; ROSEN, S. P. (Ed.). *DEVELOPMENTS IN THE QUARK THEORY OF HADRONS. VOL. 1. 1964 - 1978.* [S.l.]: [s.n.], 1964. p. 22–101.

4 GREENBERG, O. W. Spin and unitary-spin independence in a paraquark model of baryons and mesons. *Phys. Rev. Lett.*, American Physical Society, [S.l.], v. 13, p. 598–602, Nov 1964. Disponível em: https://link.aps.org/doi/10.1103/PhysRevLett.13.598. Acesso em: 8 jun. 2019.

5 HAN, M. Y.; NAMBU, Y. Three-triplet model with double SU(3) symmetry. *Phys. Rev.*, American Physical Society, [S.l.], v. 139, p. B1006–B1010, Aug 1965. Disponível em: https://link.aps.org/doi/10.1103/PhysRev.139.B1006. Acesso em: 29 mai. 2019.

6 NAMBU, Y. A Systematics of hadrons in subnuclear physics. [S.l.], p. 192–201, 1 1981.

7 BARDEEN, W. A.; FRITZSCH, H.; GELL-MANN, M. Light cone current algebra, π^0 decay, and e^+e^- annihilation. In: *Topical Meeting on the Outlook for Broken Conformal Symmetry in Elementary Particle Physics Frascati, Italy, May 4-5, 1972.* [S.l.]: [s.n.], 1972.

8 FRITZSCH, H.; GELL-MANN, M.; LEUTWYLER, H. Advantages of the Color Octet Gluon Picture. *Phys. Lett.*, [S.l.], v. 47B, p. 365–368, 1973.

9 POLITZER, H. D. Reliable Perturbative Results for Strong Interactions? *Phys. Rev. Lett.*, [S.l.], v. 30, p. 1346–1349, 1973.

10 GROSS, D. J.; WILCZEK, F. Ultraviolet Behavior of Nonabelian Gauge Theories. *Phys. Rev. Lett.*, [S.l.], v. 30, p. 1343–1346, 1973.

11 YANG, C. N.; MILLS, R. L. Conservation of Isotopic Spin and Isotopic Gauge Invariance. *Phys. Rev.*, [S.l.], v. 96, p. 191–195, 1954.

12 FEYNMAN, R. P. Quantum theory of gravitation. *Acta Phys. Polon.*, [S.l.], v. 24, p. 697–722, 1963.

13 DEWITT, B. S. Quantum theory of gravity. i. the canonical theory. *Phys. Rev.*, American Physical Society, [S.l.], v. 160, p. 1113–1148, Aug 1967. Disponível em:https://link.aps.org/doi/10.1103/PhysRev.160.1113. Acesso em: 19 jul. 2019.

14 DEWITT, B. S. Quantum Theory of Gravity. 2. The Manifestly Covariant Theory. *Phys. Rev.*, [S.l.], v. 162, p. 1195–1239, 1967.

15 FADDEEV, L. D.; POPOV, V. N. Feynman Diagrams for the Yang-Mills Field. *Phys. Lett.*, [S.l.], v. 25B, p. 29–30, 1967.

16 BECCHI, C.; ROUET, A.; STORA, R. Renormalization of the Abelian Higgs-Kibble Model. *Commun. Math. Phys.*, [S.l.], v. 42, p. 127–162, 1975.

17 BECCHI, C.; ROUET, A.; STORA, R. Renormalization of gauge theories. *Annals of Physics*, [s.l.], v. 98, n. 2, p. 287 – 321, 1976. Disponível em: http://www.sciencedirect.com/science/article/pii/0003491676901561. Acesso em: 22 jul. 2019.

18 TYUTIN, I. V. Gauge Invariance in Field Theory and Statistical Physics in Operator Formalism. [S.l.], 1975.

19 ALKOFER, R.; SMEKAL, L. von. The Infrared behavior of QCD Green's functions: Confinement dynamical symmetry breaking, and hadrons as relativistic bound states. *Phys. Rept.*, [S.l.], v. 353, p. 281, 2001.

20 CORNWALL, J. M. Dynamical Mass Generation in Continuum QCD. *Phys. Rev.*, [S.l.], D26, p. 1453, 1982.

21 CORNWALL, J. M. Positivity violations in QCD. *Mod. Phys. Lett.*, [S.l.], A28, p. 1330035, 2013.

22 BINOSI, D.; PAPAVASSILIOU, J. Pinch Technique: Theory and Applications. *Phys. Rept.*, [S.l.], v. 479, p. 1–152, 2009.

23 AGUILAR, A. C.; BINOSI, D.; PAPAVASSILIOU, J. Gluon and ghost propagators in the Landau gauge: Deriving lattice results from Schwinger-Dyson equations. *Phys. Rev.*, [S.l.], D78, p. 025010, 2008.

24 AGUILAR, A. C.; BINOSI, D.; PAPAVASSILIOU, J. The Gluon Mass Generation Mechanism: A Concise Primer. *Front. Phys.(Beijing)*, [S.l.], v. 11, n. 2, p. 111203, 2016.

25 TISSIER, M.; WSCHEBOR, N. Infrared propagators of Yang-Mills theory from perturbation theory. *Phys. Rev.*, [S.l.], D82, p. 101701, 2010.

26 TISSIER, M.; WSCHEBOR, N. An Infrared Safe perturbative approach to Yang-Mills correlators. *Phys. Rev.*, [S.l.], D84, p. 045018, 2011.

27 FISCHER, C. S.; MAAS, A.; PAWLOWSKI, J. M. On the infrared behavior of Landau gauge Yang-Mills theory. *Annals Phys.*, [S.l.], v. 324, p. 2408–2437, 2009.

28 FISCHER, C. S.; PAWLOWSKI, J. M. Uniqueness of infrared asymptotics in Landau gauge Yang-Mills theory II. *Phys. Rev.*, [S.l.], D80, p. 025023, 2009.

29 WEBER, A. Epsilon Expansion for Infrared Yang-Mills theory in Landau Gauge. *Phys. Rev.*, [S.l.], D85, p. 125005, 2012.

30 FRASCA, M. Infrared Gluon and Ghost Propagators. *Phys. Lett.*, [S.l.], B670, p. 73–77, 2008.

31 SIRINGO, F. Analytical study of Yang–Mills theory in the infrared from first principles. *Nucl. Phys.*, [S.l.], B907, p. 572–596, 2016.

32 KUGO, T.; OJIMA, I. Local Covariant Operator Formalism of Nonabelian Gauge Theories and Quark Confinement Problem. *Prog. Theor. Phys. Suppl.*, [S.l.], v. 66, p. 1–130, 1979.

33 KUGO, T. The Universal renormalization factors Z(1) / Z(3) and color confinement condition in nonAbelian gauge theory. In: *BRS symmetry. Proceedings, International Symposium on the Occasion of its 20th Anniversary, Kyoto, Japan, September 18-22, 1995.* [S.l.: s.n.], 1995. p. 107–119.

34 CHAICHIAN, M.; FRASCA, M. Condition for confinement in non-Abelian gauge theories. *Phys. Lett.*, [S.l.], B781, p. 33–39, 2018.

35 GRIBOV, V. N. Quantization of Nonabelian Gauge Theories. *Nucl. Phys.*, [S.l.], B139, p. 1, 1978. [,1(1977)].

36 ZWANZIGER, D. Local and Renormalizable Action From the Gribov Horizon. *Nucl. Phys.*, [S.l.], B323, p. 513–544, 1989.

37 ZWANZIGER, D. Fundamental modular region, Boltzmann factor and area law in lattice gauge theory. *Nucl. Phys.*, [S.l.], B412, p. 657–730, 1994.

38 VANDERSICKEL, N.; ZWANZIGER, D. The Gribov problem and QCD dynamics. *Phys. Rept.*, [S.l.], v. 520, p. 175–251, 2012.

39 DUDAL, D. et al. New features of the gluon and ghost propagator in the infrared region from the Gribov-Zwanziger approach. *Phys. Rev.*, [S.l.], D77, p. 071501, 2008.

40 DUDAL, D. et al. A Refinement of the Gribov-Zwanziger approach in the Landau gauge: Infrared propagators in harmony with the lattice results. *Phys. Rev.*, [S.l.], D78, p. 065047, 2008.

41 DUDAL, D.; SORELLA, S. P.; VANDERSICKEL, N. The dynamical origin of the refinement of the Gribov-Zwanziger theory. *Phys. Rev.*, [S.l.], D84, p. 065039, 2011.

42 DUDAL, D. et al. The BRST-invariant vacuum state of the Gribov–Zwanziger theory. *Eur. Phys. J.*, [S.l.], C79, n. 9, p. 731, 2019.

43 GAO, F. et al. Locating the Gribov horizon. *Phys. Rev.*, [S.l.], D97, n. 3, p. 034010, 2018.

44 CUCCHIERI, A.; MENDES, T.; TAURINES, A. R. Positivity violation for the lattice Landau gluon propagator. *Phys. Rev.*, [S.l.], D71, p. 051902, 2005.

