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Optimization of cooling water systems

Rio de Janeiro

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Tese apresentada, como requisito parcial para a obtenção do grau de Doutor, ao Programa de Pós-Graduação em Engenharia Química, da Universidade do Estado do Rio de Janeiro. Área de concentração: Processos Químicos, Petróleo e Meio Ambiente.

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RESUMO

LEVY, Ana Lucia Lisbona. *Otimização de sistemas de água de resfriamento*. 2022. 148 f. Tese (Doutorado em Engenharia Química) - Instituto de Química, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, 2022.

Os sistemas de água de resfriamento são amplamente utilizados em indústrias químicas, petroquímicas, refinarias e usinas de energia. Em uma configuração típica, consistem em uma torre de resfriamento, um sistema de bombeamento, trocadores de calor e tubulação de interligação entre esses elementos. A literatura sobre o projeto desses sistemas é dominada por soluções de programação matemática não linear e métodos metaheurísticos. No entanto, estas abordagens estão associadas a limitações importantes, pois a convergência de algoritmos de programação matemática não linear pode ser difícil e métodos metaheurísticos não podem garantir a otimização global. Portanto, esta tese propõe um conjunto de procedimentos de projeto que evitam as limitações das abordagens da literatura. O primeiro problema analisado corresponde à otimização do projeto do sistema hidráulico da rede de distribuição de água de resfriamento juntamente com o projeto térmico dos trocadores de calor. A utilização de técnicas matemáticas apropriadas permite a formulação do problema como um problema de programação linear inteira (PLI). Os resultados obtidos mostraram que a abordagem de projeto proposta apresenta soluções com custo total anualizado inferior aos procedimentos tradicionais usualmente empregados na prática da engenharia. O segundo problema corresponde a uma extensão do problema anterior, onde as temperaturas da água de resfriamento que sai dos trocadores de calor tornam-se variáveis de otimização. Devido à sua complexidade e número de variáveis, este problema precisa ser resolvido com uma abordagem mais especializada, englobando *Set Trimming* e um problema recursivo de programação linear inteira (MILP). Os resultados obtidos mostraram que o método iterativo apresentou uma melhoria de custo de 5,2% em relação ao modelo mais simples com distribuição de vazão fixa. O terceiro problema apresenta a otimização da torre de resfriamento considerando como variáveis discretas de otimização a altura do enchimento, o tipo de enchimento, o número de torres e a área da torre. Esse problema é resolvido usando *Set Trimming* e *Smart Enumeration*. O quarto problema integra todos os problemas anteriores. A análise resultante envolve o projeto de todos os elementos do sistema, englobando torre de resfriamento, bomba, rede de tubulação e trocadores de calor. Esse problema é resolvido usando um conjunto de técnicas diferentes, incluindo enumeração convencional, *Set trimming*, *Smart Enumeration* e um MILP recursivo. Os resultados mostraram que foi selecionado um sistema de água de resfriamento com o menor custo total anualizado e que as variações de custo para a torre de resfriamento foram mais significativas do que para a rede de água de resfriamento. A solução proposta para esses problemas é inédita. Além disso, eles ajudarão a indústria a projetar sistemas de água de resfriamento de baixo custo. Devido à magnitude dos serviços de refrigeração em uma planta de processo, a redução de custos neste tipo de sistema pode estar associada a um impacto significativo em termos financeiros.

Palavras-chave: Otimização. Sistema de água de resfriamento. Torre de água de resfriamento. Trocadores de calor.

ABSTRACT

LEVY, Ana Lucia Lisbona. *Optimization of cooling water systems*. 2022. 148 f. Tese (Doutorado em Engenharia Química) - Instituto de Química, Universidade do Estado do Rio de Janeiro, Rio de Janeiro, 2022.

Cooling water systems are widely used in chemical, petrochemical, refinery and power plant industries. In a typical configuration, they consist of a cooling tower, a pumping system, heat exchangers, and interconnection piping among these elements. The literature about the design of these systems is dominated by nonlinear mathematical programming solutions and metaheuristic methods. However, these approaches are associated with important limitations, e.g. the convergence of nonlinear mathematical programming algorithms may be difficult and metaheuristic methods cannot guarantee global optimality. Therefore, this thesis proposes a set of design procedures that avoid the limitations of the literature approaches. The first analyzed problem corresponds to the optimization of the design of the hydraulic system of the cooling water distribution network together with the thermal design of the heat exchangers. The use of appropriate mathematical techniques allows the formulation of the problem as an integer linear programming problem (ILP). The results obtained showed that the proposed design approach presents solutions with lower total annualized cost than traditional procedures usually employed in the engineering practice. The second problem corresponds to an extension of the previous problem, where the temperatures of the cooling water leaving the heat exchangers become optimization variables. Due to its complexity and number of variables, this problem needs to be solved with a more specialized approach, encompassing Set Trimming and a recursive mixed-integer linear programming (MILP) problem. The results obtained showed that the iterative method showed a cost improvement of 5.2% in relation to the simpler model with a fixed flow rate distribution. The third problem presents the optimization of the cooling tower considering as optimization discrete variables the height of the filling, the type of filling, the number of towers and the area of the tower. This problem is solved using Set Trimming and Smart Enumeration. The fourth problem integrates all the previous problems in a unique task. The resultant analysis involves the design of all elements of the system, encompassing cooling tower, pump, pipe network, and heat exchangers. This problem is solved using a set of different techniques including conventional enumeration, Set Trimming, Smart Enumeration, and a recursive MILP. The results showed that a cooling water system with the lowest annualized total cost was selected and that the cost variations for the cooling tower were more significant than for the cooling water network. The solution proposed for these problems are unprecedented. In addition, they will help the industry to design lower cost cooling water systems. Due to the magnitude of the cooling services in a process plant, cost reductions in this type of system can be associated with a significant impact in financial terms.

Keywords: Optimization. Cooling water system. Cooling water tower. Heat exchangers.

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Parameters

$\widehat{Aexc}$	excess area (%)
$\widehat{Areamin}_{srow,l}$	minimum value of the area of the heat exchanger (m^2)
Atw	sectional area of filling (m^2)
$\hat{a}$	parameter in the Eq. (174)
$\hat{b}$	parameter in the Eq. (174)
$\hat{c}$	parameter in the Eq. (174)
$\widehat{ch}$	Hazen-Williams parameter
$\hat{C}_{air}$	cost of the cooling tower associated with mass air flow (USD)
$\hat{C}_{pack}$	cost of the cooling tower associated with the filling (USD)
$\widehat{CPI_n}$	annualized cost of a pipe section with unitary length (USD/(y·m))
$\widehat{Cpu_s}$	annualized capital cost of the pump alternative *s* (USD/y)
$\widehat{Cp}_w$	heat capacity (J/(kg K))
$\hat{C}_{tower}$	investment cost of the cooling tower (USD)
$\hat{c}_{un}$	Constant in Eq. (172)
$\hat{C}_1$	correlation parameter in Eq. (110)
$\widehat{D}_n^{nom}$	nominal diameter of the commercial pipe *n* (in)
$\hat{d}$	parameter in the Eq. (174)
$\widehat{D}_n^{int}$	inner diameter of the commercial pipe *n* (m)
$\hat{d}_1$	empirical coefficient for each filling in Eq. (185)
$\hat{d}_2$	empirical coefficient for each filling in Eq. (185)
$\hat{d}_3$	empirical coefficient for each filling in Eq. (185)
$\hat{d}_4$	empirical coefficient for each filling in Eq. (185)
$\hat{d}_5$	empirical coefficient for each filling in Eq. (185)
$\hat{F}_{srow,l}$	correction factor of the LMTD for a configuration 1-N, N even
$\widehat{Fmp}$	cost factor related to the pump material
$\widehat{Fmh}_l$	cost factor related to the heat exchanger (shell/tube) material

$\widehat{Fms}$	Marshall-swift index correction factor, for the pump cost
$\widehat{Fmseq}$	Marshall-swift index correction factor, for the heatexchangers cost
$\widehat{Fmspip}$	Marshall-swift index correction factor, for the pipe cost
$\widehat{Fph_l}$	cost factor related to the pressure range of the heat exchanger
$\widehat{Ft_s}$	cost factor related to the pump type
$\hat{g}$	gravity acceleration (m/s^2)
$\hat{H}^{atm}$	air enthalpy at the ambient conditions (J/kg)
$\hat{H}_{in}$	enthalpy of air in the intlet of the tower (J/kg)
$\hat{H}_m^{sat}$	enthalpy of saturated air (J/kg)
$\hat{\imath}$	interest rate
$\widehat{Kmis}$	miscellaneous pressure loss constant in Eq. (186)
$\widehat{ks_l}$	thermal conductivity of the shell-side stream of cooler *l* (W/(m K))
$\widehat{kt_l}$	thermal conductivity of the tube-side stream of cooler *l* (W/(m K))
$\widehat{ktube}$	tube wall thermal conductivity (W/(m K))
$\hat{L}_k$	length of the pipe section *k* (m)
$\widehat{m}$	correlation parameter in Eq. (110)
$\widehat{M}_{air}$	molar mass of air (kg/kmol)
$\widehat{M}_w$	molar mass of water (kg/kmol)
$\widehat{mdrift}$	mass water flow rate from drift (kg/h)
$\widehat{ms_l}$	shell-side mass flow rate in the cooler *l* (kg/s)
$\widehat{mt_l}$	tube-side mass flow rate in the cooler *l* (kg/s)
$\hat{n}$	constant in Eq. (172)
$\hat{n}_{cycles}$	number of concentration cycles in the cooling tower
$\widehat{Nop}$	number of operating hours per year (h/yr)
Nt	number of towers
$\widehat{ny}$	number of years of the project life (yr)
$\hat{P}$	local atmospheric pressure (Pa)
$\widehat{ParPmax}_{srow,l}$	maximum value of the LMTD correction factor parameter $\hat{P}_l$
$\widehat{Pmax}_{srow,l}$	maximum value of the LMTD correction factor parameter $\hat{P}_l$
$\hat{P}_l$	LMTD correction factor parameter
$\widehat{pc}$	power cost (USD/kWh)

$\widehat{PDeq}_{srow}$	equivalent diameter of the cooler alternative *srow* (m)
$\widehat{PDs}_{srow}$	shell diameter of the cooler alternative *srow* (m)
$\widehat{Pdte}_{srow}$	outer tube diameter of the cooler alternative *srow* (m)
$\widehat{Pdti}_{srow}$	inner tube diameter of the cooler alternative *srow* (m)
$\widehat{pf}_{k,n}$	unitary head loss for a pipe diameter *n* in a pipe section *k* (m/m)
$\widehat{PL}_{srow}$	tube length of the cooler alternative *srow* (m)
$\widehat{Plpf}_{k,n_{srow}}$	tube layout of the cooler alternative *srow*
$\widehat{PNb}_{srow}$	number of baffles of the cooler alternative *srow*
$\widehat{PNpt}_{srow}$	number of tube passes of the cooler alternative *srow*
$\widehat{PNtt}_{srow}$	total number of tubes of the cooler alternative *srow*
$\widehat{Prp}_{srow}$	tube pitch ratio of the cooler alternative *srow*
$\widehat{qhe}_l$	volumetric flow rate at the cooler of the hydraulic circuit *l* (m^3/s)
$\hat{q}_k$	volumetric flow rate in the edge *k* (m^3/s)
$\widehat{qpu}_s^{design}$	volumetric flow rate at the design condition of the pump *s* (m^3/s)
$\widehat{qw}$	total volumetric water flow rate in the heat exchangers (m^3/s)
$\hat{Q}$	heat duty (W)
$\hat{Q}_l$	heat duty of the cooler *l* (W)
$\hat{r}$	annualization factor
$\hat{R}_l$	LMTD correction factor parameter
$\widehat{Rfs}$	shell-side fouling factor (m^2 K/W)
$\widehat{Rft}$	tube-side fouling factor (m^2 K/W)
$\widehat{Tci}$	outlet temperature of the cold stream (^oC)
$\widehat{Thi}$	inlet temperature of the hot stream (^oC)
$\hat{T}_{return}^{max}$	water maximum temperature in the return to the cooling tower (^oC)
$\hat{T}_{return}^{min}$	water minimum temperature in the return to the cooling tower (^oC)
$\widehat{Tbot}_{max}^{w}$	water maximum temperature in the bottom of the cooling tower (^oC)
$Ttop^{w}$	water inlet temperature in the cooling tower (top of the tower) (^oC)
$\widehat{TWB}_{in}$	wet bulb temperature (^oC)
$\widehat{vmax}$	maximum flow velocity in the pipe sections (m/s)
$\widehat{vmin}$	minimum flow velocity in the pipe sections (m/s)
$\widehat{vsmax}$	maximum flow velocity in the heat exchanger shell-side (m/s)

$\widehat{vsmin}$	minimum flow velocity in the heat exchanger shell-side (m/s)
$\widehat{vtmax}$	maximum flow velocity in the heat exchanger tube-side(m/s)
$\widehat{vtmin}$	minimum flow velocity in the heat exchanger tube-side (m/s)
$\widehat{W}_{in}$	humidity of air in the inlet of the cooling tower (kg water/kg dry air)
$\widehat{yT}_{c,l}$	binary parameter, if the cooling water is in the tube-side ($\widehat{yT}_{c,l} = 1$)
$\widehat{yT}_{h,l}$	binary parameter, if the hot stream is in the tube-side ($\widehat{yT}_{h,l} = 1$)
$\hat{z}_s$	parameter for the calculation of pump capital cost in Eq. (74)
$\hat{\alpha}_1$	parameter of quadrature point
$\hat{\alpha}_2$	parameter of quadrature point
$\hat{\alpha}_3$	parameter of quadrature point
$\hat{\alpha}_4$	parameter of quadrature point
$\widehat{\Delta Psdisp}_l$	shell-side available pressure drop in the cooler *l* (Pa)
$\widehat{\Delta Ptdisp}_l$	tube-side available pressure drop in the cooler *l* (Pa)
$\widehat{\Delta Tlm}_l$	log-mean temperature difference of the cooler *l* (ºC)
$\widehat{\Delta z}$	elevation difference between top and bottom of the cooling tower (m)
$\hat{\eta}$	pump efficiency
$\hat{\Lambda}_{l,k}$	circuit matrix
$\widehat{\mu s}_l$	viscosity of the shell-side stream in the cooler *l* (Pa·s)
$\widehat{\mu t}_l$	viscosity of the tube-side stream in the cooler *l* (Pa·s)
$\hat{\rho}_{air}$	dry air density (kg/m^3)
$\hat{\rho}_{in}$	density of humid air entering the cooling tower (kg/m^3)
$\widehat{\rho s}_l$	density of the shell-side stream in the cooler *l* (kg/m^3)
$\widehat{\rho t}_l$	density of the tube-side stream in the cooler *l* (kg/m^3)
$\hat{\rho}_w$	water density (kg/m^3)
$\hat{\tau}_{fan}$	efficiency of the fan

Binary variables

$y^{pi}_{k,n}$	binary variable, if the commercial diameter *n* is in the edge *k*, so $y^{pi}_{k,n} = 1$
$y^{pu}_{k,s}$	binary variable, if the pump *s* is in the edge *k*, so $y^{pu}_{k,s}$=1
$yrow_{srow,l}$	variable representing the set of heat exchanger design variables

$wpi_{k,n,srow,}$ product of binary variables $y_{k,n}^{pi}$ $yrow_{srow,l}$

$wpu_{k,s,srow}$ product of binary variables $y_{k,s}^{pu}$ $yrow_{srow,l}$

Continuous and discrete variables

A	heat transfer area (m^2)
Atw	cross sectional area of the tower (m^2)
C	objective function (USD/year)
Ct	total annualized cost of a cooling water tower ($/year)
C_{cap}	annualized investment cost associated with the cooling tower (USD/year)
C_{caplb}	annualized investment cost associated with the cooling tower, for the lower bound (USD/year)
C_{mk}	annualized make up water consumption cost (USD/year)
Clb	lower bound on the annualized cost (USD/year)
cop_k	operating cost of pump k (USD/year)
C_{opf}	fan operation cost (USD/year)
$C_{heatexch}$	annualized capital cost of the heat exchangers (USD/year)
C_{oper}	pipe network operating cost (USD/year)
D_k	inner diameter of the pipe section k (m)
Dte	outer tube diameter (m)
Dti	outer tube diameter (m)
Ds	shell diameter (m)
f_k	unitary head loss along pipe sections k (dimensionless)
fhe_l	head loss in the cooler l (m)
fpi_k	head loss in the pipe section k (m)
fpu_k	head of the pump k (m)
fv_l	head loss in the valve l (m)
G	air flow rate (kg of dry air/ h)
H	wet air enthalpy (J/(kg of dry air))
h'	enthalpy of saturated air at water temperature (J/(kg of dry air))
H_m	enthalpy of air in fluid conditions (J/kg)
H_{out}	enthalpy of air in the outlet of the tower (J/kg)
Hs	shell-side convective heat transfer coefficient (W/(m^2 K))

Ht	tube-side convective heat transfer coefficient (W/(m^2 K))
I_M	Merkel's integral
I_P	characteristic parameter of the cooling tower
Kfi	pressure drop coefficient in the filling
L	heat exchanger tube length (m)
L_k	tube length in the pipe section k (m)
Lay	tube layout (1 = square, 2 = triangular)
Lbc	baffle spacing (m)
Lm	mass flow rate of water (kg/s)
m_{air}	mass flowrate of dry air in the cell of the cooling tower (kg/s)
m_{airT}	total mass flowrate of dry air in the cooling tower (kg/s)
m_{hum}	average mass flow rate of humid air (kg/s)
m_{out}	mass flowrate of humid air in the outlet of the tower (kg/s)
$mbld$	mass flowrate of cooling water tower blowdown (kg/s)
$mdrift$	loss of water from the tower cell, in the form of splashes (drift) (kg/h)
mmk_{hum}	mass flowrate of makeup water (kg/s)
$mmklb$	mass flowrate of makeup water, of the lower bond (kg/s)
$mvap$	mass flowrate evaporated water (kg/s)
Nb	number of baffles
Npt	number of passes in the tube-side
N_{cell}	number of cells of the cooling tower
Ntt	total number of tubes
P	temperature ratio
P_{out}^{sat}	vapor pressure in the tower outlet (Pa)
$Pmax$	maximum value of P temperature ratio
Pot_{fan}	fan power (kwh)
$qair_{out}$	volumetric flowrate of humid air in the outlet of the tower (m^3/s)
q_{cell}	water volumetric flow rate in the cell of the cooling tower (m^3/s)
$qhemax$	maximum water volumetric flow rate in the heat exchanger (m^3/s)
q_{return}	volumetric water flowrate in the return of cooling tower (m^3/s)
q_{total}	total volumetric water flowrate (m^3/s)
Res	shell-side Reynolds number
Ret	tube-side Reynolds number

Rp	tube pitch ratio
T	temperature of the water stream (^oC)
Tco_l	temperature of the water in the outlet of the heat exchanger (^oC)
$Tcomax$	maximum water outlet temperature of the heat exchanger (^oC)
TI	pair of temperature intervals (^oC)
$T1$	water temperature at the outlet (cold water) (^oC)
$T2$	water temperature at the inlet (hot water) (^oC)
T_{out}^{air}	temperature of satured air in the outlet of the tower (^oC)
$Tbot^w$	temperature of the water leaving the cooling tower (bottom of the tower) (^oC)
T_{return}	temperature of water in the return to the cooling tower (^oC)
T_t	temperature of water in an iteration (^oC)
$T_{w,m}$	temperature of the cooling water along the quadrature points (^oC)
U	overall heat transfer coefficient (W/(m^2 K))
V	active volume of the cooling tower (m^3)
v_{air}	air velocity (m/s)
v_{hum}	air velocity in the filling (m/s)
$vair_{out}$	air velocity in the tower outlet (m/s)
v_k	flow velocity in pipe section k (m/s)
Vs	shell-side flow velocity (m/s)
Vt	tube-side flow velocity (m/s)
W_{media}	average humidity of air in the cooling tower (kg water/kg dry air)
W_{out}	humidity of the air at the outlet of the tower (kg water/kg dry air)
$wpi_{k,n,srow,l}$	product of binary variables Eq. (140)
$wpu_{k,s,srow,l}$	product of binary variables Eq. (160)
ρ_{hum}	harmonic mean of the densities of the air in the filling (kg/m^3)
ρ_{out}	density of the humid air in the outlet of the tower (kg/m^3)
ΔP	Pressure loss in the cell of cooling tower by the filling and other comp. (Pa)
ΔP_{misc}	miscellaneous head loss (Pa)
Δ_{pack}	height of the filling of cooling tower (m)
ΔP_{pack}	pressure drop in the filling (Pa)
ΔPt	tube-side pressure drop (Pa)
ΔP_t	pressure loss in the cell of the cooling tower (Pa)
ΔP_{tl}	fan head (Pa)

ΔP_{th}	total head loss in the tower (Pa)
ΔPs	shell-side pressure drop (Pa)
ΔP_{vp}	head loss by velocity pressure (Pa)
ρ_{out}	outlet air density (kg/m^3)
ρ_{hum}	average wet air density (kg/m^3)

Sets

HE	subset of heat exchangers
HY	set of hydraulic circuits
INT	node subset of interconnections
PD	water return node subset
PI	subset of pipe sections
PS	water supply node subset
PU	subset of the pumps
S	set of pairs of intervals of top and bottom cooling tower temperatures
SD	set of commercial diameters
SPU	set of available pumps
STR	set of edges
VET	set of nodes

Subscripts

K	index of the edges
L	index of the hydraulic circuits
N	index of commercial diameters
S	index of the available pump alternatives
T	index of the nodes
T	iteration counter

Superscripts

Design	design condition
Int	Internal

Nom	Nominal
Pi	Pipe
Pu	Pump

SUMMARY

INTRODUCTION

Cooling water systems are widely used in chemical, petrochemical, refinery and power plant industries. In a typical configuration, they consist of a cooling tower, a pumping system, heat exchangers, and interconnection piping between these elements.