45 CUCCHIERI, A.; MENDES, T. Constraints on the IR behavior of the gluon propagator in Yang-Mills theories. *Phys. Rev. Lett.*, [S.l.], v. 100, p. 241601, 2008.

46 DUARTE, A. G.; OLIVEIRA, O.; SILVA, P. J. Lattice Gluon and Ghost Propagators, and the Strong Coupling in Pure SU(3) Yang-Mills Theory: Finite Lattice Spacing and Volume Effects. *Phys. Rev.*, [S.l.], D94, n. 1, p. 014502, 2016.

47 OLIVEIRA, O.; SILVA, P. J. The lattice Landau gauge gluon propagator: lattice spacing and volume dependence. *Phys. Rev.*, [S.l.], D86, p. 114513, 2012.

48 DUDAL, D.; OLIVEIRA, O.; SILVA, P. J. Källén-Lehmann spectroscopy for (un)physical degrees of freedom. *Phys. Rev.*, [S.l.], D89, n. 1, p. 014010, 2014.

49 SINGER, I. M. Some Remarks on the Gribov Ambiguity. *Commun. Math. Phys.*, [S.l.], v. 60, p. 7–12, 1978.

50 MAGGIORE, N.; SCHADEN, M. Landau gauge within the Gribov horizon. *Phys. Rev.*, [S.l.], D50, p. 6616–6625, 1994.

51 CAPRI, M. A. L. et al. Properties of the Faddeev-Popov operator in the Landau gauge, matter confinement and soft BRST breaking. *Phys. Rev.*, [S.l.], D90, n. 8, p. 085010, 2014.

52 DUDAL, D. et al. Gribov no-pole condition, Zwanziger horizon function, Kugo-Ojima confinement criterion, boundary conditions, BRST breaking and all that. *Phys. Rev.*, [S.l.], D79, p. 121701, 2009.

53 SORELLA, S. P. Gribov horizon and BRST symmetry: A Few remarks. *Phys. Rev.*, [S.l.], D80, p. 025013, 2009.

54 BAULIEU, L.; SORELLA, S. P. Soft breaking of BRST invariance for introducing non-perturbative infrared effects in a local and renormalizable way. *Phys. Lett.*, [S.l.], B671, p. 481–485, 2009.

55 CAPRI, M. A. L. et al. A remark on the BRST symmetry in the Gribov-Zwanziger theory. *Phys. Rev.*, [S.l.], D82, p. 105019, 2010.

56 DUDAL, D.; SORELLA, S. P. The Gribov horizon and spontaneous BRST symmetry breaking. *Phys. Rev.*, [S.l.], D86, p. 045005, 2012.

57 DUDAL, D. et al. On bounds and boundary conditions in the continuum Landau gauge. *Eur. Phys. J.*, [S.l.], C75, n. 2, p. 83, 2015.

58 LAVROV, P.; LECHTENFELD, O.; RESHETNYAK, A. Is soft breaking of BRST symmetry consistent? *JHEP*, [S.l.], v. 10, p. 043, 2011.

59 LAVROV, P. M.; LECHTENFELD, O. Gribov horizon beyond the Landau gauge. *Phys. Lett.*, [S.l.], B725, p. 386–388, 2013.

60 MOSHIN, P. Yu.; RESHETNYAK, A. A. Finite Field-Dependent BRST-antiBRST Transformations: Jacobians and Application to the Standard Model. *Int. J. Mod. Phys.*, [S.l.], A31, p. 1650111, 2016.

61 SCHADEN, M.; ZWANZIGER, D. Living with spontaneously broken BRST symmetry. I. Physical states and cohomology. *Phys. Rev.*, [S.l.], D92, n. 2, p. 025001, 2015.

62 CUCCHIERI, A. et al. BRST-Symmetry Breaking and Bose-Ghost Propagator in Lattice Minimal Landau Gauge. *Phys. Rev.*, [S.l.], D90, n. 5, p. 051501, 2014.

63 CAPRI, M. A. L. et al. Aspects of the refined Gribov-Zwanziger action in linear covariant gauges. *Annals Phys.*, [S.l.], v. 376, p. 40–62, 2017.

64 CAPRI, M. A. L. et al. On a renormalizable class of gauge fixings for the gauge invariant operator A^2_{min}. *Annals Phys.*, [S.l.], v. 390, p. 214–235, 2018.

65 CAPRI, M. A. L. et al. Exact nilpotent nonperturbative BRST symmetry for the Gribov-Zwanziger action in the linear covariant gauge. *Phys. Rev.*, [S.l.], D92, n. 4, p. 045039, 2015.

66 ZWANZIGER, D. Quantization of Gauge Fields, Classical Gauge Invariance and Gluon Confinement. *Nucl. Phys.*, [S.l.], B345, p. 461–471, 1990.

67 LAVELLE, M.; MCMULLAN, D. Constituent quarks from QCD. *Phys. Rept.*, [S.l.], v. 279, p. 1–65, 1997.

68 CAPRI, M. A. L. et al. Local and BRST-invariant Yang-Mills theory within the Gribov horizon. *Phys. Rev.*, [S.l.], D94, n. 2, p. 025035, 2016.

69 CAPRI, M. A. L. et al. More on the nonperturbative Gribov-Zwanziger quantization of linear covariant gauges. *Phys. Rev.*, [S.l.], D93, n. 6, p. 065019, 2016.

70 CAPRI, M. A. L. et al. Nonperturbative aspects of Euclidean Yang-Mills theories in linear covariant gauges: Nielsen identities and a BRST-invariant two-point correlation function. *Phys. Rev.*, [S.l.], D95, n. 4, p. 045011, 2017.

71 SOBREIRO, R. F.; SORELLA, S. P. A Study of the Gribov copies in linear covariant gauges in Euclidean Yang-Mills theories. *JHEP*, [S.l.], v. 06, p. 054, 2005.

72 CAPRI, M. A. L. et al. Non-perturbative treatment of the linear covariant gauges by taking into account the Gribov copies. *Eur. Phys. J.*, [S.l.], C75, n. 10, p. 479, 2015.

73 CAPRI, M. A. L. et al. A non-perturbative study of matter field propagators in Euclidean Yang–Mills theory in linear covariant, Curci–Ferrari and maximal Abelian gauges. *Eur. Phys. J.*, [S.l.], C77, n. 8, p. 546, 2017.

74 CAPRI, M. A. L. et al. Renormalizability of the refined Gribov-Zwanziger action in linear covariant gauges. *Phys. Rev.*, [S.l.], D96, n. 5, p. 054022, 2017.

75 CAPRI, M. A. L.; SORELLA, S. P.; TERIN, R. C. Study of a gauge invariant local composite fermionic field. [S.l.], 2019.

76 PIGUET, O.; SORELLA, S. P. Algebraic renormalization: Perturbative renormalization, symmetries and anomalies. *Lect. Notes Phys. Monogr.*, v. 28, p. 1–134, 1995.

77 SERREAU, J.; TISSIER, M. Lifting the Gribov ambiguity in Yang-Mills theories. *Phys. Lett.*, [S.l.], B712, p. 97–103, 2012.

78 SERREAU, J.; TISSIER, M.; TRESMONTANT, A. Influence of Gribov ambiguities in a class of nonlinear covariant gauges. *Phys. Rev.*, [S.l.], D92, p. 105003, 2015.

79 TISSIER, M. Gribov copies, avalanches and dynamic generation of a gluon mass. *Phys. Lett.*, [S.l.], B784, p. 146–150, 2018.

80 EDWARDS, S. F.; ANDERSON, P. W. Theory of spin glasses. *Journal of Physics F: Metal Physics*, IOP Publishing, [s.l.], v. 5, n. 5, p. 965–974, may 1975.

81 EDWARDS, S. F.; ANDERSON, P. W. Theory of spin glasses. II. *Journal of Physics F: Metal Physics*, IOP Publishing, [s.l.], v. 6, n. 10, p. 1927–1937, oct 1976.

82 CAPRI, M. A. L. et al. Renormalizability of pure $\mathcal{N} = 1$ Super Yang-Mills in the Wess-Zumino gauge in the presence of the local composite operators A^2 and $\bar{\lambda}\lambda$. *Int. J. Mod. Phys.*, [S.l.], A33, n. 28, p. 1850161, 2018.

83 CAPRI, M. A. L. et al. Renormalizability of $\mathcal{N} = 1$ super Yang-Mills theory in Landau gauge with a Stueckelberg-like field. *Eur. Phys. J.*, [S.l.], C78, n. 10, p. 797, 2018.

84 OSPEDAL, L. P. R.; TERIN, R. C. $\mathcal{N} = 2$ Supersymmetry with Central Charge: A Twofold Implementation. *Advances in High Energy Physics*, [S.l.], p. 9, 2019.

85 CAPRI, M. A. L.; TERIN, R. C.; TOLEDO, H. C. Renormalization of a generalized supersymmetric version of the maximal abelian gauge. *Phys. Rev. D*, American Physical Society, [S.l.], v. 99, p. 025015, Jan 2019. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.99.025015. Acesso em: 16 jul. 2019.

86 FAUX, M.; SPECTOR, D. Duality and central charges in supersymmetric quantum mechanics. *Phys. Rev. D*, American Physical Society, [S.l.], v. 70, p. 085014, Oct 2004. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.70.085014. Acesso em: 14 ago. 2019.

87 CAPRI, M. A. L. et al. Local and renormalizable framework for the gauge-invariant operator $A^2_{\rm min}$ in Euclidean Yang-Mills theories in linear covariant gauges. *Phys. Rev.*, [S.l.], D94, n. 6, p. 065009, 2016.

88 DAS, A. *Lectures on quantum field theory.* [S.l.]: [s.n.], 2008.

89 WEINBERG, S. *The quantum theory of fields. Vol. 2: Modern applications.* [S.l.]: Cambridge University Press, 2013. ISBN 9781139632478, 9780521670548, 9780521550024.

90 NAKANISHI, N. Indefinite-Metric Quantum Field Theory. *Progress of Theoretical Physics Supplement*, [S.l.], v. 51, p. 1–95, 03 1972. Disponível em: https://doi.org/10.1143/PTPS.51.1. Acesso em: 29 mai. 2019.

91 LAUTRUP, B. CANONICAL QUANTUM ELECTRODYNAMICS IN COVARIANT GAUGES. *Kong. Dan. Vid. Sel. Mat. Fys. Med.*, [S.l.], v. 35, n. 11, 1967.

92 AITCHISON, I. J. R.; HEY, A. J. G. *Gauge theories in particle physics: A practical introduction. Vol. 2: Non-Abelian gauge theories: QCD and the electroweak theory.* Bristol, UK: CRC Press, 2012. Disponível em: http://www-spires.fnal.gov/spires/find/books/www?cl=QC793.3.F5A34::2012:V2. Acesso em: 28 jul. 2019.

93 HOOFT, G. 't. Renormalizable Lagrangians for Massive Yang-Mills Fields. *Nucl. Phys.*, [S.l.], B35, p. 167–188, 1971. [,201(1971)].

94 HOOFT, G. 't. Renormalization of Massless Yang-Mills Fields. *Nucl. Phys.*, [S.l.], B33, p. 173–199, 1971.

95 HOOFT, G. 't; VELTMAN, M. J. G. Regularization and Renormalization of Gauge Fields. *Nucl. Phys.*, [S.l.], B44, p. 189–213, 1972.

96 NAKANISHI, N.; OJIMA, I. Covariant operator formalism of gauge theories and quantum gravity. *World Sci. Lect. Notes Phys.*, [S.l.], v. 27, p. 1–434, 1990.

97 GRACEY, J. A. Two loop correction to the Gribov mass gap equation in Landau gauge QCD. *Phys. Lett.*, [S.l.], B632, p. 282–286, 2006. [Erratum: Phys. Lett.B686,319(2010)].

98 HUBER, M. Q.; CYROL, A. K.; SMEKAL, L. von. On Dyson-Schwinger studies of Yang-Mills theory and the four-gluon vertex. *Acta Phys. Polon. Supp.*, [S.l.], v. 8, n. 2, p. 497, 2015.

99 SOBREIRO, R. F.; SORELLA, S. P. Introduction to the Gribov ambiguities in Euclidean Yang-Mills theories. In: *13th Jorge Andre Swieca Summer School on Particle and Fields Campos do Jordao, Brazil, January 9-22, 2005*. Campos do Jordão: [s.n.], 2005.

100 BAAL, P. More (thoughts on) Gribov copies. *Nucl. Phys.*, [S.l.], B369, p. 259–275, 1992.

101 NEUBERGER, H. Nonperturbative BRS Invariance. *Phys. Lett.*, B175, p. 69–72, 1986.

102 NEUBERGER, H. Nonperturbative brs invariance and the gribov problem. *Physics Letters B*, [S.l.], v. 183, n. 3, p. 337 – 340, 1987. Disponível em: http://www.sciencedirect.com/science/article/pii/0370269387909749. Acesso em: 23 jun. 2019.

103 ZWANZIGER, D. Non-perturbative modification of the faddeev-popov formula. *Physics Letters B*, [S.l.], v. 114, n. 5, p. 337 – 339, 1982. Disponível em: http://www.sciencedirect.com/science/article/pii/0370269382903574. Acesso em: 19 nov. 2019.

104 SEMENOV-TYAN-SHANSKII, M.; FRANKE, V. A variational principle for the lorentz condition and restriction of the domain of path integration in non-abelian gauge theory. *Journal of Soviet Mathematics*, Springer, [S.l.], v. 34, n. 5, p. 1999–2004, 1986.

105 MASKAWA, T.; NAKAJIMA, H. How dense are the coulomb gauge fixing degeneracies? geometrical formulation of the coulomb gauge. *Progress of Theoretical Physics*, Oxford University Press, [S.l.], v. 60, n. 5, p. 1526–1539, 1978.

106 DELL'ANTONIO, G.; ZWANZIGER, D. Ellipsoidal Bound on the Gribov Horizon Contradicts the Perturbative Renormalization Group. *Nucl. Phys.*, [S.l.], B326, p. 333–350, 1989.

107 DELL'ANTONIO, G.; ZWANZIGER, D. Every gauge orbit passes inside the Gribov horizon. *Commun. Math. Phys.*, [S.l.], v. 138, p. 291–299, 1991.

108 ZWANZIGER, D. Nonperturbative faddeev-popov formula and the infrared limit of qcd. *Phys. Rev. D*, American Physical Society, v. 69, p. 016002, Jan 2004. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.69.016002. Acesso em: 15 jul. 2019.

109 GREENSITE, J.; OLEJNÍK, S.; ZWANZIGER, D. Coulomb energy, remnant symmetry, and the phases of non-abelian gauge theories. *Phys. Rev. D*, American Physical Society, [S.l.], v. 69, p. 074506, Apr 2004. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.69.074506. Acesso em: 08 dez. 2019.

110 DUDAL, D. et al. Landau gauge gluon and ghost propagators in the refined gribov-zwanziger framework in 3 dimensions. *Phys. Rev. D*, American Physical Society, [S.l.], v. 78, p. 125012, Dec 2008.

111 DUDAL, D. et al. The effects of gribov copies in 2d gauge theories. *Physics Letters B*, [s.l.], v. 680, n. 4, p. 377 – 383, 2009. ISSN 0370-2693. Disponível em: http://www.sciencedirect.com/science/article/pii/S0370269309010314. Acesso em: 08 nov. 2019.

112 DUDAL, D.; OLIVEIRA, O.; VANDERSICKEL, N. Indirect lattice evidence for the refined gribov-zwanziger formalism and the gluon condensate A^2 in the landau gauge. *Phys. Rev. D*, American Physical Society, [S.l.], v. 81, p. 074505, Apr 2010. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.81.074505. Acesso em: 22 out. 2019.

113 ZWANZIGER, D. Action From the Gribov Horizon. *Nucl. Phys.*, [S.l.], B321, p. 591–604, 1989.

114 ZWANZIGER, D. Renormalizability of the critical limit of lattice gauge theory by BRS invariance. *Nucl. Phys.*, [S.l.], B399, p. 477–513, 1993.

115 CUCCHIERI, A.; MENDES, T. What's up with IR gluon and ghost propagators in Landau gauge? A puzzling answer from huge lattices. *PoS*, [S.l.], LATTICE2007, p. 297, 2007.

116 CUCCHIERI, A.; MENDES, T. Constraints on the infrared behavior of the ghost propagator in yang-mills theories. *Phys. Rev. D*, American Physical Society, [S.l.], v. 78, p. 094503, Nov 2008.

117 STERNBECK, A. et al. The Gluon and ghost propagator and the influence of Gribov copies. *Nucl. Phys. Proc. Suppl.*, [S.l.], v. 140, p. 653–655, 2005. [,653(2004)].

118 LERCHE, C.; SMEKAL, L. von. Infrared exponent for gluon and ghost propagation in landau gauge qcd. *Phys. Rev. D*, American Physical Society, [S.l.], v. 65, p. 125006, Jun 2002. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.65.125006. Acesso em: 16 ago. 2019.

119 PAWLOWSKI, J. M. et al. Infrared behavior and fixed points in landau-gauge qcd. *Phys. Rev. Lett.*, American Physical Society, [S.l.], v. 93, p. 152002, Oct 2004. Disponível em: https://link.aps.org/doi/10.1103/PhysRevLett.93.152002. Acesso em: 21 jun. 2019.

120 ALKOFER, R. et al. Analytic properties of the landau gauge gluon and quark propagators. *Phys. Rev. D*, American Physical Society, [S.l.], v. 70, p. 014014, Jul 2004. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.70.014014. Acesso em: 05 jun. 2019.

121 DUDAL, D. et al. The Gribov parameter and the dimension two gluon condensate in Euclidean Yang-Mills theories in the Landau gauge. *Phys. Rev.*, [S.l.], D72, p. 014016, 2005.

122 CURCI, G.; FERRARI, R. On a Class of Lagrangian Models for Massive and Massless Yang-Mills Fields. *Nuovo Cim.*, [S.l.], A32, p. 151–168, 1976.