These systems can have large flow rates (e.g. 8000 m^3/h) and their pipes can run through many process units and large extensions of a process plant, making their costs associated with investment and operation very high. According to Kuritsyn et al. (2012), spending on electricity to cool circulating water in oil and gas refineries and petrochemical plants corresponds to about 15% of the total energy costs in these units.

Although considered to be utilities, these systems may have a direct influence on the throughput and product quality. In addition, restrictions on the granting of water to industries, the need to reduce energy consumption and revamp projects that increase the demand of cooling water, make it necessary to improve the design of cooling water systems. It is not uncommon during the operation of a plant occur cooling water distribution problems or overloaded systems that no longer have a satisfactory efficiency.

Therefore, the objective of this thesis is the development of optimization solutions for the design of cooling water systems. Several innovative solutions are provided for a set of design problems. Particularly, the proposed approach analyzes the different components of a cooling water system (cooling tower, pumps, pipes and coolers) simultaneously, thus allowing to explore the several existent tradeoffs in the design. This holistic analysis can attain important cost reductions in new projects.

Scope

The current thesis presents optimization procedures for four design problems associated with cooling water systems. These problems involve, cumulatively, specific aspects of the cooling water system. Analyzing the set of problems as a whole, it can be seen that at each stage of the thesis new variables and system components are added. In these problems, heat exchangers are considered in parallel arrangement, as it is the most used option in the industry. It was observed in the literature, several limitations of the optimization procedures applied to design each system mentioned above, particularly the analysis of the entire system, encompassing cooling tower, pump, pipes and heat exchangers. This thesis aims at contributing to fill this gap, providing roubst and computationally efficient solution procedures.

The first analyzed problem corresponds to the optimization of the design of the hydraulic system of the cooling water distribution network together with the design of the heat exchangers. The network of pipes, the pump and the heat exchangers are included in the optimization in the same framework. The use of appropriate mathematical techniques allows the formulation of the problem as an integer linear programming problem (ILP). The results of this analysis were published in the journal Industrial & Engineering Chemistry Research (LEVY et al., 2019).

The second problem corresponds to an extension of the previous problem, where the temperatures of the water leaving of the heat exchangers become optimization variables. Due to its complexity and number of variables, this problem needs to be solved with a more specialized approach, encompassing Set Trimming and a recursive MILP.

The third problem presents the optimization of the cooling tower considering as discrete variables the height of the filling, the type of filling, the number of towers and the area of the tower. This problem is solved using Set Trimming and Smart Enumeration.

The fourth problem integrates all the previous problems in a unique task. The resultant analysis involves the design of all elements of the system, encompassing cooling tower, pump, pipe network, and heat exchangers. This problem is solved using a set of different techniques including conventional enumeration, Set Trimming, Smart Enumeration, and a recursive MILP.

As verified through the bibliographic review, the solution proposed for these problems are unprecedented. In addition, they will help the industry to design lower cost cooling water systems. Due to the magnitude of the cooling services in a process plant, cost reductions in this type of system can be associated with a significant impact in financial terms.

Structure

This thesis is organized as follows. Chapter 1 presents a bibliographic review about the investigated subject. Chapter 2 presents the optimization of a cooling water network, contemplating the design of the heat exchangers and the pipe network simultaneously, where the problem corresponds to an integer linear programming (ILP). Chapter 3 presents an extension of the design problem explored in Chapter 2, instead of imposing the same outlet temperature on all coolers, the proposed formulation of the design problem includes the optimization of the distribution of the cooling water flow rate among the different coolers. Chapter 4 presents the optimization of a mechanical induced draft counter flow cooling water tower through the use of Set Trimming and Smart Enumeration. Chapter 5 presents the optimization of the whole system, encompassing the cooling water tower and the cooling water network, using the models described in the previous chapters. Finally, the conclusions are presented, followed by the References. Annex I presents the heat exchanger model employed in this (GONÇALVES et al., 2017). Annex II presents the publication made during the doctoral thesis.

1. LITERATURE REVIEW

1.1. Cooling water systems

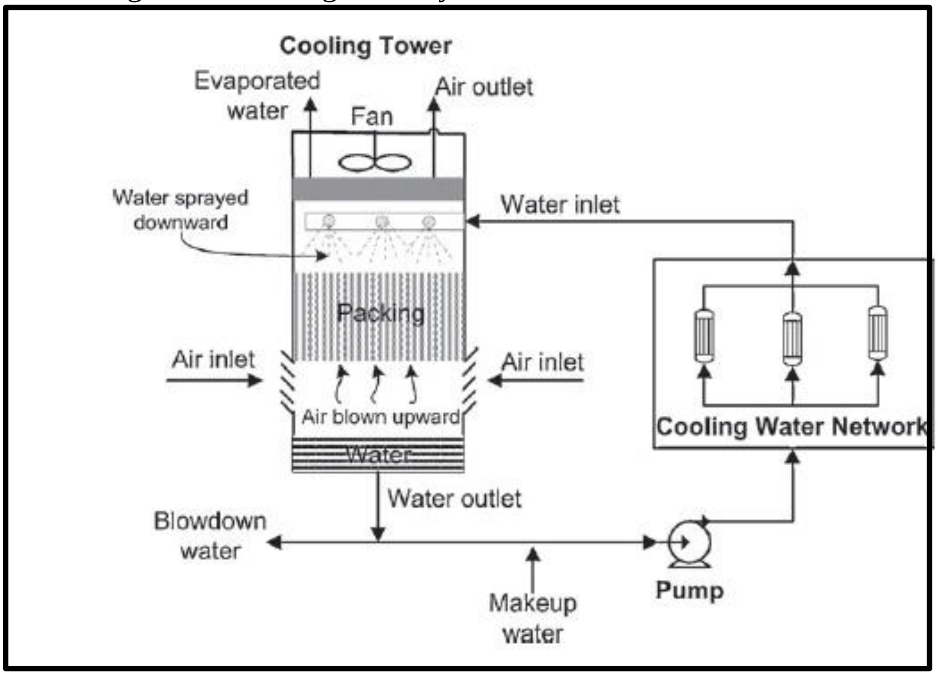
Cooling water systems are widely used to remove heat in large industrial units such as oil refineries, chemical plants and power stations (MA et al., 2017).

Water is commonly used as a cooling fluid due to its non-hazardous chemical composition, ease of handling and thermal characteristics. Cooling water systems can be open, closed or semi-open.

Open systems dispose of all heated water used in the cooling system and, therefore, their use presents restrictions from an environmental point of view. In closed systems, all water used for cooling is reused, with no discharge flow, but this type of system demands high heat exchange area associated with the final rejection of heat to the environment in the form of a dry cooling tower. Semi-open systems operate with cooling water recirculation, but since they employ the principle of evaporative cooling in so-called wet cooling towers, they require water make-up during their operation. This system is the most common alternative in the chemical process industry and will be the focus of this thesis.

Semi-open cooling water systems consist of a cooling tower, a pumping system, heat exchangers and piping. As shown in Figure 1, water from the cooling tower is routed to the heat exchangers where the process streams are cooled and the water heats up accordingly. That stream then returns to the tower, where heat exchange will take place between water and air. The cooling of the water in the tower is mainly a consequence of the evaporation process that occurs along the flow of the water through the tower filling. There is also a water blowdown (to prevent the increase of the salt concentration and precipitation) and water drift losses. The evaporation, blowdown and drift are compensated by a water make up to the tower.

Figure 1 - Cooling water system



Reference: PONCE-ORTEGA et al., 2010

It is common engineering practice to arrange the heat exchangers in the cooling system in parallel. However, there are several works in the literature suggesting their alignment in series or parallel-series combinations (PICÓN-NÚNEZ et al., 2007; AKSHAY et al., 2013; SUN et al., 2015).

Proper design and performance of cooling water systems is extremely important for maintaining the throughput and quality of industry products, besides to reduce the high investment associated with the design of such systems. Therefore, although considered a utility, these systems should be treated with great care.

1.2. Cooling towers

Cooling towers are responsible for cooling water by promoting their contact with cold air from the environment. Proper design and performance of cooling towers is essential to ensure production levels and safety of process units.

The cooling towers can be dry or wet type, the difference between them is in the form of heat transfer. In dry towers, cooling coils are used and there is no physical contact between air and water, nor evaporation of water occurs. Heat transfer occurs only by convection without water phase change. Wet cooling towers have direct contact between air and water and water cooling occurs mainly due to evaporation.

Cooling towers are defined by performance parameters such as range (difference between tower inlet and outlet water temperatures), approach (difference between tower outlet temperature and wet bulb temperature), tower characteristic ratio (ratio of water to air mass flowrate), effectiveness (ratio of the range to sum between range and approach) and evaporation rate (SINGH; DAS, 2017).

Only wet cooling towers are within the scope of this work and will be explained in detail in the following items.

1.2.1. Cooling Tower Types

Cooling towers can be classified according to (BAKER; SHRYOK, 2013):

➔ Nature of the air intake: natural, mechanical draft (induced or forced) or hybrid draft

In natural draft cooling towers, air movement occurs as a function of buoyant forces associated with the difference between the specific mass of hot air inside the tower and cold air outside (WILLIAMSON et al., 2008).

Mechanical draft air towers are characterized by the presence of fans to provide the air flow. The presence of the fans also allows the regulation of the air flow rate to compensate for atmospheric changes and load conditions. These towers have more stable performance than natural draft towers, but are associated with the energy consumption by the fans.

For forced draft towers, the fan is installed at the tower air intake. Since air has larger inlet velocities and low outlet velocities, these towers are more prone to recirculation (when hot air returns to the tower) and are therefore less stable than induced draft towers. Typically, the fans used have a centrifugal type motor. Forced draft towers are typically applicable to small services only. In induced draft towers, the fans are located at the air outlet. These towers have an air outlet velocity of 3 to 4 times the air intake velocity and are less prone to recirculation problems. These towers are widely used in large and small plants.

Hybrid draft towers are natural draft towers that also feature fans installed inside. The fans of this tower type are operated only in periods of high thermal load and adverse environmental conditions.

➔ The direction of air inlet in the tower: counterflow or crossflow.

Counterflow towers have a configuration in which air moves vertically through the filling, in the opposite direction to the falling water. The water distribution system in these towers is generally based on pressurized sprays, but in large countercurrent towers, large low pressure gravity distribution systems are used.

In towers with crossflow, the air enters in the horizontal direction crossing with the waterfall. In these towers, in general, the water to be cooled is delivered to basins located above the filling area and is distributed by the gravity filling through holes in the floor of these basins.

➔ Format of the tower: circular or polygonal.

As for the shape, the towers can be polygonal, often being built in cells, which are added to reach a certain capacity. The round towers, with mechanical draft, are built with the fans as close as possible around the center of the tower. Multifaceted towers, such as octagonal mechanical draft towers, for example, also fall under the general classification of round towers. These towers can handle large thermal loads with less footprint than required for straight towers and are significantly less subject to recirculation.

1.2.2. Heat and mass transfer in the cooling tower

The heat exchange process in a cooling tower consists of the combination of latent heat transfer resulting from water evaporation (approximately 80% of heat transfer) and sensitive heat transfer resulting from convection as a function of the difference between the temperatures of the water and air (DAEIL AQUA CO., LTD., 2004).

The rate of heat transfer to the airflow through the cooling tower depends on the ambient temperature and humidity. The thermodynamic limit associated with the cooling tower operation corresponds to the wet bulb temperature, i.e. the tower water outlet temperature will always be higher than the wet bulb temperature (KURITSYN et al., 2012). How closely the cold water temperature approaches the wet bulb temperature depends on the tower design and environmental conditions (KURITSYN et al., 2012).

The filling of the tower is responsible for increasing the contact area between water and air to make the heat exchange more efficient. The most modern fillings are in the form of plastic modules (KURITSYN et al., 2012).

Over time, the filling surfaces become covered by deposits and the heat transfer rates from water to air differ from design values. Thus, towers of similar design may have different effectiveness values over time (KURITSYN et al., 2012).

1.2.3. Merkel model

The mathematical model of cooling towers corresponds to energy and mass balances of the air and water streams. An important alternative of modeling for analysis of cooling towers involves the Merkel model (SINGHAM, 1983).

The Merkel model, applicable to counterflow cooling towers, combines the sensible and latent heat transfers into a mathematical representation where the driving force becomes the enthalpy difference.

In this model, each water drop is considered to be surrounded by an interface from which heat is transferred from water to air. Heat is then transferred from the interface to the air by convection (sensible heat transfer) and evaporation (latent heat). The Merkel model is then based on the following set of simplifying hypotheses: (i) The temperature gradient in the liquid is negligible (the interface temperature is equal to the liquid temperature); (ii) The air at the interface is saturated at the liquid temperature; and (iii) The ratio between molar fraction and humidity is equal to the ratio between air and water molar masses.

Based on these assumptions, the following equation can be formulated:

$$LmCpdT = KadV(h' - h) \tag{1}$$

where K is the mass transfer coefficient between saturated air at water temperature and the main air stream in kg/(s·m^2), a is the specific air-water interface area in m^2/m^3, h is the wet air enthalpy in J/(kg of dry air), h' is the enthalpy of saturated air at water temperature in J/(kg of dry air), V is the active volume of the cooling tower in m^3, Lm is the mass flow rate of water in kg/s, Cp is the water heat capacity in J/(kg·ºC) and T is the temperature of the water stream in ºC.

To obtain a solution, Equation (1) needs to be integrated together with the energy balance (SINGHAM, 1983):

$$LmCpdT = Gdh \tag{2}$$

where G is the air flow rate in kg of dry air/ h.

Merkel's integral is then presented by the following equation:

$$\frac{KaV}{Lm} = Cp \int_{T1}^{T2} \frac{dT}{h' - h} \tag{3}$$

where $T1$ is the water temperature at the outlet (cold water) in ºC and $T2$ is the water temperature at the inlet (hot water) in ºC.

The above considerations simplify the development and solution of cooling tower problems (although the approximations bring a certain distancing from reality).

In counterflow cooling towers, air and water conditions are equal throughout any horizontal section. Then, Equation (2) can be used to calculate the enthalpy of air at each water temperature along the tower. Calculation should begin at the base of the cooling tower, as this is the only point where air and water conditions are previously known.

1.3. Bibliographic review of cooling water systems optimization

The following items present the literature review of optimization of cooling water systems, including papers that addressed cooling water networks, cooling towers, and the system composed of the cooling water network and the cooling tower simultaneously. At the end, an overview of the literature is presented, identifying the contribution of this thesis to the state of the art.

1.3.1. Cooling water networks

This item presents the works of the literature that dealt with the optimization of the cooling water pipe network with or without the heat exchanger optimization.

Lee et al. (1997) presented the hydraulic modeling of cooling water pipe networks applied to two problems: analysis of existing cooling water systems to determine unknown pressures and flows along the network and the redistribution of cooling water between different heat exchangers in an attempt to achieve a given set of flow rates by manipulating valves and replacing pipe diameters.

Ponce-Ortega et al. (2007) studied the optimization of a cooling water system while considering the cost for heat exchangers and utilities. For this, a superstructure was built that allowed the bypass and splitting of utility streams. The superstructure also considered series and parallel arrangements of the heat exchangers. The optimization variables of the problem were: the cold utility output temperature, the temperatures for each stage of the process hot streams, the cold utility outlet temperature for each stage in each match, and the inlet temperatures for the stages. The objective function used was the total annualized cost, resulting from the sum of the annualized cost of heat exchangers and the cost of utilities. The results showed that the lowest utility cost does not always correspond to the lowest annualized total cost configuration. The formulation corresponds to a mixed integer nonlinear programming (MINLP) problem solved using the DICOPT solver in the GAMS software.

Ponce-Ortega et al. (2009) studied the optimization of a cooling water system through a disjunctive formulation. In this paper, the design equations of heat exchangers were based on Kern's thermohydraulic model (KERN, 1950) to predict the performance of shell and tube exchangers without phase change. The model was presented in the form of two compact analytical equations, which correlate pressure drop and turbulent heat transfer to the tube and shell sides. A superstructure was set up that considered series or parallel arrangements of exchangers, bypass, mixing and split of cold utility streams. The objective function used was the minimization of the total annualized cost that involved the cost of utility consumption, the investment cost for pumps and heat exchangers and pumping costs. The problem resulted in a nonconvex MINLP formulation, solved using the DICOPT solver implemented in the GAMS software.

Picón-Núnez et al. (2011) developed a methodology for optimizing cooling water systems. The hydraulic model was developed using volumetric flow rate as the primary variable. To ensure system flexibility, it was considered a system for the highest thermal demand and then by-passes were introduced to control performance under reduced loads. These calculations showed the water flow rates to the heat exchangers as a result.

Souza et al. (2014) investigated the design of cooling water pipe systems using linear programming (LP). The solution to the problem identified the lengths of subsections with standard diameters and the corresponding pump alternative that minimized investment and operating costs. The proposed methodology was also used to solve system expansion problems (revamp).

Sun et al. (2014) proposed the installation of auxiliary pumps in parallel branches of cooling water networks in order to achieve energy savings. In the usual configuration, the pumps are installed in the main cooling water delivery branch (just after the cooling tower) and must have full hydraulic head to meet the required pressure drop on all heat exchangers. Pressure drop in the branches is controlled by reducing valve openings, resulting in wasteful energy in the pumps. To circumvent this problem, a superstructure-based mathematical model was developed to minimize the total cost of the pump system containing a main pump network and an auxiliary pump network. The total cost consisted of the pump and motor investment costs plus the pump operating costs. The resulting problem is a MINLP and was solved using the annealing algorithm (a metaheuristic method). The model was tested in a case study based on a pumping network of a refinery cooling water system. The optimal number and location of the auxiliary pumps has been found to vary with the magnitude of the main pump head. The results also showed that when the main pump head equals the minimum hydraulic head of each heat exchanger, the total cost reaches the optimum location. For this case study, two auxiliary pumps had to be installed. Operating cost and total cost were reduced by 28% and 14.8%, respectively.

Souza et al. (2016) performed the optimization of a cooling water network considering the pressure drop in the pipes, the heat exchangers and the arrangement of the heat exchange equipment (parallel or series) simultaneously. All heat exchangers were specified as counterflow type. The objective function used was the total annualized cost, which included utility cost, heat exchanger investment, piping investment, and pumping cost due to pressure drop in heat exchangers and pipe sections. The problem was formulated as a MINLP. Initially, a network was studied without considering the pressure drop in the heat exchangers (base case) and then three configurations considering the pressure drop were studied. The results showed that the total annualized cost of the cooling water network was reduced by 27% compared to that obtained without considering the pressure drop in the heat exchangers.

Ma et al. (2017) performed the optimization of the pump network and the heat exchanger network of a cooling water system. For this purpose, a superstructure was set up which considered series and parallel arrangements of the heat exchangers and a network of auxiliary pumps in the branches of the heat exchangers. Flow rates, inlet and outlet temperatures and heat transfer coefficients of the hot streams were considered known. Variables included cooling water flow rates in the heat exchangers, heat exchanger area, pump hydraulic head, and pressure drop between the heat exchangers. The heat exchangers considered were of the shell-and-tube type, in counterflow. The objective function to be minimized was the total annualized cost formed by pump investment and operating costs, heat exchanger investment cost and cooling water cost. Simultaneous optimization was compared with two-step optimization, where the first step is to transform the parallel heat exchanger network into a parallel and series structure. Using the specified heat exchanger network, the flow and pumping network are optimized. The problem was formulated as a MINLP type and solved in the GAMS software using the DICOPT solver. Results showed that compared to the original network, the new configuration saved 13.4% of the total annualized cost. Simultaneous optimization reduced the total annualized cost by 6.4% compared to two-step optimization. The configuration obtained by simultaneous optimization resulted in an 18.9% reduction in the total annualized cost compared to the original parallel configuration.

Ma et al. (2018) proposed two new pumping system configurations, the so-called multi-loop system and the auxiliary pump system. In the multi-loop system, the heat exchangers were divided according to the required pressure ranges and then each pump provided head for one of the heat exchanger pressure ranges. Auxiliary pumps are installed in parallel branches of the heat exchangers to complete the hydraulic head provided by the main pump. Two case studies were performed in which the three configurations were tested (original configuration, only with main pumps, loop pumps and auxiliary pumps). The first example investigated represents a refinery, while the objective of the second case is to investigate the feasibility of pump systems with different distances to the exchangers. In the first case, the multi-loop pump network resulted in a 39.7% reduction in pumping energy and an 8.1% reduction in the total annualized cost compared to the optimal solution obtained through the conventional system. The auxiliary pump system led to a 37.8% reduction in pumping energy and 12% of the total annualized cost. The first case showed that when the system is relatively simple, the auxiliary pump system is more suitable. In the second case, the multi-loop network performed better, showing that for complex systems the multi-loop system is more suitable. The multi-loop system had a pumping cost 19.6% lower than the system pumping cost with the use of auxiliary pumps. The total annualized cost of the system with multi-loops was 6.8% lower than the system with auxiliary pumps.

1.3.2. Cooling tower

This item presents previous works that dealt with the optimization of the design of cooling towers as an isolated equipment.

Soylemezn (2001) used an iterative method to optimize a counterflow cooling tower with forced draft. The objective function was the net present value, including the operating cost associated with the tower investment cost. The tower operation cost depends on fan power. The optimal value of the heat transfer area of the tower was obtained by equating to zero the derivative of the cost equation. This value was a minimum point because the second derivative of the cost equation was always positive. The results obtained were compared with typical cooling tower designs. Depending on the tower, there is an excess of area in the projects, compared to the optimal value, of about 15% to 50%. It has been found that the cooling tower investment cost grows along with the growth of the transfer area, while the operating cost decreases with increasing the area.

Kloppers and Kröger (2004) performed the optimization of a counterflow cooling tower with natural draft, considering its geometric dimensions. The authors considered six geometric variables with significant influence on the life cycle cost of a cooling tower: cooling tower height (H6), air inlet height (H3), the shell bottom diameter (d3), the frontal area of the fill (Afr), the length or height of the fill (Lfi) and the diameter at the top of the shell (d6). Three geometric primary variables were used for optimization: H3, H6 and d3. The three other geometric variables, Afr, Lfi and d6, depend on the primary geometric variables. All other geometric variables of a cooling tower generally have a negligible influence on cooling tower performance. The objective function to be minimized was the sum of the operating and investment costs for the economic life of the cooling tower. The operation cost of the tower was defined as a function of H3, Lfi and height of the spray zone; and it is represented by the pump operation cost, which depends on the energy cost, pump power and number of hours of operation per year. The investment cost of the cooling tower was constituted by the sum of the cost of the tower concrete structure, as a function of the concrete volume used, and the cost of the filling, as a function of the filling volume used in the tower. The Wet-Cooling Tower Performance Evaluation (WCTPE) software, developed by the authors, was used in optimization in conjunction with the Leapfrog Optimization Program with Constraints (LFOPC) optimization algorithm. This algorithm is a gradient method that generates a dynamic path trajectory to any starting point toward the optimum. WCTPE software analyzes the performance of counterflow and crossflow cooling towers.

Williamson et al. (2008) carried out a study aiming to quantify the performance improvement of a natural draft cooling tower that can be obtained by optimizing the filling height and the water distribution along the tower. A two-dimensional (2D) model that allows rapid evaluation of tower performance has been combined with an evolutionary optimization algorithm to determine the optimum filling shape and water distribution profile to maximize the cooling range of a typical natural draft cooling tower. The results were compared with a symmetric numerical model with respect to the axes (i.e. a 1D model extended to two dimensions). The extended 1D model significantly reduced the computational time compared to the other numerical model, allowing a wide range of parameters to be tested with reasonable accuracy. Computational fluid dynamics (CFD) software was used to obtain the optimal curves of air enthalpy variation, water droplet size and water flow distribution along the filling height, as well as the optimal variation curves of the enthalpy, along the filling height for various tower inlet heights and the optimum curves of the difference between tower inlet and outlet water temperatures, with constant filling height and water distribution, and varying ambient conditions (temperature and relative humidity). The two-dimensional model used Poppe's method, which has the advantage that it can be applied to CFD equivalently. The extended 1D model used the Merkel method. Due to the stochastic nature of the evolutionary algorithm employed in the optimization, it had to be tested several times. The results showed that the optimal filling profile differs significantly from a uniform profile, with both water flow and filling thickness decreasing towards the center of the tower, where the air is warmer, thus with reduced cooling potential.

Serna-González et al. (2010) studied the optimization of a counterflow cooling tower with mechanical air draft. Merkel's model was used to specify the characteristic dimensions of the cooling tower, along with empirical correlations for mass transfer coefficients in the tower packing (filling) region. The water/air mass ratio, water mass flow rate, inlet and outlet temperatures, approximation of operating temperature, type of filling, type of draft, tower height and filling area, air flow pressure drop, fan energy consumption and water consumption were the optimization variables, as well as the choice of the type of filling (film, splash or tickle types) and draft (forced or induced). Disjunctive programming was used to formulate discrete choices of filling and draft types. The objective function used was the minimization of the total annualized cost, constituted by the sum of the annualized cooling tower investment cost and the annual operation costs. The annual operation cost was determined by the makeup water consumption and the energy cost by the fan operation. The resulting problem was a MINLP and was solved using the DICOPT solver in the GAMS software.

Rao and Patel (2011) explored the use of the artificial bee colony algorithm (ABC) to optimize the design of a counterflow cooling tower with mechanical air intake. The cooling tower geometry and performance were based on an adapted version of the Merkel method. The main objectives of this work were to optimize the parameters that influence the cooling tower economically and to demonstrate the efficiency of the ABC algorithm in the optimization of the cooling tower design. The optimization results using the ABC were validated by comparing them with the results obtained using the GAMS software. The objective function used was the total annualized cost plus penalties. The penalty exists when some constraint of the problem is violated. Total annualized cost was defined as the sum of annualized investment cost and annual operation cost of the cooling tower. Annualized investment cost included the fixed cost of the cooling tower, additional tower costs based on tower filling volume and air mass flowrate. The additional cost due to tower filling depended on the types of filling (different values for each type). The annual operation cost was the sum of the cost associated with water makeup and the amount spent on electricity. Results obtained from a series of examples showed that the algorithm worked properly to optimize the cooling tower design.

Rubio-Castro et al. (2011) used the Poppe method to optimize a counterflow cooling tower with mechanical draft. The differential equation model that describes the cooling process along the tower filling was reduced to algebraic equations using the fourth order Runge-Kutta algorithm and dividing the tower filling into 25 integration intervals. The objective function was defined as the minimization of the total annualized cost, which included the cost of water consumption, the cost of energy used by the fan and the cooling tower investment cost. The optimization problem was formulated as a MINLP and solved using the DICOPT solver in the GAMS software. The results obtained with the Poppe method were compared to the results obtained by the Merkel method. Poppe model consists of ordinary differential equations and algebraic equations that can be simultaneously solved to provide the values of air humidity, air enthalpy, water temperature, mass flow rate and Merkel number profiles in the cooling tower. This model can also determine air conditions at the cooling tower outlet. The Merkel model can be obtained from the Poppe model assuming that the Lewis factor is equal to one and negligible water evaporation. Both methods showed the same optimum values for the difference between the water temperature at the tower outlet and the wet bulb temperature, however significant differences were found for the other tower design and operation variables.

Rao and More (2017) explored the use of an improved Jaya algorithm (called Jaya auto-adaptive) for optimal design of forced draft cooling towers. The results obtained were compared with others in the literature and according to the authors; the algorithm used was better than the other algorithms in terms of optimal results, convergence and computational time.

Abed et al. (2018) carried out the optimization of a counter flow cooling tower with the objective of minimizing the operating cost. An ordinary differential equations (ODE) model solved using Matlab software was used. Optimization was also performed using the particle swarm optimization algorithm (PSO).

Zhang et al. (2019) performed the optimization of the counterflow cooling tower using the PSO algorithm. The cooling tower model was composed of energy and mass balances and a good fit to the experimental data was verified. Six cases were studied in order to determine the minimum total annualized cost, through single-objective particle swarm optimization (SOPSO) and one case through multiobjective particle swarm optimization (MOPSO), involving four objectives, the range, tower characteristic ratio, effectiveness and water evaporation rate, while flow rates of air and water. The results of all cases studied were compared with the literature and showed that the PSO algorithm presented satisfactory performance.

Patel et al. (2021) conducted a study of the quantitative and qualitative performance of several meta-heuristic optimization algorithms for the economic optimization of cooling tower. The studied algorithms were: Spotted Hyena Optimizer (SHO, inspired from the collaborative hunting behavior of spotted hyenas); Mouth Brooding Fish Algorithm (MBF, simulating the intelligent behavior of the mouth brooding fish to protect their offspring); Squirrel Search Algorithm (SSA, inspired from the intelligent foraging behavior of southern flying squirrels); Sailfish Optimizer (SFO, a population oriented metaheuristic algorithm inspired from the group hunting behavior of sailfish); Pathfinder Algorithm (PFA, which imitates the cooperative movement of an animal group); Harris Hawks Optimization (HHO, inspired from the supportive behavior and chasing style of Harris' hawks bird of prey); Henry Gas Solubility Optimization (HGSO, which simulates the same huddling behavior of gas mathematically to form the optimization algorithm); Marine Predator Algorithm (MPA, inspired from the foraging strategy of marine predators); Equilibrium Optimizer (EO, a metaheuristic inspired by the conservation of mass theory); Differential Evolution Algorithm (DE, which works with crossover, mutation, and selection operators); Self-Adaptive Differential Evolution algorithm (SaDE, the learning strategy and control parameters of DE algorithm are self-adapted in the SaDE variant); Linear Population Size Reduction differential Evolution (L-SHADE, a DE variant which uses a linear population size reduction for enhancing the algorithm performance). The behavior of all competitive algorithms is verified for sensitiveness of constraints. Quantitative results shows that DE, SaDE, and L-SHADE are better than recently developed algorithms considered in the present work followed by SSA and EO algorithms. The qualitative performance comparison of the obtained results also shows the dominance of DE and its variants. Convergence behavior of DE, SaDE and L-SHADE algorithms are better than the other competitive algorithms in obtaining the optimum solution. Finally, DE and its variants respond better than other competitive algorithms for sensitiveness of constraints and dominants the other approaches of this study.

1.3.3. Cooling tower and cooling water network

The works gathered in this item addresses the optimization of cooling water systems including the design of the cooling water tower and the cooling water network simultaneously.

Majozi and Moodley (2008) performed the optimization of multi-tower cooling water networks in order to minimize the total circulation water flow of the system. A superstructure was used that contemplated several cases. Case 1: No limit temperature was set for the cooling water stream returning to the tower and no dedicated tower was defined for any heat exchanger. Case 2: Temperature of the cooling water stream returning to the tower had no temperature limit and a limitation was imposed on each tower being dedicated to each cooling water consumer. Case 3: The maximum cooling water return temperature for the tower has been specified and no dedicated cooling water consumer towers have been defined. Case 4: The maximum cooling water stream temperature returning to the tower was specified and dedicated cooling towers were considered for cooling water consumers. The first case resulted in a LP problem, the second case led to a mixed integer linear programming (MILP) problem, while the other two cases resulted in MINLP problems. The problems have been resolved using the GAMS software. In all cases, there was a 40% reduction in circulating water.

Panjeshahi et al. (2009) expanded the KSD method, developed by Kim and Smith (2001), which aimed to achieve maximum water reuse (minimum water flow) for network configuration, including considerations regarding the interaction between the cooling water tower and the network of heat exchangers. The method used in the optimization was called Advanced Pinch Design (APD), based on pinch technology and the objective was to obtain minimum cost. This work also considered a water quality cycle, with the introduction of ozone treatment technology, this technique was called Enhanced Cooling Water System Design (ECWSD) and results in water and energy conservation, minimum cost and environmental impacts. The results obtained by APD and ECWSD methodologies were compared with those obtained by KSD. In this investigation, a counterflow cooling water tower with mechanical draft was considered. The model was used to predict the cooling water and air outlet conditions for a given project and operation conditions. The equations were written based on a control volume of the cooling water tower filling. The results of the cooling tower model showed that decreasing the tower flow rate has a more significant effect on effectiveness (heat removed divided by maximum heat removed) than decreasing the inlet temperature.

Ponce-Ortega et al. (2009) performed the optimization of cooling water systems including the design of the heat exchangers and cooling tower through disjunctive programming. The problem analyzed in this paper can be defined as: given a series of hot process streams with their inlet and outlet temperatures that must be attained, flow rates and physical properties, it is necessary to find the configuration of the heat exchanger network, operating conditions, design variables of each heat exchanger and cooling tower that achieves the required cooling service, coupled with a minimum of total annualized cost. The proposed superstructure for the cooling water system considers the interactions between the tower, heat exchanger network (series and parallel) and the pumping system. The cooling tower investigated was of the counterflow type. For its design, the Merkel model was used. Empirical correlations were considered for air pressure drop and total mass transfer coefficients relative to tower filling. Additional disjunctions were used to select the type of filling and the type of mechanical (induced or forced) air draft in the tower. The objective function was the total annualized cost for the cooling water system, which includes tower, heat exchanger and pump costs, as well as operating costs for the tower fan and make up water. The maximum allowed cooling water temperature has been reached to achieve good cooling tower performance. For comparison purposes, a sequential optimization was performed, resulting in a total annualized cost 7.13% higher than the optimal solution obtained with simultaneous optimization. The main reason for the higher annualized cost obtained in the sequential method is that the makeup water consumption was 118% higher. A typical parallel arrangement for heat exchangers was also considered, in which case the total annualized cost for the system was 2.22% higher than the optimal configuration.

Ponce-Ortega et al. (2010) studied the optimization of a cooling water system using the Kern model for the heat exchangers (KERN, 1950) and the Merkel model for the counterflow cooling tower, neglecting pressure losses among system components. The optimization variables of the problem were the cooling tower specifications (cooling water mass flow rate, water consumption, tower inlet and outlet temperatures, filling height and straight section area, filling type and fan energy consumption), the heat load of each heat exchanger and its design variables (heat transfer area, inner and outer tube diameters, tube arrangement, tube pitch, tube length, number of tubes and number of tube passes, baffle spacing, shell diameter and number of shells in series, tube-side and shell-side pressure drops and velocities, overall heat transfer coefficient and stream allocation) and design aspects of the pumping. A superstructure was built that contemplated all possible arrangements between the heat exchangers (series and parallel). The model was written through disjunctive programming and resulted in an MINLP problem. Three examples were employed to test the effectiveness of the optimization. In the examples considered in this study, the optimal grid structure found was relatively simple since there was cooling water bypass and one exchanger was used for each hot process stream.

Gololo and Majozi (2011) developed a technique for optimizing multi-tower cooling water networks to minimize total purge flow. A superstructure was developed in which all possibilities of cooling water recycling are explored (series and parallel heat exchangers). The cooling tower model used was the one developed by Kröger (2004). The formulation of the resulting optimization problem was a nonlinear programming (NLP). The solution procedure began by solving the relaxed model by minimizing the total cost of cooling water. The relaxed model solution was then used as a starting point to solve the exact model. The following cases were analyzed: Case 1: The maximum cooling water return temperature for the cooling tower was specified without a dedicated source or return for any cooling water consumer; Case 2: The maximum cooling water return temperature has been specified for the source and return cooling tower dedicated to cooling water consumers, a given cooling tower can only supply water for a given group of heat exchangers and this heat exchanger group can only return cooling water to this same tower. In Case 1, it was found that the total flow rate of circulating water was reduced by 22% and that the purge and makeup water flows were reduced by 7%. It was observed that in Case 2 the total circulation flow was reduced by 20%, the purge and replacement water flow rates were reduced by 4%.

Gololo and Majozi (2013) presented a mathematical technique for optimizing pressure drop in cooling water systems that feature multiple cooling towers. The problem studied can be defined as: given a group of cooling towers, with their defined characteristics (limit temperatures for each cooling water tower, heat capacities, limit temperature for each cooling tower filling, dimensions of each tower and the performance correlation coefficient for each tower), determine the minimum pressure drop for a network of a multiple cooling tower system while maintaining the minimum flow of circulating water. A superstructure with two cooling towers and two groups of heat exchangers was assembled. Heat exchangers can be operated in series or in parallel. A two-step optimization approach was used. The first stage consisted in the definition of the minimum circulation flowrate and, in the second stage, the Critical Path Algorithm (CPA) was used to synthesize the cooling water network by minimizing the pressure drop. Two cases were analyzed. The first case involved a cooling water system without dedicated water sources. As a result, a set of heat exchangers can be fed by any cooling tower and water returned to any cooling tower. The second case involved a cooling water system with dedicated cooling water towers (for water supply and return). In this case, a group of heat exchangers can only be supplied by one cooling tower. Case studies have shown that the flow of circulating cooling water can be reduced by more than 26%, with minimal pressure drop from the cooling water network.

Rubio-Castro et al. (2013) performed the optimization of a cooling water system with multiple cooling towers. A superstructure was developed that presented all the different alternatives for the networks of heat exchanger and cooling towers. Serial, parallel, series and parallel, parallel and series configurations for cooling towers were considered. The model of the cooling towers of the countercurrent type with mechanical air draft was based on the Merkel equations. The mass transfer and pressure drop were modeled through the empirical correlations of the literature. The objective function to be minimized was the total annualized cost, which included investment costs for heat exchangers and cooling towers, as well as operating costs due to the consumption of replacement water and energy consumption by water pumps, and cooling tower fans. The problem was formulated as an MINLP, in which flow rates and temperatures of the cooling water streams were the variables to be optimized. The binary variables were related to the possibility of the existence or not of heat exchangers and cooling towers in the system. The proposed model was implemented in the GAMS software. The investigated examples showed that the optimum air flow rate for the cooling tower in the traditional system (heat exchangers in parallel with a single cooling tower) is larger than for the system with multiple cooling towers. Removing the same amount of heat from the process, the cooling water systems with multiple cooling towers, with different supply temperatures, showed lower energy consumption of the tower fans and the area of the fillings than the traditional systems, which resulted in lower total costs.

Zheng et al. (2018) developed a study with the objective of simultaneously determining the optimal location of the cooling towers and the optimal configuration of the pump network in the cooling water system. According to the superstructure of the cooling water system presented, the main pumps were installed in the pipes immediately after the cooling towers and the auxiliary pumps (if necessary) would be installed later on, in the parallel branches where the operational units are located. The main and auxiliary pumps were considered to be of the centrifugal type. A cooling tower could supply cooling water to different operating units and the cooling water from the output of an operating unit can be returned to any cooling tower. The location of the cooling towers was presented as a variable defined by Cartesian coordinates, but the heat exchangers had fixed Cartesian coordinates. The objective function was the total annualized cost of the system, which consisted in the investment costs of the cooling towers, pumps and pipes and the operating costs of the cooling towers and pumps. The cost of the cooling tower was defined as a function of the filling area and height, and to simplify the computational solution for a preliminary design stage, all cooling towers were considered to have the same filling area and the same height, with the same type of filling, which is actually very common in industries. Then a fixed amount was used to express the investment cost of each cooling tower. The operating cost of a cooling tower consisted in two parts related to the air flow and the pumping of replacement water. The cost of operating the pumps considered the pumping of the recirculating water through the use of main and auxiliary pumps. Examples have been made with one and several cooling towers. The solutions showed that the cooling tower must be located close to the location with the highest concentration of demand and that the auxiliary pumps must be installed in the branches with the greatest demand, which is in line with engineering experience. In all cases, the operating cost of the main pumps, which has the largest contribution to the total cost, is significantly reduced by the optimal design of the pumping network.