123 PELAEZ, M.; TISSIER, M.; WSCHEBOR, N. Three-point correlation functions in Yang-Mills theory. *Phys. Rev.*, [S.l.], D88, p. 125003, 2013.

124 PELAEZ, M.; TISSIER, M.; WSCHEBOR, N. Two-point correlation functions of QCD in the Landau gauge. *Phys. Rev.*, [S.l.], D90, p. 065031, 2014.

125 GRACEY, J. A. et al. Two loop calculation of Yang-Mills propagators in the Curci-Ferrari model. *Phys. Rev.*, [S.l.], D100, n. 3, p. 034023, 2019.

126 BOWMAN, P. O. et al. Scaling behavior and positivity violation of the gluon propagator in full QCD. *Phys. Rev.*, [S.l.], D76, p. 094505, 2007.

127 BOER, J. de et al. On the renormalizability and unitarity of the Curci-Ferrari model for massive vector bosons. *Phys. Lett.*, [S.l.], B367, p. 175–182, 1996.

128 GRACEY, J. A. Three loop MS-bar renormalization of the Curci-Ferrari model and the dimension two BRST invariant composite operator in QCD. *Phys. Lett.*, [S.l.], B552, p. 101–110, 2003.

129 DELBOURGO, R.; JARVIS, P. D. Extended BRS invariance and OSp (4/2) supersymmetry. *Journal of Physics A: Mathematical and General*, IOP Publishing, [s.l.], v. 15, n. 2, p. 611–625, feb 1982.

130 WSCHEBOR, N. Some non-renormalization theorems in Curci-Ferrari model. *Int. J. Mod. Phys.*, [S.l.], A23, p. 2961–2973, 2008.

131 TISSIER, M.; WSCHEBOR, N. Gauged supersymmetries in yang-mills theory. *Phys. Rev. D*, American Physical Society, [S.l.], v. 79, p. 065008, Mar 2009. Disponível em: https://link.aps.org/doi/10.1103/PhysRevD.79.065008. Acesso em: 25 mai. 2019.

132 PARRINELLO, C.; JONA-LASINIO, G. A Modified Faddeev-Popov formula and the Gribov ambiguity. *Phys. Lett.*, [S.l.], B251, p. 175–180, 1990.

133 FACHIN, S.; PARRINELLO, C. Global gauge fixing in lattice gauge theories. *Phys. Rev.*, [S.l.], D44, p. 2558–2564, 1991.

134 WOLFF, U. Asymptotic freedom and mass generation in the o(3) nonlinear -model. *Nuclear Physics B*, [s.l.], v. 334, n. 3, p. 581 – 610, 1990. ISSN 0550-3213. Disponível em: http://www.sciencedirect.com/science/article/pii/0550321390903133. Acesso em: 22 jun. 2019.

135 HATA, H. Restoration of the Local Gauge Symmetry and Color Confinement in Nonabelian Gauge Theories. *Prog. Theor. Phys.*, [S.l.], v. 67, p. 1607, 1982.

136 MERMIN, N. D.; WAGNER, H. Absence of ferromagnetism or antiferromagnetism in one- or two-dimensional isotropic heisenberg models. *Phys. Rev. Lett.*, American Physical Society, [S.l.], v. 17, p. 1133–1136, Nov 1966. Disponível em: https://link.aps.org/doi/10.1103/PhysRevLett.17.1133. Acesso em: 31 jul. 2019.

137 WILSON, K. G. Renormalization group and critical phenomena. 1. Renormalization group and the Kadanoff scaling picture. *Phys. Rev.*, [S.l.], B4, p. 3174–3183, 1971.

138 WILSON, K. G. Renormalization group and critical phenomena. 2. Phase space cell analysis of critical behavior. *Phys. Rev.*, [S.l.], B4, p. 3184–3205, 1971.

139 AGUILAR, A. C.; NATALE, A. A. A dynamical gluon mass solution in a coupled system of the schwinger-dyson equations. *Journal of High Energy Physics*, Springer Science and Business Media LLC, [s.l.], v. 2004, n. 08, p. 057–057, sep 2004.

140 RODRIGUEZ-QUINTERO, J. On the massive gluon propagator, the PT-BFM scheme and the low-momentum behaviour of decoupling and scaling DSE solutions. *JHEP*, [S.l.], v. 01, p. 105, 2011.

141 HUBER, M. Q.; SMEKAL, L. von. On the influence of three-point functions on the propagators of Landau gauge Yang-Mills theory. *JHEP*, [S.l.], v. 04, p. 149, 2013.

142 PARISI, G.; SOURLAS, N. Random magnetic fields, supersymmetry, and negative dimensions. *Phys. Rev. Lett.*, American Physical Society, [S.l.], v. 43, p. 744–745, Sep 1979. Disponível em: https://link.aps.org/doi/10.1103/PhysRevLett.43.744. Acesso em: 02 out. 2018.

143 KLEIN, A.; LANDAU, L. J.; PEREZ, J. F. SUPERSYMMETRY AND THE PARISI-SOURLAS DIMENSIONAL REDUCTION: A RIGOROUS PROOF. *Commun. Math. Phys.*, [S.l.], v. 94, p. 459–482, 1984.

144 CUCCHIERI, A.; MENDES, T. Landau-gauge propagators in Yang-Mills theories at $\beta = 0$: Massive solution versus conformal scaling. *Phys. Rev.*, [S.l.], D81, p. 016005, 2010.

145 BOGOLUBSKY, I. L. et al. Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared. *Phys. Lett.*, [S.l.], B676, p. 69–73, 2009.

146 COLEMAN, S. R. There are no Goldstone bosons in two-dimensions. *Commun. Math. Phys.*, [S.l.], v. 31, p. 259–264, 1973.

147 SENECHAL, D. Mass gap of the nonlinear- model through the finite-temperature effective action. *Physical Review B*, American Physical Society (APS), [S.l.], v. 47, n. 13, p. 8353–8356, Apr 1993. Disponível em: http://dx.doi.org/10.1103/PhysRevB.47.8353. Acesso em: 22 out. 2018.

148 DUDAL, D.; VANDERSICKEL, N. On the reanimation of a local BRST invariance in the (Refined) Gribov-Zwanziger formalism. *Phys. Lett.*, [S.l.], B700, p. 369–379, 2011.

149 KONDO, K.-I. The Nilpotent BRST symmetry for the Gribov-Zwanziger theory. [S.l.], 2009.

150 DELBOURGO, R.; THOMPSON, G. MASSIVE, UNITARY, RENORMALIZABLE YANG-MILLS THEORY WITHOUT HIGGS MESONS. *Phys. Rev. Lett.*, [S.l.], v. 57, p. 2610, 1986.

151 DELBOURGO, R.; TWISK, S.; THOMPSON, G. MASSIVE YANG-MILLS THEORY: RENORMALIZABILITY VERSUS UNITARITY. *Int. J. Mod. Phys.*, [S.l.], A3, p. 435, 1988.

152 DRAGON, N.; HURTH, T.; NIEUWENHUIZEN, P. van. Polynomial form of the Stuckelberg model. *Nucl. Phys. Proc. Suppl.*, [Buckow], v. 56B, p. 318–321, 1997.

153 RUEGG, H.; RUIZ-ALTABA, M. The Stueckelberg field. *Int. J. Mod. Phys.*, [S.l.], A19, p. 3265–3348, 2004.

154 MEERLEER, T. D. et al. Fresh look at the Abelian and non-Abelian Landau-Khalatnikov-Fradkin transformations. *Phys. Rev.*, [S.l.], D97, n. 7, p. 074017, 2018.

155 CAPRI, M. A. L.; FIORENTINI, D.; SORELLA, S. P. Study of the all orders multiplicative renormalizability of a local confining quark action in the Landau gauge. *Annals Phys.*, [S.l.], v. 356, p. 320–335, 2015.

156 PIGUET, O.; SIBOLD, K. Gauge Independence in Ordinary Yang-Mills Theories. *Nucl. Phys.*, [S.l.], B253, p. 517–540, 1985.

157 PARAPPILLY, M. B. et al. Scaling behavior of quark propagator in full QCD. *Phys. Rev.*, [S.l.], D73, p. 054504, 2006.

158 GUIMARAES, M. S.; PEREIRA, A. D.; SORELLA, S. P. Remarks on the effects of the Gribov copies on the infrared behavior of higher dimensional Yang-Mills theory. *Phys. Rev.*, [S.l.], D94, n. 11, p. 116011, 2016.

159 PESKIN, M. E.; SCHROEDER, D. V. *An Introduction to quantum field theory.* Reading, USA: Addison-Wesley, 1995. Disponível em: http://www.slac.stanford.edu/ mpeskin/QFT.html. Acesso em: 18 mai. 2019.

APPENDIX A – Properties of the functional $\mathcal{H}[A,U]$.