Qi et al. (2019) carried out the optimization of a cooling water system comprising a cooler network, pump network, cooling tower and air cooler, with the objective of minimizing the total annualized cost. This work also focused on the environmental impact and, for its quantification, carbon footprint and water footprint are introduced as environmental indicators. The problem obtained was a MINLP type, solved using GAMS. A case study was carried out, obtaining an improved cooling water system structure chart and the relationship between carbon footprint, water footprint and economic performance. The existence of a trade-off among the three was verified. The result shows that total annualized cost can be reduced by 9.84%.

Zhu et al. (2020) developed a framework based on model reductions for multiscale optimization of closed wet cooling tower (CWCT) considering environmental variations. It was performed an optimal design of experiment for accurate approximation of the multivariate probability distributions by generating a finit set of samples over the intire input space. The probability distributions were propagated via multi-sample CFD simulations for constructing the physics-based and data-driven reduced models of CWCTs. Multiscale optimization model is proposed for performing integrated design and management of CWCTs and cooling water system, through the use of sampling-based stochastic programming and the heterogeneous integration of reduced models of CWCTs and other shortcut models. A comparison between the performance of the proposed approach and the deterministic approach was then performed.

1.4. Overview of literature

The analysis of the literature is organized according to the three types of problems discussed above.

1.4.1. Cooling water networks

The previous attempts to solve the design problem of cooling water networks are dominated by nonlinear mathematical programming solutions, typically MINLP problems. However, the convergence of MINLP algorithms may be difficult. Good initial estimates are not always easy to find. Additionally, if a local optimizer is employed, the solution may be trapped in a poor local optimum. As mentioned above, there is a previous work that solved the design problem using a linear formulation (Souza et al., 2014), but it is limited to the pipe network, without including the heat exchangers.

Aiming at avoiding the limitations of nonlinear formulations, but still addressing the complete cooling water network, including pump, pipe sections and heat exchangers, this thesis presents a new formulation for the design problem based on a mixed-integer linear programming (MILP). The linearity of the formulation allows attaining the global optimum, without the need of a good initial estimate or a specialized global solver.

1.4.2. Cooling tower

The literature review of the design optimization of cooling towers showed the dominance of two types of methods: metaheuristic methods and mathematical programming. However, there are important limitations related to these approaches.

As discussed above, Mathematical Programming presents nonconvergence problems for nonlinear models, such as those models employed for the optimization of cooling towers. Additionally, if local solvers are used, the solution may be trapped in a local optimum. Metaheuristic methods are more robust and can migrate among different local optima towards the global optimum, but global optimality cannot be guaranteed. Other problem associated with metaheuristic methods is the dependency of their computational performance with a proper tuning of the algorithm control parameters.

Based on these issues that hinders the design optimization of cooling towers using the approaches available in the literature, this thesis presents a solution procedure for the design problem that avoids the cited drawbacks: Set Trimming + Smart Enumeration (Costa; Bagajewicz, 2019). This approach always attains the global optimum, without the need of initial estimates.

1.4.3. Cooling tower and cooling water network

The engineering design practice for cooling water systems is to solve the design problems of each element that make up the system separately. Each heat exchanger is designed considering a maximum pressure drop associated with the process streams, the pipe network design for the selection of pumps and diameters is performed using a fixed value for pressure drop of the heat exchangers, and the cooling tower is designed considering fixed values of the inlet and outlet temperatures of the cooling water.

However, this approach results in a non-optimal solution, as it does not explore the relations between the investment costs of heat exchangers, piping and the operating costs of the pump. Additionally, the network is composed of a group of interconnected pipes and the total pressure drop associated with the head of the pump encompasses the hydraulic interactions of all elements of the network simultaneously. The tradeoffs related to the cooling water temperatures are also not present in the traditional analysis.

Because of this set of limitations, several papers in the literature have addressed the optimal design problem of the entire cooling system. However, with the exception of Ponce-Ortega et al. (2010), the previous papers in the literature employed simplified models for the heat exchangers. Ponce-Ortega et al. (2010) included a heat exchanger model based on the Kern correlations, but it ignores the pressure drop in the pipe sections. Avoiding the limitations presented in the literature, this thesis proposes a general formulation, including all the elements: cooling tower, pump, pipe sections and heat exchangers, represented by the Kern model.

The numerical solution of the problem is another issue observed in the literature. The most general formulations are represented by MINLP problems that are solved using mathematical programming algorithms, which are affected by the limitations already discussed.

Therefore, this thesis presents a robust algorithimic procedure that can solve the design problem without concergence drawbacks. The proposed procedure is based on the split of the search space, followed by a recursive convergence of the design of the cooler water network, using a conventional MILP algorithm, and the design of the cooling tower, using Set Trimming + Smart Enumeration. Despite global optimality cannot be guaranteed, the robustness of the proposed approach allows an easier utilization, even for users without specific knowledge about mathematical optimization tools and how to need good initial estimates.

2. OPTIMIZATION OF COOLING WATER SYSTEMS INCLUDING THE DESIGN OF HEAT EXCHANGERS

This chapter presents the optimization of a cooling water network, contemplating the design of the heat exchangers and the pipe network simultaneously. The objective function is to minimize the total annualized cost and the constraints, including the modeling of the pipe network, the heat exchangers and cost-related equations. The solution of the problem presents optimal values for the variables related to each heat exchanger, pipe section and pump.

The problem corresponds to an integer linear programming (ILP), so its solution is the global optimum, not requiring good initial estimates for convergence, as in nonlinear problems.

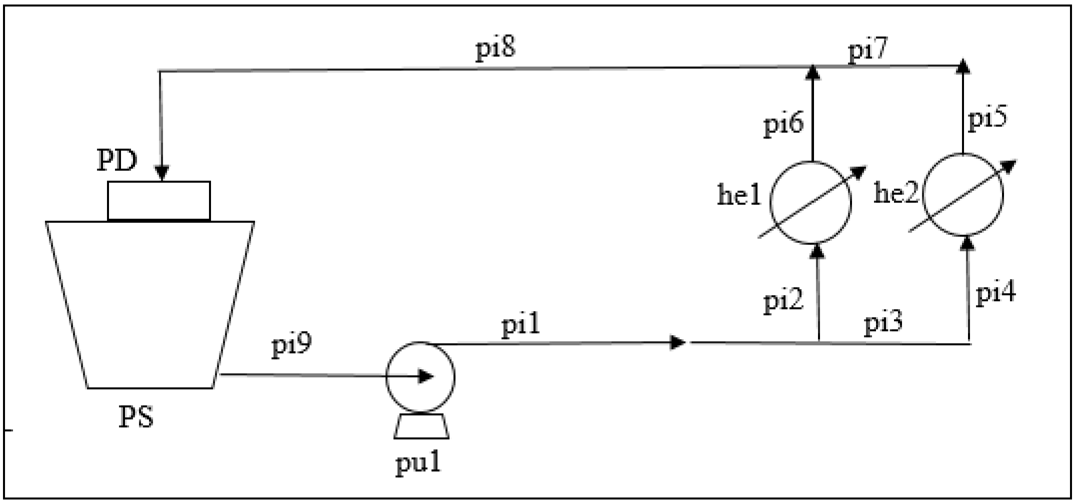
2.1. Problem description

The proposed analysis of the cooling water system assumes that the flow rates of the cooling water and process streams in the coolers are previously known, i.e. the mass and energy balances of the cooling system are the starting point of the optimization problem. Additionally, the layout of the system, represented by the set of lengths of the different pipe sections, are also considered known.

The design variables of the cooling water system addressed in the current problem are: the diameters of the pipe sections, the pump selection, the head losses of the valves associated with each heat exchanger to guarantee the hydraulic balance according to the design flow rate, and the heat exchangers geometry (tube length, tube diameter, tube layout, tube pitch ratio, number of baffles, shell diameter, and number of tube passes).

The elements that compose the system are represented using graph theory (MAH, 1990). The edges ($k \in STR$) represent the heat exchangers ($HE \subset STR$), pipe sections ($PI \subset STR$), and pumps ($PU \subset STR$). The nodes ($t \in VET$) represent the cooling tower basin ($PS \subset VET$), the cooling tower top ($PD \subset VET$), and the interconnections among the edges ($INT \subset VET$). The cooling tower basin node represents the cooling water supply to the network, and the cooling tower top represents the cooling water return to the tower. Figure 1 contains an illustration of a typical cooling water system with its interconnected elements. The network edges are organized according to a set of independent hydraulic circuits ($l \in HY$). A hydraulic circuit is a path along the digraph starting at the cooling water supply node and ending at the cooling water return. All edges belong to at least one hydraulic circuit. Each heat exchanger is associated with a unique circuit. It is assumed that each hydraulic circuit can be associated with a valve/restriction plate. The network circuits are described by the matrix $\hat{\Lambda}_{l,k}$, such that, if $\hat{\Lambda}_{l,k} = 1$, then the edge k belongs to the circuit l, otherwise, $\hat{\Lambda}_{l,k} = 0$. For example, Table 1 contains the representation of the hydraulic circuits of the cooling water system depicted in Figure 2.

Figure 2- Cooling water system representation.



Fonte: A autora, 2022

Table 1 - Hydraulic circuits of the cooling water system present in Figure 1

Circuit	**Edges**
1	pi9, pi1, pi2, he1, pi6, pi8
2	pi9,pi1, pi3 , pi4, he2, pi5, pi7, pi8

Fonte: A autora, 2022

2.2. Representation of the cooling water system

Because the layout of the system and the necessary volumetric flow rate in each cooler are already established ($\widehat{qhe}_l$), the flow rates along each network element ($\hat{q}_k$) can be calculated, prior to the optimization:

$$\hat{q}_k = \sum_{l \in HY} \widehat{qhe}_l \hat{\Lambda}_{l,k} \qquad k \in STR \tag{4}$$

The diameters of the pipe sections must be chosen according to a set of available commercial options ($n \in SD$). The values of the standard inner and nominal diameters are represented by $\widehat{D}_n^{int}$ and $\widehat{D}_n^{nom}$ (the nominal diameter is identified in inches, according to the industrial practice), respectively. In turn, pump selection is based on a set of available commercial options ($s \in SPU$).

Each available pump option s is represented by the corresponding head at the design flow rate. In the presentation of the model, the problem parameters, which are fixed prior to the optimization, are represented with the symbol "^" on top.

The cooling water system model is composed of the pipe network model, which describes the hydraulic behavior of the system; the heat exchanger model, which represents its thermofluidodynamic behavior; and economic equations, which characterize the performance of each alternative solution. The following subsection presents the formulation of the optimization problem, describing the constraints related to each component of the problem and the objective function.

2.3. Optimization problem formulation

2.3.1. Pipe network model constraints

The hydraulic behavior of the pipe network is represented by a mechanical energy balance along each hydraulic circuit (thepressure drop and kinetic energy in the spray were dismissed here because of its relatively small value):

$$\sum_{k \in PU} \hat{\Lambda}_{l,k} fpu_k - \widehat{\Delta z} - \sum_{k \in HE} \hat{\Lambda}_{l,k}\, fhe_k - \sum_{k \in PI} \hat{\Lambda}_{l,k} fpi_k \hat{L}_k - fv_l = 0 \qquad l \in HY \tag{5}$$

where fpu_k is the head of the pump k, $\widehat{\Delta z}$ is the elevation difference between the top and the bottom of the cooling tower, fhe_k is the head loss along the heat exchanger k, $\hat{L}_k$ is the length of the pipe section k, fpi_k the unitary head loss along the pipe section k, and fv_l is the head loss in the valve/restriction plate associated with the circuit l.

The head loss in the heat exchangers is calculated as follows:

$$fhe_l = \frac{\Delta Pt_l\, \widehat{yT}_{c,l} + \Delta Ps_l\, \widehat{yT}_{h,l}}{\hat{\rho}_w \hat{g}} \qquad l \in HY \tag{6}$$

where $\hat{\rho}_w$ is the cooling water density, $\hat{g}$ is the gravity acceleration, ΔPt_l and ΔPs_l are the pressure drops in the heat exchanger tube-side and shell-side, respectively, and $\widehat{yT}_{c,l}$ and $\widehat{yT}_{h,l}$ are parameters that indicate if the cooling water is in the tube-side ($\widehat{yT}_{c,l} = 1$) or if the hot process stream is in the tube-side ($\widehat{yT}_{h,l} = 1$).

Next, the head loss of pipe section k is calculated using the Hazen−Williams equation (SAVIC; WALTERS, 1967):

$$fpi_k = \frac{10.67}{\widehat{ch}^{1.85}} \frac{\hat{q}_k^{1.852}}{D_k^{4.8704}} \qquad k \in PI \qquad 7)$$

where D_k is the inner diameter and $\widehat{ch}$ is the Hazen−Williams constant.

In turn, the pipe inner diameter (D_k) is a discrete variable, according to the commercially available values. Therefore, it can be represented by binary variables:

$$D_k = \sum_{n \in SD} \widehat{D}_n^{int} y_{k,n}^{pi} \qquad k \in PI \qquad (8)$$

where $y_{k,n}^{pi}$, is a binary variable that it is equal to 1 if the pipe section k is composed of a commercial diameter n; otherwise, it is equal to 0. Because the problem solution must present a unique diameter for each pipe section:

$$\sum_{n \in SD} y_{k,n}^{pi} = 1 \qquad k \in PI \qquad (9)$$

The substitution of Eq. (8) into Eq. (7) of unitary head loss yields a linear constraint:

$$fpi_k = \sum_{n \in SD} \widehat{pf}_{k,n} y_{k,n}^{pi} \qquad k \in PI \qquad 10)$$

where:

$$\widehat{pf}_{k,n} = 10.67 \frac{\hat{q}_k^{1.852}}{c^{1.85}\ (\widehat{D}_n^{int})^{4.8704}} \qquad k \in PI \tag{11}$$

The head of a pump k is also represented by a linear relation:

$$fpu_k = \sum_{s \in SPU} \widehat{fpu}_{k,s} y_{k,s}^{pu} \qquad k \in PU \tag{12}$$

where $\widehat{fpu}_{k,s}$ is the head of the pump model *s* at the design flow rate of the edge *k*, and $y_{k,s}^{pu}$ is a binary variable that it is equal to 1, if the pump k corresponds to the available option *s*; otherwise, it is equal to 0.

Because only one pump option must be chosen:

$$\sum_{s \in SPU} y_{k,s}^{pu} = 1 \qquad k \in PU \tag{13}$$

In order to avoid erosion and fouling, maximum and minimum flow velocity bounds are imposed:

$$\widehat{vmax} - v_k \geq 0 \qquad k \in PI \tag{14}$$

$$\widehat{vmin} - v_k \leq 0 \qquad k \in PI \tag{15}$$

where the fluid velocity can be expressed by the following equation:

$$v_k = \frac{4\hat{q}_k}{\pi D_k^2} \qquad k \in PI \tag{16}$$

Substitution of Eq. (8) and (16) into Eqs. (14) and (15) yields the following linear relations:

$$\widehat{vmax} - \frac{4}{\pi}\sum_{n \in SD} \frac{\hat{q}_k}{(\widehat{D}_n^{int})^2} y_{k,n}^{pi} \geq 0 \qquad k \in PI \tag{17}$$

$$\widehat{vmax} - \frac{4}{\pi}\sum_{n \in SD} \frac{\hat{q}_k}{(\widehat{D}_n^{int})^2} y_{k,n}^{pi} \geq 0 \qquad k \in PI \tag{18}$$

According to engineering practice, the pipe diameter at the pump suction must be equal or higher than the diameter at the pump discharge:

$$y_{kps,n}^{pi}\widehat{D}_n^{int} \geq y_{kpd,n}^{pi}\widehat{D}_n^{int} \qquad n \in SD \tag{19}$$

where *kps* and *kpd* are indices associated with the pump suction and discharge, respectively.

2.3.2. Heat exchanger model

The heat exchanger model is based on Gonçalves et al. (2017) where the design problem of E-type shell-and-tube heat exchangers without phase change is formulated as an integer linear programming (ILP). The allocation of the process and cooling water streams in the tube-side or in the shell-side is considered a designer decision established prior to the optimization.

The design variables are the tube inner and outer diameters (*dti* and *dte*), tube length (*L*), tube layout (*lay*), tube pitch ratio (*rp*), number of passes in the tube-side (*Ntp*), shell diameter (*Ds*), and number of baffles (*Nb*).

The presentation of the constraints related to the heat exchanger model is organized here in two parts. First, the original nonlinear thermofluid dynamic model and design equations are presented. For the sake of simplicity, only the main equations of the original nonlinear model are presented here; further details can be found in Gonçalves et al. (2017). After this, a reformulation is implemented to obtain a linear model.

The tube-side and shell-side heat-transfer coefficients are calculated using the Dittus−Boelter correlation (INCROPERA et al., 2002) for the tube-side and the Kern model (KERN, 1950) for the shell-side. The pressure drop in the tube-side is calculated using the

Darcy–Weisbach equation (SAUNDERS, 1988) and the Kern model for the shell-side (KERN, 1950).

The allocation of cooling and process water streams to the tube-side or shell-side is considered a designer decision prior to the optimization. This information is represented by binary parameters $\widehat{yT_c}$ e $\widehat{yT_h}$ (if water is on the tube-side then $\widehat{yT_c}$ =1, if the hot stream is in the tube-side so $\widehat{yT_h}\ = 1$).

The relations between physical properties, fouling factor and mass flow rate of cooling water and hot process streams with the corresponding values in the tube-side and the shell side are given by:

$$\widehat{\rho t} = \hat{\rho}_w\, \widehat{yT_c} + \hat{\rho}_h\, \widehat{yT_h} \tag{20}$$

$$\widehat{\rho s} = \hat{\rho}_w\, \widehat{yT_h} + \hat{\rho}_h\, \widehat{yT_c} \tag{21}$$

$$\widehat{Cpt} = \widehat{Cp}_w\, \widehat{yT_c} + \widehat{Cp}_h\, \widehat{yT_h} \tag{22}$$

$$\widehat{Cps} = \widehat{Cp}_w\, \widehat{yT_h} + \widehat{Cp}_h\, \widehat{yT_c} \tag{23}$$

$$\widehat{\mu t} = \hat{\mu}_w\, \widehat{yT_c} + \hat{\mu}_h\, \widehat{yT_h} \tag{24}$$

$$\widehat{\mu s} = \hat{\mu}_w\, \widehat{yT_h} + \hat{\mu}_h\, \widehat{yT_c} \tag{25}$$

$$\widehat{kt} = \hat{k}_w\, \widehat{yT_c} + \hat{k}_h\, \widehat{yT_h} \tag{26}$$

$$\widehat{ks} = \hat{k}_w\, \widehat{yT_h} + \hat{k}_h\, \widehat{yT_c} \tag{27}$$

$$\widehat{Rft} = \widehat{Rf}_w\, \widehat{yT_c} + \widehat{Rf}_h\, \widehat{yT_h} \tag{28}$$

$$\widehat{Rfs} = \widehat{Rf}_w\, \widehat{yT_h} + \widehat{Rf}_h\, \widehat{yT_c} \tag{29}$$

$$\widehat{mt} = \widehat{m}_w\, \widehat{yT_c} + \widehat{m}_h\, \widehat{yT_h} \tag{30}$$

$$\widehat{ms} = \widehat{m}_w\, \widehat{yT_h} + \widehat{m}_h\, \widehat{yT_c} \tag{31}$$

The design variables are discrete according to their physical nature and/or available commercial options, so they are represented in the optimization problem by using binary variables.

However, instead of adopting a group of different binary variables to describe each original discrete variable, the approach employed here involves a single binary group. Each individual binary variable (y_{srow}) identifies a candidate heat exchanger, which represents the combination of design variable values. Previous results have indicated that this approach can provide a considerable reduction in computational effort (GONÇALVES et al., 2017; SOUZA et al., 2018).

Starting from the original nonlinear thermofluidodynamic equations that represent the behavior of heat exchangers, the nature of the binary variables and the use of proper mathematical manipulation allow the representation of the problem in a linear form. Details of this transformation using the techniques presented by Gonçalves et al. (2017) can be found in the Annex 1. It is important to mention that this procedure does not involve any mathematical approximation, that is, the solution of the linear problem is identical to the solution of the original nonlinear problem.

The representation of model-related constraints is organized here into two parts. First, the original nonlinear equations of the thermofluidodynamic model and design equations are presented (for simplicity, the representation of nonlinear equations does not include the indices that identify each heat exchanger individually). Then the complete final structure of the constraints is shown in its linear form.