In this Appendix we review some useful properties of the functional, $\mathcal{H}[A,U]$

$$\mathcal{H}[A,U] \equiv \mathrm{Tr} \int d^4x \, A^U_\mu A^U_\mu = \mathrm{Tr} \int d^4x \left(U^\dagger A_\mu U + \frac{i}{g} U^\dagger \partial_\mu U \right) \left(U^\dagger A_\mu U + \frac{i}{g} U^\dagger \partial_\mu U \right) . \tag{420}$$

For a given gauge field configuration A_μ, $\mathcal{H}[A,U]$ is a functional characterized along the gauge orbit of A_μ. Let us define $\mathcal{A}$ the space of connections A^a_μ with finite Hilbert norm $||A||$, *i.e.*

$$||A||^2 = \mathrm{Tr} \int d^4x \, A_\mu A_\mu = \frac{1}{2} \int d^4x A^a_\mu A^a_\mu < +\infty \, , \tag{421}$$

and $\mathcal{U}$ being the space of local gauge transformations U in a way that the Hilbert norm $||U^\dagger \partial U||$ is also finite, namely

$$||U^\dagger \partial U||^2 = \mathrm{Tr} \int d^4x \, \left(U^\dagger \partial_\mu U \right) \left(U^\dagger \partial_\mu U \right) < +\infty \, . \tag{422}$$

Let us present the following proposition (100, 66, 106, 107)

- Proposition

  The functional $\mathcal{H}[A,U]$ reach's its absolute minimum on the gauge orbit of A_μ.

This proposition establishes that there exists a $h \in \mathcal{U}$ in a way that

$$\delta \mathcal{H}[A,h] = 0 \, , \tag{423}$$

$$\delta^2 \mathcal{H}[A,h] \geq 0 \, , \tag{424}$$

$$\mathcal{H}[A,h] \leq \mathcal{H}[A,U] \, , \qquad \forall \, U \in \mathcal{U} \, . \tag{425}$$

The operator $A^2_{\rm min}$ is described by

$$A^2_{\rm min} = \min_{\{U\}} \mathrm{Tr} \int d^4x \, A^U_\mu A^U_\mu = \mathcal{H}[A,h] \, . \tag{426}$$

Observing the two conditions (423) and (424), one can compute $\delta \mathcal{H}[A,h]$ and $\delta^2 \mathcal{H}[A,h]$ we set[33]

$$v = h e^{ig\omega} = h e^{ig\omega^a T^a} \, , \tag{427}$$

[33] The case of the gauge group $SU(N)$ is considered here.

$$\left[T^a, T^b\right] = i f^{abc}\, T^c \,, \qquad \mathrm{Tr}\left(T^a T^b\right) = \frac{1}{2}\delta^{ab} \,, \tag{428}$$

with ω being an infinitesimal Hermitian matrix and we can evaluate the linear and quadratic terms of the expansion of the functional $f_A[v]$ in power series of ω. Let us first achieve an expression for A^v_μ

$$\begin{aligned} A^v_\mu &= v^\dagger A_\mu v + \frac{i}{g} v^\dagger \partial_\mu v \\ &= e^{-ig\omega} h^\dagger A_\mu h e^{ig\omega} + \frac{i}{g} e^{-ig\omega}\left(h^\dagger \partial_\mu h\right) e^{ig\omega} + \frac{i}{g} e^{-ig\omega} \partial_\mu e^{ig\omega} \\ &= e^{-ig\omega} A^h_\mu e^{ig\omega} + \frac{i}{g} e^{-ig\omega} \partial_\mu e^{ig\omega} \,. \end{aligned} \tag{429}$$

Expanding up to the order ω^2, one has

$$\begin{aligned} A^v_\mu &= \left(1 - ig\omega - g^2\frac{\omega^2}{2}\right) A^h_\mu \left(1 + ig\omega - g^2\frac{\omega^2}{2}\right) + \frac{i}{g}\left(1 - ig\omega - g^2\frac{\omega^2}{2}\right) \partial_\mu \left(1 + ig\omega - g^2\frac{\omega^2}{2}\right) \\ &= \left(1 - ig\omega - g^2\frac{\omega^2}{2}\right)\left(A^h_\mu + igA^h_\mu\omega - g^2 A^h_\mu \frac{\omega^2}{2}\right) + \\ &+ \frac{i}{g}\left(1 - ig\omega - g^2\frac{\omega^2}{2}\right)\left(ig\partial_\mu\omega - \frac{g^2}{2}\left(\partial_\mu\omega\right)\omega - \frac{g^2}{2}\omega\left(\partial_\mu\omega\right)\right) \\ &= A^h_\mu + igA^h_\mu\omega - \frac{g^2}{2}A^h_\mu\omega^2 - ig\omega A^h_\mu + g^2\omega A^h_\mu\omega - \frac{g^2}{2}\omega^2 A^h_\mu \\ &+ \frac{i}{g}\left(ig\partial_\mu\omega - \frac{g^2}{2}\left(\partial_\mu\omega\right)\omega - \frac{g^2}{2}\omega\partial_\mu\omega + g^2\omega\partial_\mu\omega\right) + O(\omega^3) \,, \end{aligned} \tag{430}$$

from this expression, we establish

$$A^v_\mu = A^h_\mu + ig[A^h_\mu, \omega] + \frac{g^2}{2}[[\omega, A^h_\mu], \omega] - \partial_\mu\omega + i\frac{g}{2}[\omega, \partial_\mu\omega] + O(\omega^3) \,, \tag{431}$$

We now evaluate

$$
\begin{aligned}
\mathcal{H}[A,v] &= \mathrm{Tr}\int d^4x A_\mu^U A_\mu^U \\
&= \mathrm{Tr}\int d^4x\ \Big[\Big(A_\mu^h + ig[A_\mu^h,\omega] + \frac{g^2}{2}[[\omega,A_\mu^h],\omega] - \partial_\mu\omega + i\frac{g}{2}[\omega,\partial_\mu\omega] + O(\omega^3)\Big)\times \\
&\quad \Big(A_\mu^h + ig[A_\mu^h,\omega] + \frac{g^2}{2}[[\omega,A_\mu^h],\omega] - \partial_\mu\omega + i\frac{g}{2}[\omega,\partial_\mu\omega] + O(\omega^3)\Big)\Big] \\
&= \mathrm{Tr}\int d^4x\ \Big\{A_\mu^h A_\mu^h + igA_\mu^h[A_\mu^h,\omega] + g^2 A_\mu^h\omega A_\mu^h\omega - \frac{g^2}{2}A_\mu^h A_\mu^h\omega^2 - \frac{g^2}{2}A_\mu^h\omega^2 A_\mu^h - A_\mu^h\partial_\mu\omega \\
&+ i\frac{g}{2}A_\mu^h[\omega,\partial_\mu\omega] + ig[A_\mu^h,\omega]A_\mu^h - g^2[A_\mu^h,\omega][A_\mu^h,\omega] - ig[A_\mu^h,\omega]\partial_\mu\omega + g^2\omega A_\mu^h\omega A_\mu^h \\
&- \frac{g^2}{2}A_\mu^h\omega^2 A_\mu^h - \frac{g^2}{2}\omega^2 A_\mu^h A_\mu^h - \partial_\mu\omega A_\mu^h - ig\partial_\mu\omega[A_\mu^h,\omega] + \partial_\mu\omega\partial_\mu\omega + i\frac{g}{2}[\omega,\partial_\mu\omega]A_\mu^h\Big\} \\
&+ O(\omega^3) \\
&= \mathcal{H}[A,h] - \mathrm{Tr}\int d^4x\ \Big\{A_\mu^h,\partial_\mu\omega\Big\} + \mathrm{Tr}\int d^4x\ \Big(g^2A_\mu^h\omega A_\mu^h\omega - \frac{g^2}{2}A_\mu^hA_\mu^h\omega^2 - \frac{g^2}{2}A_\mu^h\omega^2A_\mu^h \\
&- g^2[A_\mu^h,\omega][A_\mu^h,\omega] + g^2\omega A_\mu^h\omega A_\mu^h - \frac{g^2}{2}A_\mu^h\omega^2A_\mu^h - \frac{g^2}{2}\omega^2A_\mu^hA_\mu^h\Big) + \mathrm{Tr}\int d^4x\ (\partial_\mu\omega\partial_\mu\omega \\
&+ i\frac{g}{2}[\omega,\partial_\mu\omega]A_\mu^h - ig\partial_\mu\omega[A_\mu^h,\omega] - ig[A_\mu^h,\omega]\partial_\mu\omega + i\frac{g}{2}A_\mu^h[\omega,\partial_\mu\omega]\Big) + O(\omega^3) \\
&= \mathcal{H}[A,h] + 2\int d^4x\, tr\left(\omega\partial_\mu A_\mu^h\right) + \int d^4x\, tr\left\{2g^2\omega A_\mu^h\omega A_\mu^h - 2g^2A_\mu^hA_\mu^h\omega^2\right. \\
&- \left. g^2\left(A_\mu^h\omega - \omega A_\mu^h\right)\left(A_\mu^h\omega - \omega A_\mu^h\right)\right\} + \int d^4x\, tr\Big(\partial_\mu\omega\partial_\mu\omega + i\frac{g}{2}\omega\partial_\mu\omega A_\mu^h - i\frac{g}{2}\partial_\mu\omega\omega A_\mu^h \\
&- ig\partial_\mu\omega A_\mu^h\omega + ig\partial_\mu\omega\omega A_\mu^h - igA_\mu^h\omega\partial_\mu\omega + ig\omega A_\mu^h\partial_\mu\omega + i\frac{g}{2}A_\mu^h\omega\partial_\mu\omega - i\frac{g}{2}A_\mu^h\partial_\mu\omega\omega\Big) \\
&+ O(\omega^3) \\
&= \mathcal{H}[A,h] + 2\mathrm{Tr}\int d^4x\left(\omega\partial_\mu A_\mu^h\right) + \mathrm{Tr}\int d^4x\ \Big(\partial_\mu\omega\partial_\mu\omega + ig\omega\partial_\mu\omega A_\mu^h - ig\partial_\mu\omega\omega A_\mu^h \\
&- 2ig\partial_\mu\omega A_\mu^h\omega + 2ig\partial_\mu\omega\omega A_\mu^h\Big) + O(\omega^3)\ .
\end{aligned} \tag{432}
$$

Thus

$$
\begin{aligned}
\mathcal{H}[A,v] &= \mathcal{H}[A,h] + 2\mathrm{Tr}\int d^4x\ \left(\omega\partial_\mu A_\mu^h\right) + \mathrm{Tr}\int d^4x\ \Big(\partial_\mu\omega\partial_\mu\omega + ig\omega\partial_\mu\omega A_\mu^h - ig\partial_\mu\omega\omega A_\mu^h \\
&- ig\left(\partial_\mu\omega\right)A_\mu^h\omega + ig\left(\partial_\mu\omega\right)\omega A_\mu^h\Big) + O(\omega^3) \\
&= \mathcal{H}[A,h] + 2\mathrm{Tr}\int d^4x\ \left(\omega\partial_\mu A_\mu^h\right) + \mathrm{Tr}\int d^4x\ \left\{\partial_\mu\omega\left(\partial_\mu\omega - ig\left[A_\mu^h,\omega\right]\right)\right\} + O(\omega^3)\ .
\end{aligned} \tag{433}
$$

Finally

$$
\mathcal{H}[A,v] = \mathcal{H}[A,h] + 2\mathrm{Tr}\int d^4x\ \left(\omega\partial_\mu A_\mu^h\right) - \mathrm{Tr}\int d^4x\, \omega\partial_\mu D_\mu(A^h)\omega + O(\omega^3)\ , \tag{434}
$$

so that

$$\begin{aligned} \delta\mathcal{H}[A,h] &= 0 \;\Rightarrow\; \partial_\mu A^h_\mu \;=\; 0\;, \\ \delta^2\mathcal{H}[A,h] &> 0 \;\Rightarrow\; -\partial_\mu D_\mu(A^h) \;>\; 0\;. \end{aligned} \tag{435}$$

Thereby, it is possible to assert that the set of field configurations which satisfies the conditions (435), *i.e.* defining relative minima of the functional $f_A[U]$, belong to the well-known Gribov region Ω, which is defined as

$$\Omega = \{A_\mu |\, \partial_\mu A_\mu = 0 \text{ and } -\partial_\mu D_\mu(A) > 0\}\;. \tag{436}$$

Let us prove that the transversality condition, $\partial_\mu A^h_\mu = 0$, can be solved for $h = h(A)$ as a power series in A_μ. We outset from

$$A^h_\mu = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h\;, \tag{437}$$

with

$$h = e^{ig\phi} = e^{ig\phi^a T^a}\;. \tag{438}$$

Let us expand h in powers of ϕ

$$h = 1 + ig\phi - \frac{g^2}{2}\phi^2 + O(\phi^3)\;. \tag{439}$$

From equation (437), one has

$$A^h_\mu = A_\mu + ig[A_\mu,\phi] + g^2\phi A_\mu \phi - \frac{g^2}{2}A_\mu\phi^2 - \frac{g^2}{2}\phi^2 A_\mu - \partial_\mu\phi + i\frac{g}{2}[\phi,\partial_\mu] + O(\phi^3)\;. \tag{440}$$

Therefore, condition $\partial_\mu A^h_\mu = 0$ is rewritten as

$$\begin{aligned} \partial^2\phi &= \partial_\mu A + ig[\partial_\mu A_\mu,\phi] + ig[A_\mu,\partial_\mu\phi] + g^2\partial_\mu\phi A_\mu\phi + g^2\phi\partial_\mu A_\mu\phi + g^2\phi A_\mu\partial_\mu\phi \\ &- \frac{g^2}{2}\partial_\mu A_\mu\phi^2 - \frac{g^2}{2}A_\mu\partial_\mu\phi\phi - \frac{g^2}{2}A_\mu\phi\partial_\mu\phi - \frac{g^2}{2}\partial_\mu\phi\phi A_\mu - \frac{g^2}{2}\phi\partial_\mu\phi A_\mu - \frac{g^2}{2}\phi^2\partial_\mu A_\mu \\ &+ i\frac{g}{2}[\phi,\partial^2\phi] + O(\phi^3)\;. \end{aligned} \tag{441}$$

This equation is determined iteratively for ϕ as a power series in A_μ, *i.e.*

$$\phi = \frac{1}{\partial^2}\partial_\mu A_\mu + i\frac{g}{\partial^2}\left[\partial A,\frac{\partial A}{\partial^2}\right] + i\frac{g}{\partial^2}\left[A_\mu,\partial_\mu\frac{\partial A}{\partial^2}\right] + \frac{i}{2}\frac{g}{\partial^2}\left[\frac{\partial A}{\partial^2},\partial A\right] + O(A^3)\;, \tag{442}$$

in order that

$$
\begin{aligned}
A^h_\mu &= A_\mu - \frac{1}{\partial^2}\partial_\mu \partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu \frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] \\
&+ ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3) \ . \qquad (443)
\end{aligned}
$$

Expression (443) can be cast in a different but useful way, as eq.(186). Thus

$$
\begin{aligned}
A^h_\mu &= \left(\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \frac{1}{\partial^2}\partial A\right]\right) + O(A^3) \\
&= A_\mu - ig\left[\frac{1}{\partial^2}\partial A, A_\mu\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu \frac{1}{\partial^2}\partial A\right] - \frac{\partial_\mu}{\partial^2}\partial A + ig\frac{\partial_\mu}{\partial^2}\partial_\nu\left[\frac{1}{\partial^2}\partial A, A_\nu\right] \\
&- i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\partial_\nu\left[\frac{\partial A}{\partial^2}, \frac{\partial_\nu}{\partial^2}\partial A\right] + O(A^3) \\
&= A_\mu - \frac{\partial_\mu}{\partial^2}\partial A + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + \frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu \frac{1}{\partial^2}\partial A\right] + ig\frac{\partial_\mu}{\partial^2}\left[\frac{\partial_\nu}{\partial^2}\partial A, A_\nu\right] \\
&+ i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\frac{\partial A}{\partial^2}, \partial A\right] + O(A^3) \qquad (444)
\end{aligned}
$$

which is exactly the expression (443). The transverse field obtained in (186) has the property of being gauge invariant order by order in the coupling constant g. Let us perform the transformation properties of ϕ_ν under a gauge transformation

$$
\delta A_\mu = -\partial_\mu \omega + ig[A_\mu, \omega] \ . \qquad (445)
$$

We have, up to the order $O(g^2)$,

$$
\begin{aligned}
\delta\phi_\nu &= -\partial_\nu \omega + ig\left[\frac{1}{\partial^2}\partial A, \partial_\nu \omega\right] - i\frac{g}{2}\left[\omega, \partial_\nu \frac{1}{\partial^2}\partial A\right] - i\frac{g}{2}\left[\frac{\partial A}{\partial^2}, \partial_\nu \omega\right] + O(g^2) \\
&= -\partial_\nu \omega + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu \omega\right] + i\frac{g}{2}\left[\partial_\nu \frac{1}{\partial^2}\partial A, \omega\right] + O(g^2) \ . \qquad (446)
\end{aligned}
$$

Thereby

$$
\delta\phi_\nu = -\partial_\nu\left(\omega - i\frac{g}{2}\left[\frac{\partial A}{\partial^2}, \omega\right]\right) + O(g^2) \ , \qquad (447)
$$

from which the gauge invariance of A^h_μ is determined.