2.3.2.1. Original nonlinear equations

The shell-side convective heat transfer coefficient is evaluated using the Kern model (KERN, 1950), relating Nusselt (*Nus*), Reynolds (*Res*), and Prandtl ($\widehat{Prs}$) numbers:

$$Nus = 0.36\, Res^{0.55} \widehat{Prs}^{1/3} \tag{32}$$

$$Nus = \frac{hs\ Deq}{\widehat{ks}} \tag{33}$$

$$Res = \frac{Deq\ vs\ \widehat{\rho s}}{\widehat{\mu s}} \tag{34}$$

$$\widehat{Prs} = \frac{\widehat{Cps}\ \widehat{\mu s}}{\widehat{ks}} \tag{35}$$

where *hs* is the shell-side convective heat transfer coefficient, *vs* is the flow velocity, and *Deq* is the equivalent diameter. The thermophysical properties are density, $\widehat{\rho s}$, heat capacity, $\widehat{Cps}$, dynamic viscosity, $\widehat{\mu s}$, and thermal conductivity, $\widehat{ks}$.

The evaluation of the equivalent diameter depends on the tube layout:

$$Deq = \frac{4\ ltp^2}{\pi\ dte} - dte \qquad \text{(square pattern)} \tag{36}$$

$$Deq = \frac{3.46\ ltp^2}{\pi\ dte} - dte \qquad \text{(triangular pattern)} \tag{37}$$

where *ltp* is the tube pitch.

The expression of the shell-side flow velocity is:

$$vs = \frac{\widehat{ms}}{\widehat{\rho s}\ Ar} \tag{38}$$

where $\widehat{ms}$ is the mass flow rate. The flow area in the shell-side flow is given by (KERN, 1950):

$$Ar = Ds\ FAR\ lbc \tag{39}$$

where *lbc* is the baffle spacing. The expression of the free-area ratio, *FAR*, is:

$$FAR = \frac{ltp - dte}{ltp} = 1 - \frac{1}{rp} \tag{40}$$

The head loss in the shell-side flow is also based on the Kern model (KERN, 1950):

$$\frac{\Delta Ps}{\widehat{\rho s}\ \hat{g}} = fs \frac{Ds(Nb + 1)}{Deq} \left(\frac{vs^2}{2\ \hat{g}} \right) \tag{41}$$

where ΔPs is the shell-side pressure drop, *fs* is the shell-side friction factor and *Nb* is the number of baffles.

The shell-side friction factor is given by:

$$fs = 1.728\, Res^{-0.188} \tag{42}$$

The relation between the number of baffles and the baffle spacing is:

$$Nb = \frac{L}{lbc} - 1 \tag{43}$$

The tube-side convective heat transfer coefficient is evaluated relating Nusselt (*Nut*), Reynolds (*Ret*), and Prandtl ($\widehat{Prt}$) numbers:

$$Nut = 0.023\, Ret^{0.8} \widehat{Prt}^{n} \tag{44}$$

$$Nut = \frac{ht\, dti}{\widehat{kt}} \tag{45}$$

$$Ret = \frac{dti\, vt\, \widehat{\rho t}}{\widehat{\mu t}} \tag{46}$$

$$\widehat{Prt} = \frac{\widehat{Cpt}\, \widehat{\mu t}}{\widehat{kt}} \tag{47}$$

where *ht* is the tube-side convective heat transfer coefficient, *vt* is the flow velocity, $\widehat{\rho t}$ is the density, $\widehat{Cpt}$ is the heat capacity, $\widehat{\mu t}$ is the dynamic viscosity, $\widehat{kt}$ is the thermal conductivity, and the parameter *n* is equal to 0.4 for heating and 0.3 for cooling.

The expression of the flow velocity in the tube-side is:

$$vt = \frac{4\, \widehat{mt}}{Ntp\, \pi\, \widehat{\rho t}\, dti^2} \tag{48}$$

where $\widehat{mt}$ is the mass flow rate and *Ntp* is the number of tubes per pass.

The pressure drop in the tube-side flow is given by (SAUNDERS, 1988):

$$\frac{\Delta Pt}{\widehat{\rho t}\ \hat{g}} = \frac{ft\ \ Npt\ \ L\ \ vt^2}{2\ \hat{g}\ dti} + \frac{K\ \ Npt\ \ vt^2}{2\ \hat{g}} \qquad 49)$$

where *ft* is the tube-side friction factor. The parameter *K*, associated with the pressure drop in the heads, is equal to 0.9 for one tube pass and 1.6 for two or more tube passes.

The Darcy friction factor for turbulent flow is given by (SAUNDERS, 1988):

$$ft = 0.014 + \frac{1.056}{Ret^{0.42}} \qquad 50)$$

The LMTD method is used and, considering a design margin, "excess area" ($\widehat{Aexc}$), the heat-transfer rate equation is represented by the following relation:

$$UA \geq \left(1 + \frac{\widehat{Aexc}}{100}\right) \frac{\hat{Q}}{\widehat{\Delta Tlm}\ F} \qquad 51)$$

where *U* is the overall heat-transfer coefficient, *A* is the heat transfer area, $\hat{Q}$ is the heat load, $\widehat{\Delta Tlm}$ is the logarithmic mean temperature difference, and *F* is the LMTD correction factor. In

turn, the heat transfer area is given by:

$$A = Ntt\ \ \pi\ dte\ L \qquad 52)$$

where *Ntt* is the total number of tubes. Simplified expressions to represent the relation between the total number of tubes and the design variables can be found in Kakaç and Liu (2012):

$$Ntt = \frac{\pi\ D_s^2}{4}\ \frac{CTP}{CL\ ltp^2} \qquad 53)$$

where *CL* depends on the tube layout, *CL* = 1 for the square arrangement and *CL* = 0.866 for the triangular arrangement; *CTP* depends on the number of passes in the tubes, *CTP* = 0.93 for a single pass and *CTP* = 0.90 for multiple passes.

The overall heat-transfer coefficient is given by:

$$U = \frac{1}{\dfrac{dte}{dti\ ht} + \dfrac{\widehat{Rft}\ dte}{dti} + \dfrac{dte \ln(\frac{dte}{dti})}{2\ \widehat{ktube}} + \widehat{Rfs} + \dfrac{1}{hs}} \qquad 54)$$

where $\widehat{ktube}$ is the thermal conductivity of the tube wall, and $\widehat{Rft}$ and $\widehat{Rfs}$ are the tube-side and shell-side fouling factors, respectively.

The correction factor of the LMTD is equal to 1, for a single pass in the tubes, and the following expression is employed for an even number of tube-side passes:

$$F = \frac{(\hat{R}^2 + 1)^{0.5} \ln\left(\dfrac{(1-\hat{P})}{(1-\hat{R}\,\hat{P})}\right)}{(\hat{R}-1)\ln\left(\dfrac{2-\hat{P}\left(\hat{R}+1-(\hat{R}^2+1)^{0.5}\right)}{2-\hat{P}\left(\hat{R}+1+(\hat{R}^2+1)^{0.5}\right)}\right)} \qquad 55)$$

where:

$$\hat{R} = \frac{\widehat{Thi} - \widehat{Tho}}{\widehat{Tco} - \widehat{Tci}} \qquad 56)$$

$$\hat{P} = \frac{\widehat{Tco} - \widehat{Tci}}{\widehat{Thi} - \widehat{Tci}} \qquad 57)$$

Bounds on pressure drops are represented by:

$$\Delta Ps \leq \widehat{\Delta Psdisp} \qquad 58)$$

$$\Delta Pt \leq \widehat{\Delta Ptdisp} \qquad 59)$$

where ΔPs and ΔPt are the pressure drops on the shell-side and tube-side, and $\Delta \widehat{Psdisp}$ and $\Delta \widehat{Ptdisp}$ are the corresponding admissible values, respectively.

Additional bounds are applied to velocities and Reynolds numbers:

$$vs \geq \widehat{vsmin} \quad 60)$$

$$vs \leq \widehat{vsmax} \quad 61)$$

$$vt \geq \widehat{vtmin} \quad 62)$$

$$vt \leq \widehat{vtmax} \quad 63)$$

$$Res \geq 2 \cdot 10^3 \quad 64)$$

$$Ret \geq 10^4 \quad 65)$$

Design standards impose the following geometric bounds (TABOREK, 2008 a, b):

$$lbc \geq 0.2\,Ds \quad 66)$$

$$lbc \leq 1.0\,Ds \quad 67)$$

$$L \geq 3\,Ds \quad 68)$$

$$L \leq 15\,Ds \quad 69)$$

2.3.2.2. Optimization constraints after reorganization in linear form

The final structure of the set of linear constraints is represented below, adding an extra index for each individual heat exchanger ($l \in HY$).

Because only one option of heat exchanger must be selected:

$$\sum_{srow} yrow_{srow,l} = 1 \qquad l \in HY \tag{70}$$

After substitution of discrete representation and reformulation, the heat transfer rate equation (Eq. (51)), becomes:

$$\hat{Q}_l \left(\sum_{srow} \frac{\widehat{Pdte}_{srow}}{\widehat{Pht}_{srow,l}\, \widehat{Pdti}_{srow}} yrow_{srow,l} + \widehat{Rft} \sum_{srow} \frac{\widehat{Pdte}_{srow}}{\widehat{Pdti}_{srow}} yrow_{srow,l} + \frac{\sum_{srow} \widehat{Pdte}_{srow}\, yrow_{srow} \ln\left(\frac{\widehat{Pdte}_{srow}}{\widehat{Pdti}_{srow}}\right)}{2\, \widehat{ktube}} + \widehat{Rfs} + \sum_{srow} \frac{1}{\widehat{Phs}_{srow,l}} yrow_{srow,l} \right) \leq \left(\frac{100}{100+\widehat{Aexc}}\right) \left(\pi \sum_{srow} \widehat{PNtt}_{srow}\, \widehat{Pdte}_{srow}\, \widehat{PL}_{srow,l} yrow_{srow,l} \right) \widehat{\Delta Tlm}_l\, \hat{F}_{srow,l} \quad l \in HY \tag{71}$$

where $\widehat{Pdte}_{srow}$, $\widehat{Pdti}_{srow}$, and $\widehat{PL}_{srow,l}$ are values of the outer and inner tube diameters and tube length and $\widehat{PNtt}_{srow}$ is the total number of tubes calculated prior to the optimization for each solution alternative. The other parameters in Eq. (71) are expressed by:

$$\widehat{Pht}_{srow,l} = \frac{\widehat{kt}_l\, 0.023 \left(\frac{4\widehat{mt}_l}{\pi\, \widehat{\mu t}_l}\right)^{0.8} \widehat{Prt}_l^{\,n}}{\widehat{Pdti}_{srow}^{1.8}} \left(\frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}}\right)^{0.8} \tag{72}$$

$$\widehat{Phs}_{srow,l} = \frac{\widehat{ks}_l\, 0.36 \left(\frac{\widehat{ms}_l}{\widehat{\mu s}_l}\right)^{0.55} \widehat{Prs}_l^{\,1/3}}{\widehat{PDeq}_{srow}^{0.45}} \left(\frac{(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\, \widehat{PFAR}_{srow}\, \widehat{PL}_{srow}}\right)^{0.55} \tag{73}$$

$$\widehat{PFAR}_{srow} = 1 - \frac{1}{\widehat{Prp}_{srow}} \tag{74}$$

$$\widehat{pDeq}_{srow} = \frac{\widehat{aDeq}_{srow} \widehat{Prp}_{srow}^2 \widehat{Pdte}_{srow}^2}{\pi \widehat{Pdte}_{srow}} - \widehat{Pdte}_{srow} \tag{75}$$

$$\widehat{aDeq}_{srow} = \begin{cases} 4 & \text{if } \widehat{Play}_{srow} = 1 \\ 3.46 & \text{if } \widehat{Play}_{srow} = 2 \end{cases} \quad (76)$$

$$\hat{F}_{srow,l} = \begin{cases} \dfrac{\left(\hat{R}_l^{\,2}+1\right)^{0.5} \ln\left(\dfrac{(1-\hat{P}_l)}{(1-\hat{R}_l\,\hat{P}_l)}\right)}{(\hat{R}_l-1)\ln\left(\dfrac{2-\hat{P}_l\left(\hat{R}_l+1-\left(\hat{R}_l^{\,2}+1\right)^{0.5}\right)}{2-\hat{P}_l\left(\hat{R}_l+1+\left(\hat{R}_l^{\,2}+1\right)^{0.5}\right)}\right)} & \text{if } \widehat{PNpt}_{srow} \neq 1 \\ 1 & \text{if } \widehat{PNpt}_{srow} = 1 \end{cases} \quad (77)$$

where $\widehat{PNpt}_{srow}$ is the value of the number of tube passes; $\widehat{PNb}_{srow}$ is the value of the number of baffles; $\widehat{Prp}_{srow}$ is the value of the tube pitch ratio; $\widehat{Play}_{srow}$ is the indication of the tube layout; and $\hat{R}_l$ and $\hat{P}_l$ are parameters for the evaluation of the correction factor of the LMTD (INCROPERA; DE WITT, 2002).

The constraints related to bounds on pressure drops, velocities, Reynolds numbers, and geometry are represented in an alternative way, where the violation of these constraints for each solution candidate can be identified prior to the optimization, and for these options, an exclusion constraint is added, as presented below. According to Souza et al. (2018), this procedure allows a better computational performance.

Pressure drop bounds (equivalent to Eqs. (58) and (59)):

$$yrow_{srow,l} = 0 \quad \text{para } srow \in SDPsmaxout \text{ e } l \in HY \quad (78)$$

$$yrow_{srow,l} = 0 \quad \text{para } srow \in SDPtmaxout \text{ e } l \in HY \quad (79)$$

The sets $SDPsmaxout_l$ and $SDPtmaxout_l$ are given by (where $\hat{\varepsilon}$ is a small positive number):

$$SDPsmaxout_l = \{srow \;/\; \widehat{P\Delta Ps}_{srow,l} \geq \widehat{\Delta Psdisp} + \hat{\varepsilon}\} \quad (80)$$

$$SDPtmaxout_l = \{srow \;/\; \widehat{P\Delta Ptturb1}_{srow,l} + \widehat{P\Delta Ptturb2}_{srow,l} + \widehat{P\Delta Ptcab}_{srow,l}\hat{K}_{srow} \geq \widehat{\Delta Ptdisp} - \hat{\varepsilon}\} \quad (81)$$

where:

$$\widehat{P\Delta Ps}_{srow,l} = 0.864 \frac{\widehat{ms}_l^{\,1.812}\,\widehat{\mu s}^{\,0.188}}{\widehat{\rho s}} \left(\frac{(\widehat{PNb}_{srow}+1)^{2.812}}{\widehat{PDs}_{srow}^{0.812}\left(\widehat{PFAR}_{srow}\widehat{PL}_{srow}\right)^{1.812}\widehat{PDeq}_{srow}^{1.188}} \right) \quad 82)$$

$$P\Delta\widehat{Ptturb1}_{srow,l} = \left(\frac{0.112\ \widehat{mt}_l^{\,2}}{\pi^2 \widehat{\rho t}_l} \right) \left(\frac{\widehat{PNpt}_{srow}^3 \widehat{PL}_{srow}}{\widehat{PNtt}_{srow}^2 \widehat{Pdti}_{srow}^{\,5}} \right) \quad 83)$$

$$P\Delta\widehat{Ptturb2}_{srow,l} = 0.528 \left(\frac{4^{1.58}\ \widehat{mt}_l^{\,1.58}\ \widehat{\mu t}_l^{0.42}}{\pi^{1.58}\ \widehat{\rho t}_l} \right) \frac{\widehat{PNpt}_{srow}^{2.58} \widehat{PL}_{srow}}{\widehat{PNtt}_{srow}^{1.58}\widehat{Pdti}_{srow}^{4.58}} \quad 84)$$

$$P\Delta\widehat{Ptcab}_{srow,l} = \left(\frac{8\ \widehat{mt}_l^{\,2}}{\pi^2\ \widehat{\rho t}_l} \right) \frac{\widehat{PNpt}_{srow}^3}{\widehat{PNtt}_{srow}^2 \widehat{Pdti}_{srow}^{\,4}} \quad 85)$$

It is important to note that Eqs (78) and (79) are not included in the formulation for the cooling water streams. In this case, the optimal pressure drop will be obtained through the trade-off between capital and operating costs in the context of the entire system solution.

Flow velocity bounds (equivalent to Eqs (60) to (63)):

$$yrow_{srow,l} = 0 \quad \text{para } srow \in (Svsminout \cup Svsmaxout) \text{ e } l \in HY \quad 86)$$

$$yrow_{srow,l} = 0 \quad \text{para } srow \in (Svtminout \cup Svtmaxout) \text{ e } l \in HY \quad 87)$$

where the sets *Svsminout*, *Svsmaxout*, *Svtminout*, and *Svtmaxout* are given by:

$$Svsminout_l = \{ srow \ / \frac{\widehat{ms}_l}{\widehat{\rho s}} \frac{(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\ \widehat{PFAR}_{srow}\ \widehat{PL}_{srow}} \leq \widehat{vsmin} - \hat{\varepsilon} \} \quad 88)$$

$$Svsmaxout_l = \{ srow \ / \ \frac{\widehat{ms}_l}{\widehat{\rho s}} \frac{(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\ \widehat{PFAR}_{srow}\ \widehat{PL}_{srow}} \geq \widehat{vsmax} + \hat{\varepsilon} \} \quad 89)$$

$$Svtminout_l = \{ srow \ / \ \frac{4\ \widehat{mt}_l}{\pi\ \widehat{\rho t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\ \widehat{Pdti}_{srow}^{\,2}} \leq \widehat{vtmin} - \hat{\varepsilon} \} \quad 90)$$

$$Svtmaxout_l = \{ srow \ / \frac{4\ \widehat{mt}_l}{\pi\ \widehat{\rho t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\ \widehat{Pdti}_{srow}^{\,2}} \geq \widehat{vtmax} + \hat{\varepsilon} \} \quad 91)$$

Reynolds number bounds (equivalent to Eqs. (64) and (65)):

$$yrow_{srow,l} = 0 \quad \text{para } srow \in Resminout \text{ e } l \in HY$$ 92)

$$yrow_{srow,l} = 0 \quad \text{para } srow \in Retminout \text{ e } l \in HY$$ 93)

where the sets *Resminout* and *Retminout* are given by:

$$Resminout = \{srow / \frac{\widehat{ms}_l}{\widehat{\mu s}} \frac{\widehat{PDeq}_{srow}(\widehat{PNb}_{srow}+1)}{\widehat{PDs}_{srow}\,\widehat{PFAR}_{srow}\,\widehat{PL}_{srow}} \le 2{\cdot}10^3 - \hat{\varepsilon}\}$$ 94)

$$Retminout = \{srow \ / \frac{4\,\widehat{mt}_l}{\pi\,\widehat{\mu t}} \frac{\widehat{PNpt}_{srow}}{\widehat{PNtt}_{srow}\,\widehat{Pdti}_{srow}} \le 10^4 - \hat{\varepsilon}\}$$ 95)

Baffle spacing bounds (equivalent to Eqs. (66) and (67)):

$$yrow_{srow,l} = 0 \quad for\ srow \in (SLNbminout \cup SLNbmaxout)$$ 96)

where the sets *SLNbminout* and *SLNbmaxout* are given by:

$$SLNbminout_l = \{srow / \frac{\widehat{PL}_{srow}}{\widehat{PNb}_{srow}+1} \le 0.2\widehat{PDs}_{srow} - \hat{\varepsilon}\} \quad l \in HY$$ 97)

$$SLNbmaxout_l = \{srow / \frac{\widehat{PL}_{srow}}{\widehat{PNb}_{srow}+1} \ge 1.0\widehat{PDs}_{srow} + \hat{\varepsilon}\} \quad l \in HY$$ 98)

Tube length/shell diameter ratio bounds (equivalent to Eqs. (68) and (69)):

$$yrow_{srow,l} = 0 \quad \text{for } srow \in (SLDminout \cup SLDmaxout)$$ 99)

where the sets $SLDminout_l$ and $SLDmaxout_l$ are given by:

$$SLDminout_l = \{srow \ / \ \widehat{PL}_{srow} \ \leq 3\widehat{PD}s_{srow} - \hat{\varepsilon}\} \quad l \in HY$$

100)

$$SLDmaxout_l = \{srow \ / \ \widehat{PL}_{srow} \ \geq 15\widehat{PD}s_{srow} + \hat{\varepsilon}\} \quad l \in HY$$

101)

Additionally, it is possible to include a constraint associated with a lower bound on the heat-transfer area, which can accelerate the solution convergence:

$$yrow_{srow,l} = 0 \quad \text{for } srow \in SAminout$$

102)

where the set of heat exchangers with area lower than a given lower bound ($\widehat{Amin}_l$) is expressed by:

$$SAminout_l = \{srow \ /\pi \ \widehat{PNtt}_{srow}\widehat{Pdte}_{srow}\widehat{PL}_{srow} \leq \widehat{Amin}_l - \hat{\varepsilon}\} \quad l \in HY$$

103)

The lower bound on the heat-transfer area can be determined by:

$$\widehat{Amin}_l = \frac{\hat{Q}_l}{\widehat{Umax}\widehat{\Delta Tlm}_l} \qquad l \in HY \tag{104}$$

where:

$$\widehat{Umax} = \frac{1}{\frac{1}{\widehat{htmax}}\widehat{drmin} + \widehat{Rft} \cdot \widehat{drmin} + \frac{\widehat{Pdte}_{srow} \ln\left(\widehat{drmin}\right)}{2\,\widehat{ktube}} + \widehat{Rfs} + \frac{1}{\widehat{hsmax}}} \tag{105}$$

$$\widehat{htmax} = \max\left(\widehat{Pht}_{srow}\right)$$

(106)

$$\widehat{hsmax} = \max\left(\widehat{Phs}_{srow}\right)$$

(107)

$$\widehat{drmin} = \min\left(\frac{\widehat{Pdte}_{srow}}{\widehat{Pdti}_{srow}}\right) \tag{108}$$

2.3.3. <u>Economic model constraints</u>

The annualized capital cost of the pipe sections is given by:

$$C_{pipe} = \widehat{Fmspip} \sum_{k \in PI} \sum_{n \in SD} \widehat{Cpi}_n \hat{L}_k \, y^{pi}_{k,n} \tag{109}$$

where $\widehat{Fmspip}$ is a Marshall-swift index correction factor and the unitary cost of a standard pipe *n* ($\widehat{Cpi}_n$) is calculated using the proposal of Narang et al. (2009):

$$\widehat{Cpi}_n = \left(\hat{C}_1/0.3048\right)\left(\hat{D}^{nom}_n/12\right)^{\widehat{m}} \tag{110}$$

where $\hat{C}_1$ and $\widehat{m}$ are correlation parameters

The annualized capital cost of the pump is given by:

$$C_{pump} = \sum_{k \in PU} \sum_{s \in SPU} \widehat{Cpu}_s y^{pu}_{k,s} \tag{111}$$

where the parameter that expresses the capital cost of each pump ($\widehat{Cpu}_s$) is calculated by a set of equations presented by Couper et al. (2005), for $s \in SPU$:

$$Cpu_s = \hat{r} \, \widehat{F}_I \, \widehat{Fms} \, \widehat{Fmp} \, \widehat{Ft}_s \left(1.39 exp\left(8.833 - 0.6019 ln(\hat{z}_s) + 0.0519\left(ln(\hat{z}_s)\right)^2\right)\right) \tag{112}$$

$$\hat{z}_s = 28710 \widehat{qpu}^{design}_s \sqrt{\widehat{fpu}^{design}_s} \tag{113}$$

where $\hat{r}$ is the annualization factor, $\widehat{Fms}$ is the Marshall-swift index correction factor, $\widehat{Fmp}$ is a cost factor related to the pump material, $\widehat{Ft}_s$ is a cost factor related to the pump type, $\widehat{qpu}_s^{design}$ is the design flow rate of the pump s, and $\widehat{fpu}_s^{design}$ is the corresponding head at the design flow rate.

The expression for evaluation of $\widehat{Ft}_s$ is:

$$\widehat{Ft}_s = exp\left(\sum_{\hat{j}} \hat{b}_j \left(ln(\hat{z}_s)\right)^{(\hat{j}-1)}\right) \qquad 114)$$

where $\hat{b}_j$ are correlation parameters.

The pump curve relates $\widehat{qpu}_s^{design}$ and $\widehat{fpu}_s^{design}$:

$$\widehat{fpu}_s^{design} = \sum_i \hat{a}_{i,s}\left(\widehat{qpu}_s^{design}\right)^i \qquad (115)$$

The expression of the annualization factor is:

$$\hat{r} = \frac{\hat{\imath}(1+\hat{\imath})^{\widehat{ny}}}{(1+\hat{\imath})^{\widehat{ny}} - 1} \qquad (116)$$

where $\hat{\imath}$ is the interest rate and $\widehat{ny}$ is the number of years of the project life.

The operational costs (C_{oper}) associated with the pipe network operation are given by:

$$C_{oper} = \sum_{k \in PU} cop_k \qquad 117)$$

$$cop_k = \left(\frac{\hat{q}_k\, \hat{\rho}_w \hat{g}\, fpu_k}{\hat{\eta}}\right)\frac{\widehat{N}_{OP}\widehat{pc}}{10^3} \qquad 118)$$

where $\widehat{N}_{OP}$ is the number of operating hours per year, $\widehat{pc}$ the energy price, and $\hat{\eta}$ the pump and driver efficiency.

The annualized total capital cost of the heat exchangers is given by the sum of the individual equipment costs:

$$C_{heatexc} = \sum_{l \in HY} Cheatexc_l \qquad 119)$$

where the evaluation of the cost of each unit is based on the equations presented by Cooper et al. (2005):

$$Cheatexc_l = 1.218\,\hat{r}\,\widehat{Fmh}_l\,\widehat{Fph}_l\,\widehat{Fmseq}\exp\Big(8.821 - 0.30863\big(\ln\big(10.76\,\widehat{pA}_{srow,l}\big)\big) + 0.0681\big(\ln\big(10.76\,\widehat{pA}_{srow,l}\big)\big)^2 - 1.1156 + 0.0906\big(\ln 10.76\,\widehat{pA}_{srow,l}\big)\Big) \quad l \in HY \qquad (120)$$

where $\widehat{Fmh}_l$ is a cost factor related to the heat exchanger material, $\widehat{Fph}_l$ is a cost factor related to pressure range, $\widehat{Fmseq}$ is the Marshall-swift index correction factor, and $\widehat{pA}_{srow,l}$ is the area of the corresponding heat exchanger:

$$\widehat{pA}_{srow,l} = \pi\,\widehat{PNtt}_{srow}\,\widehat{Pdte}_{srow}\,\widehat{PL}_{srow} \quad l \in HY \qquad 121)$$

2.3.4. Objective function

The objective function is the minimization of the total annualized cost, including the capital costs of the pipe sections, pumps, and heat exchangers and the operational costs associated with the pumps:

$$\min C = C_{pipe} + C_{pump} + C_{heatexc} + C_{oper} \qquad 122)$$

2.4. Results

The performance of the proposed approach, based on the simultaneous design of the pipe network and the coolers, is illustrated through its comparison with conventional procedures, where the design steps of the coolers and the pipe network are conducted separately, this procedure is adopted in the practice.

The procedure to design the heat exchangers separately from the pipe network is tested here using two different alternatives:

(A) The coolers are designed seeking to minimize the heat transfer surface according to maximum pressure drop constraints, and then the pipe network is optimized considering the total annualized cost of the pipe sections and pumps (this approach will be called here: two-step design A).

(B) The coolers are first optimized using the total annualized cost associated with the corresponding capital and operational costs, and then the pipe network is designed through the minimization of the total annualized cost of the pipe sections and pumps (this approach will be

called here: two-step design B).

In both alternatives described above, the heat exchanger design was solved using the formulation proposed by Gonçalves et al. (2017). Then, the set of values of the cooling water pressure drops obtained in the optimization of each cooler is employed in the optimization of the pipe network. These pressure drops are considered fixed in the hydraulic optimization of the pipe network using the formulation presented above to minimize the corresponding annualized cost of the pipe network.

The different design procedures were compared through a set of design examples. In the examples, the entire network is considered in the same elevation, with the exception of the cooling tower top that is 2 m higher. The pipes are made of carbon steel, with STD Schedule, A106 pipe. The pipe commercial diameters are based on the standard ASME/ANSI B.36.10/19. The capital costs of the pipes are calculated using the coefficients $\hat{C}_1$ and $\hat{m}$ in Eq. (110) equal to 7.0386 and 1.4393. The Hazen−Williams parameter $(\widehat{ch})$ is considered equal to 100 for all pipe sections. The parameters of the pump, heat exchangers, and pipe capital cost correlations are shown in Table 2. The set of standard values of the discrete variables for the design of the heat exchangers is shown in Table 3. The thermal conductivity of the tubes of the heat exchangers is 50 W/(m·K). The flow velocity in the tube side of the heat exchangers must be between 1 and 3 m/s, and the corresponding bounds in the shell-side are 0.5 and 2 m/s. Physical properties of the cooling water are shown in Table 4. The rest of the problem parameters are shown in Table 5.

Table 2 - Pump, heat exchangers and pipe capital cost correlation parameters (COUPER, 2003)

Factor	Value
F_{ph}	1
F_{ms}	1.308
F_{mseq}	1.308
F_{mspip}	1.144
F_{mh}	1
F_{mp}	1.35
b_1	5.1029
b_2	-1.2217
b_3	0.0771

Table 3 - Standard values of the discrete design variables of the coolers (KAKAÇ et al., 2012)

Variable	Values
Outer tube diameter, $\widehat{pdte}_{sd}$ (m)	0.019, 0.025, 0.032, 0.038, 0.051
Tube length, $\widehat{pL}_{sL}$ (m)	1.220,1.829,2.439, 3.049, 3.659, 4.877, 6.098
Number of baffles, $\widehat{pNb}_{sNb}$	1, 2, … , 20
Number of tube passes, $\widehat{pNpt}_{sNpt}$	1, 2, 4, 6
Tube pitch ratio, $\widehat{prp}_{srp}$	1.25, 1.33, 1.50
Shell diameter, $\widehat{pDs}_{sDs}$ (m)	0.787, 0.838, 0.889, 0.940, 0. 991, 1.067, 1.143, 1.219, 1.372, 1.524
Tube layout, $\widehat{play}_{slay}$	1 = square, 2 = triangular

Table 4 - Physical properties of the cooling water (INCROPERA; DE WITT, 2002)

Property	Value
Density (kg/m^3)	995
Thermal capacity (J/(kg·K))	4187
Viscosity (mPa·s)	0.72
Thermal conductivity (W/(m·K))	0.59

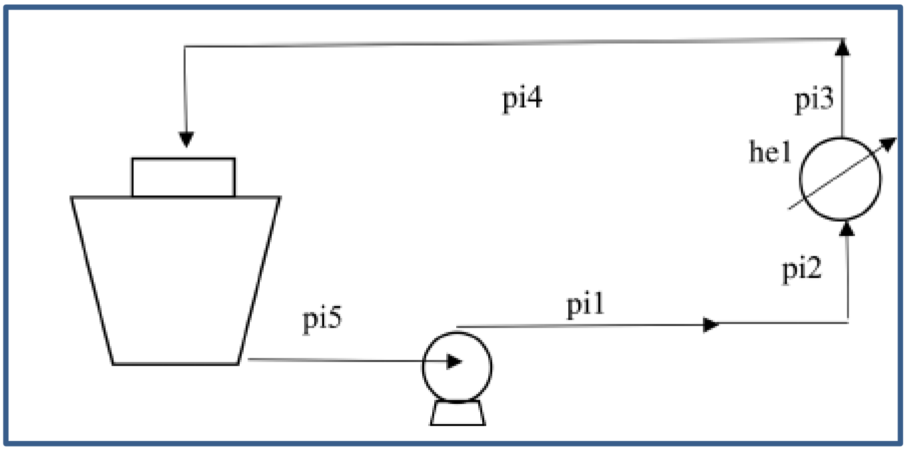
Table 5 - Problem parameters (COUPER, 2003)

Parameter	Value
Maximum flow velocity in the pipes (m/s)	3.0
Minimum flow velocity in the pipes (m/s)	1.0
Pump efficiency	0.80
Number of operating hours per year (h/y)	8760
Energy cost (USD/kWh)	0.130 8
Interest rate	0.05
Project horizon (y)	10

2.4.1.1.Example 1: Cooling water system with one heat exchanger

Figure 3 presents the structure of the cooling water system of Example 1.

Figure 3 - Example 1: Cooling water system structure



Fonte: A autora, 2022

Table 6 presents the set of hydraulic heads associated to the available pumps, Table 7 presents the physical properties of the hot stream that flows in the shell-side of the cooler, Table 8 presents the features of the thermal task of the cooler, and Table 9 presents the lengths of the pipe sections.

Table 6 - Example 1: Pump head options (rated condition)

	Options
Pump head (m)	3, 4, 6, 7, 10, 12, 14, 18, 20, 22, 30, 33, 35, 37, 40

Fonte: A autora, 2022

Table 7 - Example 1: Physical properties of the streams

Property	Value
Density (kg/m^3)	1080
Heat capacity (J/(kg·K))	3601
Viscosity (mPa·s)	1.30
Thermal conductivity (W/(m·K))	0.58

Fonte: A autora, 2022

Table 8 - Example 1: Cooling task data

	he1 (hot/cold side)
Mass flow rate (kg/s)	11.00 / 37.84
Inlet temperature (^{o}C)	90.0 / 30.0
Outlet temperature (^{o}C)	50.0 / 40.0
Fouling factor (m^{2}K/W)	0.0001 / 0.0004
Allowable pressure drop (kPa)	100 / 100

Fonte: A autora, 2022

Table 9 - Example 1: Pipe section lengths

Pipe section	Length (m)
pi1	198
pi2	15
pi3	15
pi4	200
pi5	2

Fonte: A autora, 2022

The results of the optimization in each approach are displayed in Tables 10-14. Table 10 contains the optimal design variables of heat exchanger he1, Table 11 displays the thermofluid dynamic behavior of heat exchanger he1, Table 12 displays the diameters and head losses in the pipe sections, Table 13 shows the pump head and head loss in the valve, and Table 14 presents the objective function and its corresponding components.

Table 10 – Example 1: Heat exchanger design results (he1)

	Simultaneous design	Two-step design (a)	Two-step design (b)
Area (m^2)	62.7	57.4	62.7
Tube diameter (m)	0.019	0.025	0.019
Tube length (m)	3.049	3.049	3.049
Number of baffles	19	20	19
Number of tube passes	2	6	2
Tube pitch ratio	1.25	1.25	1.25
Shell diameter (m)	0.489	0.54	0.489
Tube layout	2	2	2
Total number of tubes	344	236	344
Baffle spacing (m)	0.152	0.145	0.152

Fonte: A autora, 2022

Table 11 - Example 1: Thermo-fluid dynamic results (he1)

	Simultaneous design	Two-step design (a)	Two-step design (b)
Shell-side flow velocity (m/s)	0.683	0.650	0.683
Tube-side flow velocity (m/s)	1.135	2.522	1.135
Shell-side coefficient (W/(m^2K))	4212.6	3600.6	4212.6
Tube-side coefficient (W/(m^2K))	5407.5	9567.7	5407.5
Overall coefficient (W/(m^2K))	925.0	1007.1	925.0
Shell-side pressure drop (Pa)	57458	43217	57458
Tube-side pressure drop (Pa)	9275	91550	9275

Fonte: A autora, 2022

Table 12 - Example 1: Diameters and head losses of the pipes

Pipe	Simultaneous design		Two-step design (a)		Two-step design (b)	
Section	Diameter (in)	Head loss (m)	Diameter (in)	Head loss (m)	Diameter (in)	Head loss (m)
pi1	8	2.329	8	2.329	8	2.329
pi2	5	1.643	6	0.670	6	0.670
pi3	6	0.670	6	0.670	5	1.643
pi4	8	2.352	8	2.352	8	2.352
pi5	8	2.329	8	2.352	8	2.329

Fonte: A autora, 2022

Table 13 - Example 1: Pump head and valve head loss

Design	Pump head (m)	Valve head loss (m)
Simultaneous design	10	0.032
Two-step design (a)	18	0.575
Two-step design (b)	10	0.032

Fonte: A autora, 2022

Table 14 – Example 1: Annualized costs

Cost ($/year)	Simultaneous design	Two-step design (a)	Two-step design (b)
Pump cost	662.10	726.58	662.10
Heat exchanger cost	6080.93	5731.90	6080.93
Pipe cost	6154.28	6188.01	6154.28
Operation cost	5313.13	9563.64	5313.13
Total cost	18210.44	22210.13	18210.44

Fonte: A autora, 2022

Table 14 indicates that the two-step design (a) presented a higher value of the objective function in relation to the simultaneous design and the two-step design (b), which obtained the same results.

According to Table 11, the design solution of the heat exchanger he1 in the two-step design A is associated to a pressure drop near to the maximum available value (91.55 kPa x 100 kPa), i.e. the optimization sought to minimize the heat transfer area through the exploration of the available pressure drop. In fact, Table 10 indicates that the two-step design A presented a heat exchanger area that is 8.4% lower than the other approaches. However, the increase of the pressure drop in heat exchanger he1 penalized the total pressure drop of the system. Consequently, the operational cost of two-step design A is 22% higher than the other approaches.

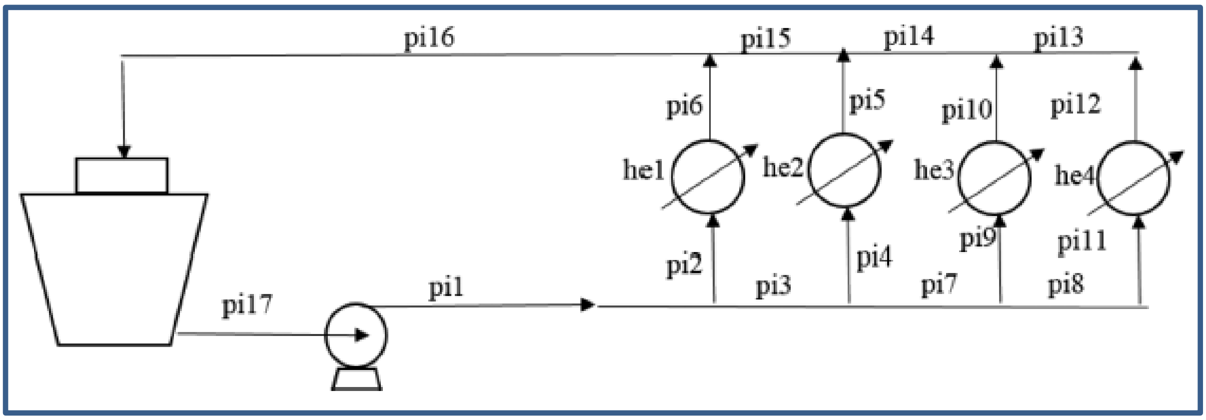
The simultaneous design and the two-step design B yielded equivalent solutions, i.e. there was no difference to explore the trade-off between capital and operational costs simultaneously or in two steps in this example. The two separated tradeoffs involving capital and operational costs for the heat exchanger and the pipe network superposed without affect the optimal point, therefore there was no difference between the results of the simultaneous design and the two-step design B.

2.4.2. Example 2: Cooling water system with four heat exchangers

The network architecture is presented on Figure 4. Table 15 presents the set of hydraulic heads associated to the available pump alternatives, Table 16 presents the physical properties of the hot streams that flows in the shell-side with the exception of heat exchanger he2 where the hot stream flows in the tube-side, Table 17 presents the features of the thermal task of each cooler (in this example, no maximum pressure drops constraints were imposed for any stream in the simultaneous and two-step design B approaches), and Table 18 presents the lengths of the pipe sections.

•

Figure 4 - Example 2: Cooling water system structure



Fonte: A autora, 2022

Table 15 - Example 2: Pump head options

	Options
Pump head (m)	6, 7, 10, 12, 14, 18, 20, 22, 25, 28, 30, 33, 35, 37, 40

Fonte: A autora, 2022

Table 16 - Example 2: Physical properties of the streams

	Hot stream			
	he1	**he2**	**he3**	**he4**
Density (kg/m^3)	1080	750	786	1080
Thermal capacity (J/(kg·K))	3601	2840	2177	3601

Viscosity (mPa·s)	1.3	0.34	1.89	1.30
Thermal conductivity (W/(m·K))	0.58	0.19	0.12	0.58

Fonte: A autora, 2022

Table 17 - Example 2: Cooling tasks data

	he1 hot/cold	**he2 hot/cold**	**he3 hot/cold**	**he4 hot/cold**
Mass flow rate (kg/s)	21.94 /75.48	27.8 / 56.57	30 / 77.99	14 / 36.12
Inlet temperature (°C)	90 / 30	70 / 30	100 / 30	80 / 30
Outlet temperature (°C)	50 / 40	40 / 40	50 / 40	50 / 40
Fouling factor (m^2K/W)	0.0001/ 0.0004	0.0002/ 0.0004	0.0002/ 0.0004	0.0001/ 0.0004
Allowable pressure drop (kPa)	100 / 100	70 / 50	60 / 100	100 / 50

Fonte: A autora, 2022

Table 18 - Example 2: Pipe lengths

Pipe section	**Length (m)**	**Pipe section**	**Length (m)**
pi1	98	**pi10**	8
pi2	12	**pi11**	10
pi3	20	**pi12**	10
pi4	15	**pi13**	18
pi5	15	**pi14**	25
pi6	12	**pi15**	20
pi7	25	**pi16**	100
pi8	18	**pi17**	2
pi9	8		

Fonte: A autora, 2022

Table 19 presents the number of equations and variables of this MILP with four heat exchangers.