Finally, performing the expression of $A^2_{\rm min}$ as a power series in A_μ, one has

$$
\begin{aligned}
A^2_{\rm min} &= \mathrm{Tr}\int d^4x\, A^h_\mu A^h_\mu \\
&= \mathrm{Tr}\int d^4x \left[\phi_\mu\left(\delta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\partial^2}\right)\phi_\nu\right] \\
&= \mathrm{Tr}\int d^4x \left[\left(A_\mu - ig\left[\frac{1}{\partial^2}\partial A, A_\mu\right]+\frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\mu\frac{1}{\partial^2}\partial A\right]\right)\times\right. \\
&\left.\left(\delta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\partial^2}\right)\left(A_\nu - ig\left[\frac{1}{\partial^2}\partial A, A_\nu\right]+\frac{ig}{2}\left[\frac{1}{\partial^2}\partial A, \partial_\nu\frac{1}{\partial^2}\partial A\right]\right)\right] \\
&= \frac{1}{2}\int d^4x \left[A^a_\mu\left(\delta_{\mu\nu}-\frac{\partial_\mu\partial_\nu}{\partial^2}\right)A^a_\nu - 2gf^{abc}\frac{\partial_\nu\partial A^a}{\partial^2}\frac{\partial A^b}{\partial^2}A^c_\nu - gf^{abc}A^a_\nu\frac{\partial A^b}{\partial^2}\frac{\partial_\nu\partial A^c}{\partial^2}\right] \\
&+ O(A^4)\ . \qquad (448)
\end{aligned}
$$

We finish this Appendix making a simple remark, due to gauge invariance, $A^2_{\rm min}$ can be rewritten in a manifestly invariant way in terms of $F_{\mu\nu}$ and the covariant derivative D_μ (66).

APPENDIX B – Remarks on the localization of the BRST-invariant RGZ action

In this appendix, we explicitly show that the BRST invariant local formulation of the RGZ action in terms of A^h_μ in Landau gauge is equivalent to the original construction presented in (40). We begin with the BRST invariant Refined Gribov-Zwanziger action in the Landau gauge expressed as

$$\begin{aligned} S^L_{RGZ} &= S_{YM} + \int d^4x \Bigg[\left(ib^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) + \varphi^{ac}_\mu \partial_\nu D^{ab}_\nu (A^h) \varphi^{bc}_\mu \\ &- \bar{\omega}^{ac}_\mu \partial_\nu D^{ab}_\nu (A^h) \omega^{bc}_\mu + \frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu - M^2 \left(\bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu - \bar{\omega}^{ab}_\mu \omega^{ab}_\mu \right) \\ &+ \tau^a \partial_\mu A^{h,a}_\mu + \bar{\eta}^a \partial_\mu D^{ab}_\mu (A^h) \eta^b \Bigg] . \end{aligned} \tag{449}$$

The partition function is written as

$$Z = \int [\mathcal{D}\Phi] \, \mathrm{e}^{-S^L_{RGZ}} \,, \tag{450}$$

where $\Phi = \{A, b, \bar{c}, c, \bar{\varphi}, \varphi, \bar{\omega}, \omega, \xi, \tau, \bar{\eta}, \eta\}$. Integrating out the fields $(b, \tau, \bar{\eta}, \eta)$ one obtains

$$Z = \int \left[\mathcal{D}\tilde{\Phi}\right] \delta\left(\partial_\mu A^a_\mu\right) \delta\left(\partial_\mu A^{h,a}_\mu\right) \det\left(-\partial_\mu D^{ab}_\mu (A^h)\right) \mathrm{e}^{-\int d^4x (\dots)} \,, \tag{451}$$

with $(\dots)$ a shorthand notation for the remaining terms in the action (449) and $\tilde{\Phi} = \{A, \bar{c}, c, \bar{\varphi}, \varphi, \bar{\omega}, \omega, \xi\}$. In order to deal with the delta function $\delta\left(\partial_\mu A^{h,a}_\mu\right)$ imposing the transversality condition $\partial_\mu A^{h,a}_\mu = 0$, we make use of

$$\delta(f(x)) = \frac{\delta(x - x_0)}{|f'(x_0)|} \,, \tag{452}$$

with $f(x_0) = 0$. Of course, this relation holds if $f'(x_0)$ exists and is nonvanishing. It is possible to construct an iterative solution for $\partial_\mu A^{h,a}_\mu = 0$ as described in Appendix (A). Such a solution ξ_0 is expressed as

$$\xi_0 = \frac{1}{\partial^2} \partial_\mu A_\mu + \frac{ig}{\partial^2} \left[\partial_\mu A_\mu, \frac{\partial_\nu A_\nu}{\partial^2} \right] + \frac{ig}{\partial^2} \left[A_\mu, \partial_\mu \frac{\partial_\nu A_\nu}{\partial^2} \right] + \frac{ig}{2} \frac{1}{\partial^2} \left[\frac{\partial_\mu A_\mu}{\partial^2}, \partial_\nu A_\nu \right] + \mathcal{O}(A^3) \,, \tag{453}$$

where we have employed the matrix notation of Appendix (A). The important feature of (453) is that all terms always contain the divergence of the gauge field, *i.e.* $\partial_\mu A^a_\mu$.

Hence, the analogue of (452) is

$$\delta(\partial_\mu A^{h,a}_\mu) = \frac{\delta(\xi - \xi_0)}{\det\left(-\partial_\mu D^{ab}_\mu(A^h)\right)}\,, \tag{454}$$

where, due to the restriction of the domain of integration in the functional integral to the Gribov region, we have taken into account that $\det\left(-\partial_\mu D^{ab}_\mu(A^h)\right) > 0$. Hence, plugging (454) into (451) yields

$$Z = \int \left[\mathcal{D}\tilde{\Phi}\right] \delta\left(\partial_\mu A^a_\mu\right) \delta(\xi - \xi_0) \mathrm{e}^{-\int d^4x(\ldots)}\,. \tag{455}$$

Moreover, one easily sees from (453) that, due to the presence of $\delta(\partial_\mu A^a_\mu)$, it follows that $\xi_0 = 0$. Therefore,

$$Z = \int \left[\mathcal{D}\tilde{\Phi}\right] \delta\left(\partial_\mu A^a_\mu\right) \delta(\xi) \mathrm{e}^{-\int d^4x(\ldots)}\,. \tag{456}$$

Finally, reminding that $A^h_\mu = (h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h)$ with $h = \mathrm{e}^{ig\xi^a T^a}$, integration over ξ gives $A^h_\mu = (h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h) \to A$, so that the original formulation of the Refined Gribov-Zwanziger action in the Landau gauge as presented in (40).

APPENDIX C – Discrete Symmetries of the Dirac Theory

In addition to the continuous Ward Identities described in subsection (5.7), there are three other symmetries which helped us to constrain the final counterterm (407). The so-called CPT symmetries: specified as parity, time-reversal and charge conjugation. These symmetries enabled us to restrict a large set of fermionic terms. In the next subsections, we will present in details, following (159), their action on the fermionic bilinears so as to render our renormalization procedure as much clear as possible [34].

C.1 Parity

Parity is a space-time symmetry, which assigns $(x_4, x_i) \to (x_4, -x_i)$, where $i = 1, 2, 3$. Therefore, to establish the transformations laws under parity (P) for each Dirac bilinear fields, one has the transformations for ψ and $\bar{\psi}$ characterized as

$$P\psi(x_4, x_i)P = -i(\gamma)_4\psi(x_4, -x_i)\,, \tag{457}$$

$$P\bar{\psi}(x_4, x_i)P = iP\psi^\dagger(x_4, x_i)P\gamma_4 = i(P\psi(x_4, x_i)P)^\dagger\gamma_4 = i\bar{\psi}(x_4, -x_i)\gamma_4\,. \tag{458}$$

For the fermionic gauge-invariant quantities ψ^h and $\bar{\psi}^h$, one has the same symmetries, namely

$$P\psi^h(x_4, x_i)P = -i\gamma_4\psi^h(x_4, -x_i)\,, \tag{459}$$

$$P\bar{\psi}^h(x_4, x_i)P = i\bar{\psi}^h(x_4, -x_i)\gamma_4\,. \tag{460}$$

Then the scalar bilinear transforms as

$$P\bar{\psi}\psi P = \bar{\psi}(x_4, -x_i)(i\gamma_4)(-i\gamma_4)\psi(x_4, -x_i) = \bar{\psi}\psi(x_4, -x_i), \tag{461}$$

while for the vector one, we have

$$P\bar{\psi}\gamma_\mu\psi P = \bar{\psi}(i\gamma_4)\gamma_\mu(-i\gamma_4)\psi(x_4, -x_i) = \begin{cases} \bar{\psi}\gamma_\mu\psi(x_4, -x_i), & \mu = 4; \\ -\bar{\psi}\gamma_\mu\psi(x_4, -x_i), & \mu = 1, 2, 3. \end{cases} \tag{462}$$

[34] In order to be most detailed as possible, in this appendix we will work with all the possible Dirac bilinear fields, such as $\bar{\psi}\psi$, $\bar{\psi}\gamma^\mu\psi$, $i\bar{\psi}[\gamma^\mu, \gamma^\nu]\psi$, $\bar{\psi}\gamma^\mu\gamma^5\psi$, and $\bar{\psi}\gamma^5\psi$.