Table 19 – Number of equations and variables of the MILP problem with four heat exchangers

Constraints	4 104 644
Variables	1 478 828
Discrete variables	1 478 789

Fonte: A autora, 2022

The results of the optimization in each approach are displayed in Tables 20-27. Tables 20-23 contain the design variables of the heat exchangers, Tables 24-27 displays the thermofluid dynamic behavior of heat exchangers, Table 28 displays the optimal pump heads and valve head losses, and Table 29 presents the objective function and its corresponding components.

Table 20 - Example 2: Heat exchanger design results (he1)

	Simultaneous design	Two-step design (a)	Two-step design (b)
Area (m^2)	123.5	113.9	123.5
Tube diameter (m)	0.019	0.019	0.019
Tube length (m)	3.049	2.439	3.049
Number of baffles	15	14	18
Number of tube passes	2	4	2
Tube pitch ratio	1.25	1.25	1.25
Shell diameter (m)	0.686	0.737	0.686
Tube layout	2	2	2
Total number of tubes	677	781	677
Baffle spacing (m)	0.191	0.163	0.160

Fonte: A autora, 2022

Table 21 - Example 2: Heat exchanger design results (he2)

	Simultaneous design	Two-step design (a)	Two-step design (b)
Area (m^2)	246.6	227.8	211.5
Tube diameter (m)	0.019	0.019	0.019
Tube length (m)	6.098	4.877	6.098
Number of baffles	8	8	9
Number of tube passes	6	6	6
Tube pitch ratio	1.25	1.25	1.25
Shell diameter(m)	0.737	0.737	0.635
Tube layout	1	2	2
Total number of tubes	676	781	580
Baffle spacing (m)	0.678	0.542	0.610

Fonte: A autora, 2022

Table 22 - Example 2: Heat exchanger design results (he3)

	Simultaneous design	Two-step design (a)	Two-step design (b)
Area (m^2)	197.5	207.3	197.5
Tube diameter (m)	0.019	0.019	0.019
Tube length (m)	4.877	3.049	4.877
Number of baffles	17	9	17
Number of tube passes	2	4	2
Tube pitch ratio	1.25	1.25	1.25
Shell diameter(m)	0.686	0.889	0.686

Tube layout	2	2	2
Total number of tubes	677	1137	677
Baffle spacing (m)	0.271	0.305	0.271

Fonte: A autora, 2022

Table 23 - Example 2: Heat exchanger design results (he4)

	Simultaneous design	**Two-step design (a)**	**Two-step design (b)**
Area (m^2)	75.3	61.129	75.3
Tube diameter (m)	0.019	0.019	0.019
Tube length (m)	3.659	2.439	3.659
Number of baffles	15	17	17
Number of tube passes	2	4	2
Tube pitch ratio	1.25	1.25	1.25
Shell diameter(m)	0.489	0.540	0.489
Tube layout	2	2	2
Total number of tubes	344	419	677
Baffle spacing (m)	0.229	0.136	0.261

Fonte: A autora, 2022

Table 24 - Example 2: Thermo-fluid dynamic results (he1)

	Simultaneous design	**Two-step design (a)**	**Two-step design (b)**
Shell-side flow velocity (m/s)	0.770	0.848	0.923
Tube-side flow velocity (m/s)	1.151	1.995	1.151
Shell-side coefficient (W/(m^2K))	4522.2	4744.3	4970.5
Tube-side coefficient (W/(m^2K))	5466.1	8488.9	5466.1
Overall coefficient (W/(m^2K))	941.2	1027.4	959.2
Shell-side pressure drop (Pa)	81442	96038	132043
Tube-side pressure drop (Pa)	9507.3	44457	9507.4

Fonte: A autora, 2022

Table 25 - Example 2: Thermo-fluid dynamic results (he2)

	Simultaneous design	**Two-step design (a)**	**Two-step design (b)**
Shell-side flow velocity (m/s)	0.570	0.712	0.734
Tube-side flow velocity (m/s)	1.689	1.462	1.969
Shell-side coefficient (W/(m^2K))	3819.9	4980.4	5064.3
Tube-side coefficient (W/(m^2K))	3471.5	3092.8	3924.0
Overall coefficient (W/(m^2K))	776.2	787.4	844.7
Shell-side pressure drop (Pa)	16106	35163	35587
Tube-side pressure drop (Pa)	71172	45165	94475

Fonte: A autora, 2022

Table 26 - Example 2: Thermo-fluid dynamic results (he3)

	Simultaneous design	Two-step design (a)	Two-step design (b)
Shell-side flow velocity (m/s)	1.027	0.704	1.027
Tube-side flow velocity (m/s)	1.189	1.416	1.189
Shell-side coefficient (W/(m^{2}K)	1207.2	980.9	1207.2
Tube-side coefficient (W/(m^{2}K)	5611.3	6452.8	5611.3
Overall coefficient (W/(m^{2}K)	566.9	518.9	566.9
Shell-side pressure drop (Pa)	125836	45721	125836
Tube-side pressure drop (Pa)	14798	27824	14798

Fonte: A autora, 2022

Table 27 - Example 2: Thermo-fluid dynamic results (he4)

	Simultaneous design	Two-step design (a)	Two-step design (b)
Shell-side flow velocity (m/s)	0.580	0.886	0.507
Tube-side flow velocity (m/s)	1.084	1.780	1.084
Shell-side coefficient (W/(m^{2}K)	3848.6	4860.5	3576.1
Tube-side coefficient (W/(m^{2}K)	5210.4	7747.7	5210.4
Overall coefficient (W/(m^{2}K)	899.3	1018.4	883.5
Shell-side pressure drop (Pa)	34131	91457	23446
Tube-side pressure drop (Pa)	9850.4	35949	9850.4

Fonte: A autora, 2022

Table 28 – Example 2: Pump head and valve head losses

Design	Pump head (m)	Head loss he1(m)	Head loss he2(m)	Head loss he3(m)	Head loss he4(m)
Simultaneous design	6	0.450	0.017	0.024	0.027
Two-step design (a)	10	0.466	0.614	0.752	0.034
Two-step design (b)	10	2.522	0.102	0.259	0.033

Fonte: A autora, 2022

Table 29- Example 2: Annualized costs

Cost ($/year)	Simultaneous design	Two-step design (a)	Two-step design (b)

Pump cost	1455.21	1752.81	1752.81
Heat exchanger cost	48350.33	46232.95	46253.56
Pipe cost	15606.02	13706.12	12354.19
Operational cost	20739.07	34565.12	34565.12
Total cost	86150.64	96257.02	94925.69

Fonte: A autora, 2022

Table 29 indicates a behavior of the two-step design A approach similar to the previous example. The heat exchanger cost is minimized according to the available pressure drop at the expense of the operational costs, which results in a higher total annualized cost.

Unlike in the case of one exchanger, the two-step design B approach yielded a solution associated with a higher total annualized cost when compared with the simultaneous solution. The optimization using the simultaneous approach and the two-step design B yielded heat exchangers with the same area, with exception of heat exchanger he2, as it is displayed in Tables 20-23. In relation of this heat exchanger, the simultaneous design selected an option with a larger area (Table 21: 246.6 m^2 x 211.5 m^2), but lower shell-side pressure drop, where the cooling water flows (Table 25: 16.1 kPa x 36.6 kPa). Despite the penalty of a more expensive heat exchanger, the analysis of the selected pumps for each case in Table 28 indicates that this selection allowed identifying an optimal solution associated with a pump with lower power, which yielded a reduction of the operational costs, as it can be observed in Table 29. The optimization of the total annualized cost of the heat exchangers separately in the two-step design problem B penalized the operational costs, which implied in a higher value of the total annualized cost of the entire system.

3. OPTIMIZATION OF COOLING WATER SYSTEMS TOGETHER WITH THE DISTRIBUTION OF THE FLOW RATES

The engineering practice states that the temperature of the return of the cooling water to the cooling tower must be the highest possible to reduce costs. Typically, this guidance is applied to the design of coolers (for example, in Brazil, the design of a cooler using cooling water is associated with a cooling water outlet temperature between 40 ºC and 45 ºC). The hypothesis of a maximum return temperature of the cooling water in all coolers was adopted in the formulation of the design optimization presented in the previous chapter.

Aiming at attaining additional cost reductions, this chapter presents an extension of the design problem explored in Chapter 2. Instead of imposing the same outlet temperature to all coolers, the proposed formulation of the design problem includes the optimization of the distribution of the cooling water flow rate among the different coolers. Therefore, the coolers may have different cooling water outlet temperatures. The proposed problem solution presents the optimal values of the design variables related to each heat exchanger, the cooling water outlet temperature of each heat exchanger, the pipe diameters and the pump type.

The inclusion of the outlet temperatures in the formulation presented in the previous chapter yielded a MINLP formulation that was associated with severe convergence limitations. Attempts to eliminate these convergence limitations through the reformulation of the problem in a linear form were also unsuccessful, because the resultant dimension of the problem was too large.

These obstacles were overcome through the development of an optimization procedure, presented in this chapter, which employs different techniques sequentially.

3.1. Problem description

The optimization problem corresponds to the minimization of the total annualized cost of a cooling water system, encompassing the heat exchangers, the pipe sections of the network for cooling water distribution and the pump. The problem design variables are the pipe diameters of the pipe sections, the pump size, the head losses of the valves associated with each heat exchanger, and the heat exchanger geometry (tube length, tube diameter, tube layout, tube pitch ratio, number of baffles, shell diameter, and number of tube passes), the mass flow rates of the cooling water in each heat exchanger and in each pipe section, the temperatures of the water streams at the outlet of the heat exchangers and the return temperature to the cooling tower.

These variables were also present in the optimization problem of Chapter 2, with the exception of the mass flow rates and temperatures of the cooling water streams. The inclusion of the mass flow rates and temperatures as variables can provide additional cost reductions, but increases the design problem complexity considerably. The cooling water flow rates in the heat exchangers are related to the outlet temperatures and the pressure losses on the heat exchangers and pipe sections. Modifications in the cooling water temperature influences the heat exchanger geometry. Therefore, several additional tradeoffs are present in this problem configuration.

3.2. Solution procedure

The proposed solution procedure involves four steps, as follows (a more detailed description of each step is presented later):

1) Reduction the number of heat exchanger candidates for each thermal task

The cooling water system is associated with several hot streams that must be cooled. Each hot stream corresponds to a thermal task and the design solution must indicate the dimensions of the heat exchangers associated with the minimum cost. Therefore, the search space of the proposed optimization problem contains a large set of heat exchanger candidates, characterized by all possible combinations of the available discrete values of each design variable (shell diameter, tube diameter, tube length, etc.). In order to reduce the number of options, this step apply the Set Trimming technique (COSTA and BAGAJEWICZ, 2019) to eliminate heat exchanger candidates for each thermal task that are not feasible, independently of the cooling water flow rate (that it is not established a priori). This elimination is based on geometrical constraints and constraints related to the hot streams (that are already established for each thermal task).

2) Identification of the water flow rate for each heat exchanger candidate

This step determines the cooling water flow rate for each heat exchanger candidate in each thermal task to match the desired outlet temperature of the process stream. It corresponds to a solution of a system of nonlinear algebraic equations of the heat exchanger model, according to the LMTD method.

After this step, each remaining heat exchanger associated with each task is related to a cooling water flow rate.

3) Further reduction the number of heat exchanger candidates for each thermal task

The cooling water flow rates evaluated in the previous step allow eliminating more solution candidates. This additional elimination step is conducted using again the Set Trimming technique, but now exploring other constraints. The constraints applied here are associated with the values of the cooling water flow rates that were evaluated in the previous step.

Alternatively, the elimination of candidates using Set Trimming could be applied in a single step, after the evaluation of the cooling water flow rate for all candidates. However, this alternative would be more computationally intensive. The determination of the cooling water temperature is a relatively time consuming step and the previous application of Set trimming reduces the number of nonlinear systems of equations that must be solved.

4) Identification of the optimal solution of the cooling water system in the remaining search space

The set of heat exchanger candidates for each task and the corresponding cooling water flow rates can be inserted into a MILP problem equivalent to of the one presented in the previous chapter. Attempts to solve this problem in several examples indicated that the corresponding dimension of the resultant problem is too large for the available computational resources.

This obstacle was overcome through a decomposition procedure involving two MILPs problems.

The next sections of this chapter present a more detailed discussion of each step described above.

3.2.1. Step 1: Reduction the number of heat exchanger candidates for each thermal task

This step applies the Set Trimming technique (COSTA and BAGAJEWICZ, 2019) to reduce the number of heat exchanger options (formed by combinations of the available values of the discrete variables) for each thermal task. Each heat exchanger candidate is characterized by a set of discrete values of the corresponding design variables (shell diameter, tube length, tube diameter, etc.).

Set Trimming is an optimization technique based on the use of inequality constraints to gradually reduce the search space. The search space explored in the Set Trimming is represented by a set composed of all the possible combinations of the discrete values of the variables, i.e. each candidate solution corresponds to a certain combination of the available values of the discrete variables.

Based on this combinatorial representation of the search space, Set Trimming eliminates infeasible candidates through the application of the inequality constraints sequentially. According to the sequential nature of the process, the remaining feasible candidates of the application of a constraint are then tested to the next one. This sequential pattern brings significant reductions of the computational effort in relation to an exhaustive enumeration, because the number of candidates tested in each constraint tends to decrease.

An important aspect of Set Trimming is the exploration of computational routines that can evaluate large sets of data. The application of Set Trimming is not based on slow computational loops (e.g. “for loops”) that examine individually the constraints for each alternative. Set Trimming involves the manipulation of sets (e.g. “element-wise” operations in Scilab/Matlab, arrays in Python, indexed operations in GAMS, etc.).

This method eliminates several drawbacks that affect other optimization techniques: guarantees global optimality; does not depend on good initial estimates; guarantees convergence; and does not require any tuning of algorithm parameters.

Lemos et al. (2020) performed the global design optimization of shell-and-tube heat exchangers using Set Trimming. The results showed that Set Trimming demanded shorter computational times when compared to mathematical programming techniques.

The current step in the proposed optimization procedure is based on the application of some constraints explored in Lemos et al. (2020) through Set Trimming. All constraints that do not involve the cooling water flow rate are explored here (differently from the optimization problem of the previous chapter, the cooling water flow rates are problem variables here, therefore, their values are not known).

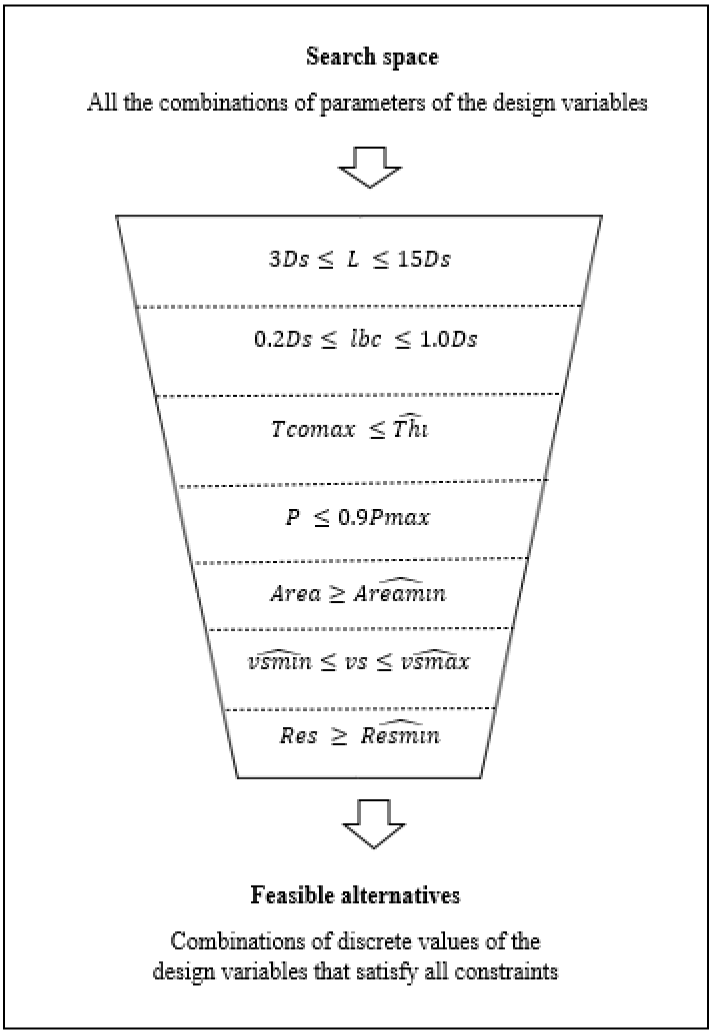
The set of constraints and the corresponding ordering in which they are applied are presented below (assuming that the hot stream flows in the shell-side). Aiming at reducing the computational effort these constraints are ordered according to a crescent computational effort (therefore, the constraints that involve a higher computational effort are applied to a fewer number of candidates).

Set Trimming order:

1) Bounds on the tube length / shell diameter ratio: Eqs. (68-69)
2) Bounds on the baffle spacing: Eqs. (66-67)
3) Upper bound on the water temperatures in the outlet of heat exchangers: See below
4) Upper bound on the P temperature ratio of the LMTD method: See below
5) Lower bound on the heat transfer area: See below
6) Bounds on hot stream flow velocities: Eqs. (60-61)
7) Lower bound on hot stream Reynolds number: Eq. (64)

The Figure 5 presents a schematic of how the Set Trimming procedure was performed.

Figure 5 - Schematic of set trimming procedure



Fonte: A autora, 2022

3.2.1.1. Upper bound on the water temperatures in the outlet of heat exchangers

Aiming at avoiding erosion and vibration, the flow velocity of the streams in the heat exchangers must be below a maximum value (see Eq. (61)). Therefore, the maximum cooling water flow rate in a candidate heat exchanger ($qhemax$) can be calculated by:

$$qhemax = \left(\frac{3.14}{4}\right)\left(\frac{Ntt\ dti^2}{Npt}\right)\widehat{vtmax}$$

123)

Based on this maximum value of cooling water flow rate, it is possible to evaluate the corresponding outlet temperature of the cooling water ($Tcomax$):

$$Tcomax = \frac{\hat{Q}}{qhemax\ \hat{\rho}_w \widehat{Cp}_w} + \widehat{Tci}$$

124)

In any heat exchanger, the outlet temperature of the cold stream is thermos/dynamically limited by the inlet temperature of the hot stream. Therefore, this condition become a trimming step:

$$Tcomax \leq \widehat{Thi}$$

125)

3.2.1.2. Upper bound on the P temperature ratio of the LMTD method

In the design of shell-and-tube heat exchangers, it is assumed that the correction factor of the LMTD must be higher than a minimum value, typically 0.75. Smith (2005) presented an similar limit based on the asymptotic value of the P temperature ratio of the LMTD method (*Pmax*), which can be calculated considering *Tcomax*, as follows:

$$Pmax = \frac{Tcomax - \widehat{Tci}}{\widehat{Thi} - \widehat{Tci}} \qquad 126)$$

For heat exchanger with multiple passes, the P temperature ratio (Eq. 57) is limited by a maximum value. Therefore, this condition becomes a trimming step:

$$P \leq 0.9Pmax \qquad 127)$$

3.2.1.3. Lower bound on the heat transfer area

This trimming corresponds to the elimination of heat exchangers with area (evaluated by Eq. (52)) lower than the area lower bound evaluated by the Eq. (104), using $Tcomax$ as the outlet temperature of the cooling water.

3.2.2. Step 2: Identification of the water flow rate for each heat exchanger candidate

In this step, the water outlet temperature was calculated for each heat exchanger. The Kern model is used, according to the work of Gonçalves et al. (2017). The mathematical model for evaluation of the water outlet temperature (*Tco*) and the corresponding water volumetric flow rate (*qhe*) is composed of the energy balance of the cooling water stream and the heat transfer rate expression. The determination of *Tco* and *qhe* is conducted through the solution of an optimization problem.