Here, the vector displays the same minus sign on the spatial components as does the space-time vector x^μ. Analogously, the symmetries of the pseudo-scalar and pseudo-vector [35] are given by

$$P i\bar{\psi}\gamma_5\psi P \;=\; i\bar{\psi}(i\gamma_4)\gamma_5(-i\gamma_4)\psi(x_4,-x_i) = -i\bar{\psi}\gamma_5\psi(x_4,-x_i)\,, \tag{463}$$

$$P\bar{\psi}\gamma_\mu\gamma_5\psi P \;=\; \bar{\psi}(i\gamma_4)\gamma_\mu\gamma_5(-i\gamma_4)\psi(x_4,-x_i) = \begin{cases} -\bar{\psi}\gamma_\mu\gamma_5\psi, & \mu = 4; \\ +\bar{\psi}\gamma_\mu\gamma_5\psi, & \mu = 1,2,3. \end{cases} \tag{464}$$

It is important to remember that the word "pseudo" ensures an extra minus sign in the parity symmetry case. Therefore, to study the fermionic counterpart we used the following bilinear quantity

$$\bar{\psi}\left[\gamma_\mu,\gamma_\nu\right]\psi \;=\; 2\bar{\psi}\sigma_{\mu\nu}\psi\ , \tag{465}$$

where under parity symmetry, we have

$$iP\bar{\psi}\left[\gamma_\mu,\gamma_\nu\right]\psi P \;=\; i\bar{\psi}(x_4,-x_i)\gamma_4\left[\gamma_\mu,\gamma_\nu\right]\gamma_4\psi(x_4,-x_i)\,. \tag{466}$$

Using the commutation relations for the gamma matrices: $\gamma_4\left[\gamma_4,\gamma_i\right]\gamma_4 = -\left[\gamma_4,\gamma_i\right]$, $\gamma_4\left[\gamma_i,\gamma_j\right]\gamma_4 = \left[\gamma_i,\gamma_j\right]$and $\sigma_{\mu\nu} = \frac{1}{2}\left[\gamma_\mu,\gamma_\nu\right]$, one gets

$$iP\bar{\psi}(x_4,x_i)\sigma_{\mu\nu}\psi(x_4,x_i)P \;=\; \begin{cases} -\bar{\psi}(x_4,-x_i)\sigma_{4j}\psi(x_4,-x_i), & \mu = i = 4, \nu = j = 1,2,3; \\ \bar{\psi}(x_4,-x_i)\sigma_{ij}\psi(x_4,-x_i) & \mu = i, \nu = j = 1,2,3. \end{cases} \tag{467}$$

C.2 Time-Reversal

The time-reversal is a discrete symmetry which acts as $\psi(x_4,x_i) \to \psi(-x_4,x_i)$, with $i = 1,2,3$, where the time Euclidean time component has been identified with x_4. The time-reversal transformations for the Dirac and the fermionic gauge-invariant fields are:

$$T\psi(x_4,x_i)T \;=\; (-\gamma_1\gamma_3)\psi(-x_4,x_i)\,, \tag{468}$$

$$T\bar{\psi}T \;=\; (T\psi T)^\dagger(-i\gamma_4)^* = \psi^\dagger(-x_4,x_i)(-\gamma_1\gamma_3)^\dagger(i\gamma_4) = \bar{\psi}(-x_4,x_i)(\gamma_1\gamma_3), \tag{469}$$

[35] Both bilinears have an extra negative sign in the parity transformation.

$$T\psi^h(x_4,x_i)T = (-\gamma_1\gamma_3)\psi^h(-x_4,x_i)\,, \tag{470}$$

$$T\bar{\psi}^h T = (T\psi^h T)^\dagger(-i\gamma_4)^* = \bar{\psi}^h(-x_4,x_i)(\gamma_1\gamma_3). \tag{471}$$

The transformation law for the scalar bilinear is described by

$$T\bar{\psi}\psi(x_4,x_i)T = \bar{\psi}(\gamma_1\gamma_3)(-\gamma_1\gamma_3)\psi(-x_4,x_i) = \bar{\psi}\psi(-x_4,x_i)\ , \tag{472}$$

while for the pseudo-scalar one gets:

$$Ti\bar{\psi}\gamma_5\psi T = -i\bar{\psi}(\gamma_1\gamma_3)\gamma_5(-\gamma_1\gamma_3)\psi(-x_4,x_i). \tag{473}$$

For the vector, one has

$$T\bar{\psi}\gamma_\mu\psi T = \bar{\psi}(\gamma_1\gamma_3)(\gamma_\mu)^*(-\gamma_1\gamma_3)\psi = \begin{cases}\bar{\psi}\gamma_\mu\psi(-x_4,x_i), & \mu=4;\\ -\bar{\psi}\gamma_\mu\psi(-x_4,x_i), & \mu=1,2,3.\end{cases} \tag{474}$$

Under time-reversal the pseudo-vector has the same transformation of the vector, given by

$$T\bar{\psi}\gamma_5\psi T = \bar{\psi}(\gamma_1\gamma_3)(\gamma_5)^*(-\gamma_1\gamma_3)\psi = \begin{cases}\bar{\psi}\gamma_5\psi(-x_4,x_i), & \mu=4,\\ -\bar{\psi}\gamma_5\psi(-x_4,x_i) & \mu=1,2,3.\end{cases} \tag{475}$$

The case $\bar{\psi}\,[\gamma_\mu,\gamma_\nu]\,\psi = 2\bar{\psi}\sigma_{\mu\nu}\psi$ is characterized by

$$\begin{aligned} T\bar{\psi}\sigma_{\mu\nu}\psi T &= \frac{1}{2}\bar{\psi}(-x_4,x_i)(\gamma_1\gamma_3)\,[\gamma_\mu,\gamma_\nu]^*\,(-\gamma_1\gamma_3)\psi(-x_4,x_i)\\ &= \begin{cases}\bar{\psi}(-x_4,x_i)\sigma_{0j}\psi(-x_4,x_i), & \mu=0,\nu=j=1,2,3;\\ -\bar{\psi}(-x_4,x_i)\sigma_{ij}\psi(-x_4,x_i) & \mu=i,\nu=j=1,2,3.\end{cases} \end{aligned} \tag{476}$$

C.3 **Charge Conjugation**

Finally, let us present the charge conjugation C, which acts on the the Dirac and fermionic gauge-invariant fields in the following way

$$C\psi(x)C = (-\bar{\psi}\gamma_4\gamma_2)^T\,, \tag{477}$$

$$\begin{aligned} C\bar{\psi}(x)C &= C\psi^\dagger C(-i\gamma_4) = (-i\gamma_2\psi)^T(-i\gamma_4)\\ &= (-i(-i\gamma_4)\gamma_2\psi)^T = (-\gamma_4\gamma_2\psi)^T \end{aligned} \tag{478}$$

$$C\psi^h(x)C = (-\bar{\psi}^h\gamma_4\gamma_2)^T\,, \tag{479}$$

$$C\bar{\psi}^h(x)C = (-\gamma_4\gamma_2\psi^h)^T\,. \tag{480}$$

Let us now consider the bilinears. For the scalar type, one has

$$\begin{aligned} C\bar{\psi}\psi C &= (-\gamma_4\gamma_2\psi)^T(-\bar{\psi}\gamma_4\gamma_2)^T = (\gamma_4)^{\alpha\beta}(\gamma_2)^\delta_\beta\psi_\delta\bar{\psi}^\vartheta(\gamma_4)^\varsigma_\vartheta(\gamma_2)_{\varsigma\alpha} \\ &= -\bar{\psi}^\vartheta(\gamma_4)^\varsigma_\vartheta(\gamma_2)_{\varsigma\alpha}(\gamma_4)^{\alpha\beta}(\gamma_2)^\delta_\beta\psi_\delta = \bar{\psi}\gamma_2\gamma_4\gamma_4\gamma_2\psi \\ &= -\bar{\psi}\psi\,. \end{aligned} \tag{481}$$

For the pseudo-scalar:

$$Ci\bar{\psi}\gamma^5\psi C = i(-i\gamma^0\gamma^2\psi)^T\gamma^5(-i\bar{\psi}\gamma^0\gamma^2)^T = i\bar{\psi}\gamma^5\psi. \tag{482}$$

The $\gamma^0$and γ^2 are symmetric matrices while $\gamma^1$and γ^3 are antisymmetric. Thus for the vector and pseudo-vector one gets:

$$C\bar{\psi}\gamma^\mu\psi C = \bar{\psi}\gamma^\mu\psi; \tag{483}$$

$$C\bar{\psi}\gamma^\mu\gamma^5\psi C = -\bar{\psi}\gamma^\mu\gamma^5\psi. \tag{484}$$

For the bilinear $i\bar{\psi}\left[\gamma^\mu,\gamma^\nu\right]\psi = 2\bar{\psi}\sigma^{\mu\nu}\psi$, we get

$$C\bar{\psi}\sigma_{\mu\nu}\psi C = \frac{1}{2}(-\gamma_4\gamma_2\psi)^T\sigma_{\mu\nu}(-\bar{\psi}\gamma_4\gamma_2)^T = -\bar{\psi}\gamma_4\gamma_2(\sigma_{\mu\nu})^T\gamma_4\gamma_2\psi, \tag{485}$$

using again the symmetry properties of the gamma matrices. Finally,

$$C\bar{\psi}(x_4,x_i)\sigma_{\mu\nu}\psi(x_4,x_i)C = \bar{\psi}\sigma_{\mu\nu}\psi. \tag{486}$$