The expression of the energy balance is given by:

$$qhe\ \hat{\rho}_w \widehat{Cp}_w \left(Tco - \widehat{Tci}\right) = \hat{Q} \tag{128}$$

The heat transfer rate expression is equivalent to Eq. (71), but the logarithmic mean temperature difference (ΔTlm) and the correction factor (F) are continuous variables, as they are a function of *Tco*, which is the continuous variable to be determined in this step for each heat exchanger. The correction factor *F* now presents the same form of Eq. (77), with the difference that now *P* and *R* are continuous variables (see Eqs. (56) and (57)) which depend on the temperature of the water leaving the exchanger *Tco*.

The corresponding expression of the heat transfer rate equation becomes:

$$\hat{Q}\left(\frac{\widehat{Pdte}}{ht\ \widehat{Pdti}} + \widehat{Rft}_l\ \frac{\widehat{Pdte}}{\widehat{Pdti}} + \frac{\widehat{Pdte}\ln\left(\frac{\widehat{Pdte}}{\widehat{Pdti}}\right)}{2\ \widehat{ktube}} + \widehat{Rfs} + \frac{1}{\widehat{Phs}}\right)$$
$$\geq \left(\frac{100\ + Aexc}{100}\right)\left(\pi\widehat{PNtt}\ \widehat{Pdte}\ \widehat{PL}\ \Delta Tlm\left((1 - \widehat{yNpt})F + \widehat{yNpt}\right)\right) \tag{129}$$

where ht, and F are represented by:

$$Pht = \frac{\widehat{kt}\ 0.023\ \left(\frac{4mt}{\pi\ \widehat{\mu t}}\right)^{0.8}\ \widehat{Prt}^N}{\widehat{Pdti}_{srow}^{1.8}}\left(\frac{\widehat{PNpt}}{\widehat{PNtt}}\right)^{0.8} \tag{130}$$

$$F\ = \frac{(R^2 +\ 1)^{0.5}\ \ln\left(\frac{(1-P)}{(1-\ R\ P)}\right)}{(R-1)\ln\left(\frac{2-P(R+1-\ (R^2 +\ 1)^{0.5})}{2-P(R+1+(R^2 +\ 1)^{0.5})}\right)} \tag{131}$$

$$R = \frac{\widehat{Thi} - \widehat{Tho}}{Tco - \widehat{Tci}} \tag{132}$$

$$P = \frac{Tco - \widehat{Tci}}{\widehat{Thi} - \widehat{Tci}} \qquad 133)$$

The parameter $\widehat{yNpt}$ in Eq. (129) indicates the corresponding heat exchanger configuration of the current candidate: $\widehat{yNpt} = 1$ when $\widehat{PNpt} = 1$ and zero, otherwise.

The determination of Tco and qhe are conducted through the minimization of the excess area (*Aexc*) subjected to the Eqs. (128) and (129). Additionally, it is necessary to add an additional constraint, in relation to the excess area:

$$Aexc \geq 0 \qquad 134)$$

3.2.3. Step 3: Further reduction the number of heat exchanger candidates for each thermal task

The current step in the proposed optimization procedure is based on the application of some constraints explored in Lemos et al. (2020) through Set Trimming. The constraints explored here are those that involve the cooling water flow rate, because now its value is known for each heat exchanger.

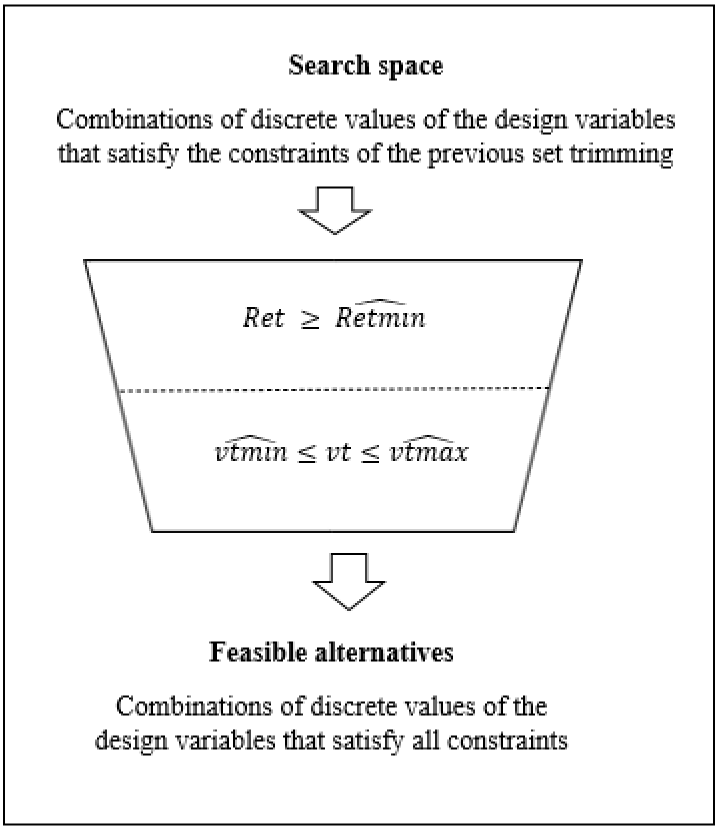
The set of constraints and the corresponding order in which they are applied in the Set Trimming are presented below (assuming that the hot stream flows in the shell-side). Aiming at reducing the computational effort, these constraints are ordered according to a crescent computational effort (therefore, the constraints that involve a higher computational effort are tested in a fewer number of candidates).

Set Trimming order:

1) Lower bound on Reynolds number in the tubes: Eq. (65).
2) Bounds on the flow velocity in the tubes: Eqs. (62-63)

The Figure 6 presents a schematic of how the set trimming procedure was performed.

Figure 6 - Schematic of set trimming procedure



Fonte: A autora, 2022

3.2.4. <u>Step 4: Identification of the optimal solution of the cooling water system in the remaining search space</u>

The final step is the solution of the MILP problem resultant from the extension of the original MILP problem in the previous chapter to consider different values of the outlet temperature and flow rate of the cooling water in each heat exchanger.

This new MILP problem is describe below. In order to avoid unnecessary repetition, the presentation will be focused on the equations that needed to be modified in relation to the previous MILP formulation. The rest of equations that are not modified in relation to the previous problem will only be mentioned.

The design problem variables of the current MILP problem are the pipe diameters of the pipe sections, the pump selection, the head losses of the valves associated with each heat exchanger to guarantee the hydraulic balance according to the design flow rate, and the heat exchanger geometry (tube length, tube diameter, tube layout, tube pitch ratio, number of baffles, shell diameter, and number of tube passes). In this model, the *srow* index is used again (see Chapter 2), but now the water temperature at the outlet of each exchanger receives this index as well ($\widehat{pTco}_{srow,l}$), since each heat exchanger geometry is associated with a calculated temperature.

The same indexes presented in Chapter 2 are used and the networks edges are also organized according to a set of independent hydraulic circuits ($l \in HY$).

Since the layout of the system and the necessary volumetric flow rate in each cooler are already established ($\widehat{qhe_{srow,l}}$), the flow rates along each network element (q_k) can be calculated by:

$$qhe_l = \sum_{srow} \widehat{qhe}_{srow,l}\, yrow_{srow,l} \qquad srow, l \in HY \tag{135}$$

$$q_k = \sum_{l \in HY} qhe_l \hat{\Lambda}_{l,k} \qquad k \in STR \tag{136}$$

The hydraulic behavior of the pipe network is represented by a mechanical energy balance, Eq. (5).

Meanwhile, the head loss in the heat exchangers are calculated as follows:

$$\Delta Pt_l = \sum_{srow} \widehat{\Delta Pt}_{srow,l}\, yrow_{srow,l} \qquad srow, l \in Retminfeas \tag{137}$$

$$fhe_l = \frac{\Delta Pt_l}{\hat{\rho}_w \hat{g}} \qquad l \in HY \tag{138}$$

where ΔPt_l is the pressure drop in the heat exchanger tube-side, and the head loss of pipe section k is calculated using the Hazen-Williams equation, Eq. (7), but now with q_k as a variable.

The pipe diameter is represented by Eq. (8) and the selection of an unique diameter for each pipe section is represented by Eq. (9).

The substitution of Eqs (8), (137) and (138) into Eq. (7) yields the following equation:

$$fpi_k = \sum_{srow} \sum_{n \in SD} \sum_{l \in HY} \frac{10.67}{\hat{c}^{1.85}} \frac{\hat{\Lambda}_{l,k} \widehat{qhe}_{srow,l}{}^{1.852}}{\widehat{D}_n^{int\,4.8704}} y_{k,n}^{pi}\, yrow_{srow,l} \quad k \in PI \tag{139}$$

The product of binary variables can be grouped in a continuous nonnegative variable: $wpi_{k,n,srow,l}$

$$wpi_{k,n,srow,l} = y_{k,n}^{pi}\, yrow_{srow,l} \qquad k \in PI \tag{140}$$

Therefore, Eq. (44) becomes:

$$fpi_k = \sum_{srow} \sum_{n \in SD} \sum_{l \in HY} \frac{10.67}{\hat{c}^{1.85}} \frac{\hat{\Lambda}_{l,k} \widehat{qhe}_{srow,l}{}^{1.852}}{\widehat{D}_n^{int\,4.8704}} wpi_{k,n,srow,l} \quad k \in PI \tag{141}$$

which is complemented by the following relations that represents the product of variables in a linear form:

$$wpi_{k,n,srow,l} \le y_{k,n}^{pi} \tag{142}$$

$$wpi_{k,n,srow,l} \le yrow_{srow,l} \tag{143}$$

$$wpi_{k,n,srow,l} \ge y_{k,n}^{pi} + yrow_{srow,l} - 1 \tag{144}$$

The head of a pump k is also represented by a linear relation, presented by Eq. (12) Because only one pump option must be chosen, it is used the Eq. (13).

In order to avoid erosion and fouling, maximum and minimum flow velocity bounds are imposed, Eqs. (14), (15) and (16). However, q_k is a variable in Eq. (16).

The substitution of Eqs (7), (137) and (138) into Eqs (14) and (15) yields the following equations:

$$\widehat{vmax} - \frac{4}{\pi}\sum_{srow}\sum_{n \in SD}\sum_{l \in HY}\frac{\hat{\Lambda}_{l,k}\,\widehat{qhe}_{srow,l}}{(\widehat{D}_n^{int})^2}y_{k,n}^{pi}\,yrow_{srow,l} \geq 0 \qquad k \in PI \qquad (145)$$

$$\widehat{vmin} - \frac{4}{\pi}\sum_{srow}\sum_{n \in SD}\sum_{l \in HY}\frac{\hat{\Lambda}_{l,k}\widehat{qhe}_{srow,l}}{(\widehat{D}_n^{int})^2}y_{k,n}^{pi}\,yrow_{srow,l} \leq 0 \qquad k \in PI \qquad (146)$$

The substitution of the product of binary variables yields:

$$\widehat{vmax} - \frac{4}{\pi}\sum_{srow}\sum_{n \in SD}\sum_{l \in HY}\frac{\hat{\Lambda}_{l,k}\,\widehat{qhe}_{srow,l}}{(\widehat{D}_n^{int})^2}wpi_{k,n,srow,l} \geq 0 \qquad k \in PI \qquad (147)$$

$$\widehat{vmin} - \frac{4}{\pi}\sum_{srow}\sum_{n \in SD}\sum_{l \in HY}\frac{\hat{\Lambda}_{l,k}\,\widehat{qhe}_{srow,l}}{(\widehat{D}_n^{int})^2}wpi_{k,n,srow,l} \leq 0 \qquad k \in PI \qquad (148)$$

According to engineering practice, the pipe diameter at the pump suction must be equal or higher than the diameter at the pump discharge, Eq. (19).

The temperature of the water that returns to the cooling water tower is, for each combination of temperatures of the water in the outlet of the heat exchangers, determined by the following equation:

$$\sum_{l \in HY} qhe_l(Tco_l - T_{return}) = 0 \qquad (149)$$

Equation (149) can be replaced by the following:

$$\hat{T}_{return}^{max}\,q_{return} \geq \sum_{l \in HY} qhe_l Tco_l \qquad (150)$$

$$\hat{T}_{return}^{min}\, q_{return} \le \sum_{l \in HY} qhe_l Tco_l \tag{151}$$

$$q_{return} = \sum_{l \in HY} qhe_l \tag{152}$$

Tco_l can be calculated by:

$$Tco_l = \sum_{srow} \widehat{pTco}_{srow,l} yrow_{srow,l} \tag{153}$$

where $\widehat{pTco}_{srow,l}$ is the temperature of the water in the outlet of each heat exchanger, which was obtained in a previous step of the algorithm.

The substitution of Eq. (137) and (153) in the equations (150) to (152) yields:

$$\hat{T}_{return}^{max}\, q_{return} \ge \sum_{srow} \sum_{l \in HY} \hat{\Lambda}_{l,k}\, \widehat{qhe}_{srow,l} \widehat{Tco}_{srow,l}\, yrow_{srow,l} \qquad 154)$$

$$\hat{T}_{return}^{min}\, q_{return} \le \sum_{srow} \sum_{l \in HY} \hat{\Lambda}_{l,k}\, \widehat{qhe}_{srow,l} \widehat{Tco}_{srow,l}\, yrow_{srow,l} \qquad 155)$$

$$q_{return} = \sum_{srow} \sum_{l \in HY} \hat{\Lambda}_{l,k}\, \widehat{qhe}_{srow,l}\, yrow_{srow,l} \qquad 156)$$

where q_{return} is the return flow rate to the cooling tower, $\hat{T}_{return}^{max}$ is the corresponding maximum temperature and $\hat{T}_{return}^{min}$ is the minimum temperature.

Since, only one option of heat exchanger must be select, then it is used Eq. (70).

3.2.5. Economic model

The annualized capital cost of the pipe sections is given by Eqs. (109) and (110). The annualized capital cost of the pump is given by Eqs. (111) to (115). The annualization factor is given by Eq. (116).

The operational costs associated to the pipe network operation is given by Eq. (117). The operational cost, cop_k, is given by:

$$cop_k = \left(\frac{q_{total}\hat{\rho}_w\hat{g}\ fpu_k}{\hat{\eta}}\right)\frac{\hat{N}_{OP}\widehat{pc}}{10^3} \qquad 157)$$

where q_{total} is defined by:

$$q_{total} = q_{return} \qquad 158)$$

The substitution of Eqs (12) and (158) in Eq. (157) yields:

$$cop_k = \sum_{srow}\sum_{s\in SPU}\sum_{l\in HY}\left(\frac{\hat{\Lambda}_{l,k}\ \widehat{qhe}_{srow,l}\hat{\rho}_w\hat{g}\ fpu_k}{\hat{\eta}}\right)\frac{\hat{N}_{OP}\widehat{pc}}{10^3}y^{pu}_{k,s}yrow_{srow,l} \qquad 159)$$

The product of binary variables can be grouped in a continuous nonnegative variable: $wpu_{k,s,srow,l}$

$$wpu_{k,s,srow,l} = y^{pu}_{k,s}\ yrow_{srow,l} \qquad k \in PU \qquad (160)$$

Therefore:

$$cop_k = \sum_{srow}\sum_{s\in SPU}\sum_{l\in HY}\left(\frac{\hat{\Lambda}_{l,k}\ \widehat{qhe}_{srow,l}\hat{\rho}_w\hat{g}\ fpu_k}{\hat{\eta}}\right)\frac{\hat{N}_{OP}\widehat{pc}}{10^3}wpu_{k,s,srow,l} \qquad 161)$$

Additionally:

$$wpu_{k,s,srow,l} \le y^{pu}_{k,s} \qquad (162)$$

$$wpu_{k,s,srow,l} \le yrow_{srow,l} \qquad (163)$$

$$wpu_{k,s,srow,l} \ge y^{pu}_{k,s} + yrow_{srow,l} - 1 \qquad (164)$$

The total annualized capital cost of the heat exchangers is given by Eqs. (119) to (121).

Because of the combinatorial explosion resultant from the fact that the flow rates and temperatures are not fixed in this problem, the dimension of the resultant MILP cannot be solved using conventional computational resources. This drawback was overcome through the decomposition of the MILP problem in two MILP subproblems (MILP 1 and MILP 2). The cooling water temperatures at the heat exchanger outlets were calculated in Step 2 and they appear in this step as parameters.

The problems MILP 1 and MILP 2 have the same equations and only differ in the parameters that are fixed in each iteration. The solution of the problem is conducted by a convergence loop involving the two MILP problems, as follows.

Initial estimate: Using the MILP from the problem in Chapter 2, the initial values for the pipe diameters are obtained, which will be used as input data for the next step.

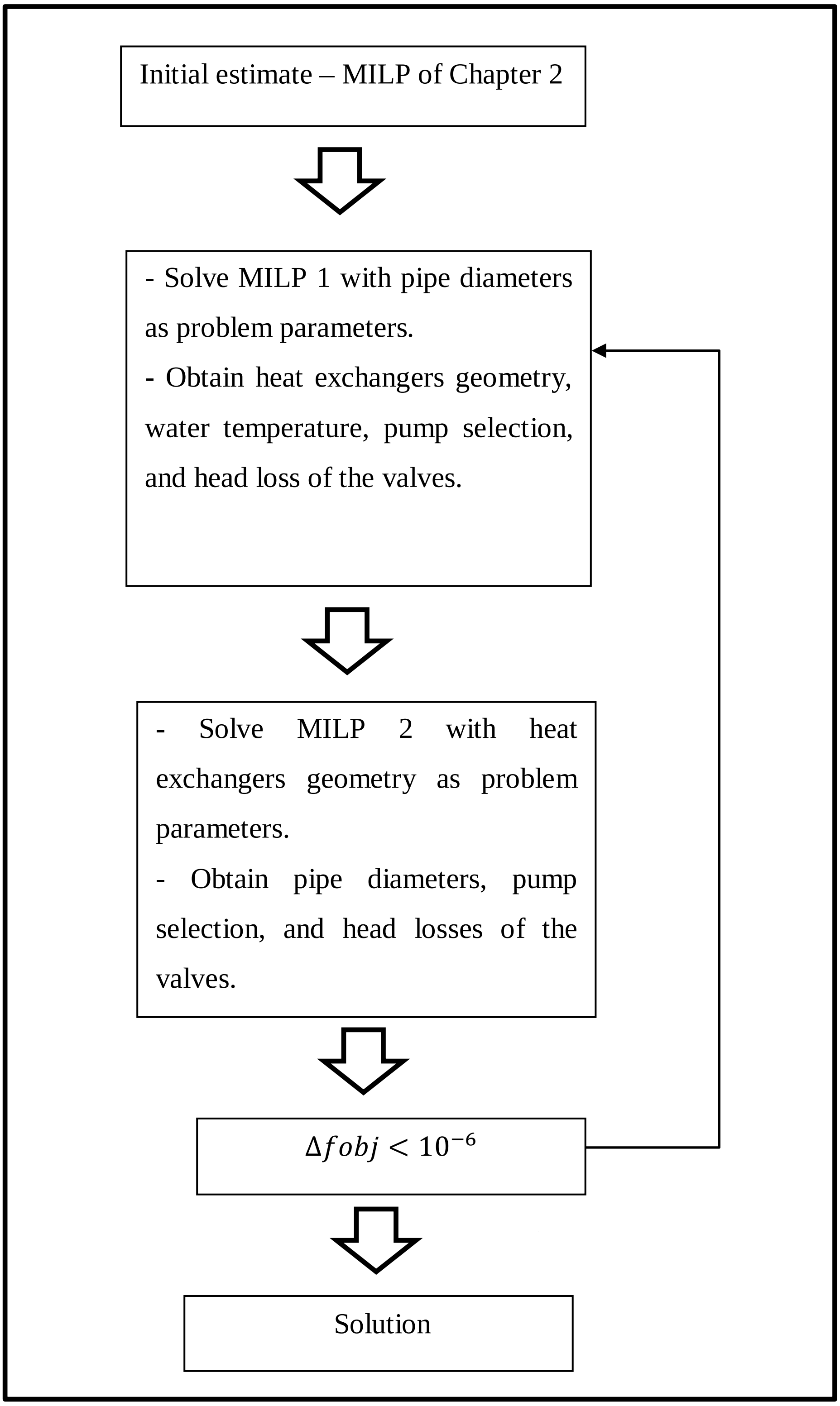
MILP 1: It uses as input data the pipe diameters obtained in the previous step. The solution of this MILP problem determines the geometry of the heat exchangers (tube length, tube diameter, tube layout, tube pitch ratio, number of baffles, shell diameter, and number of tube passes), outlet water temperature in the heat exchangers, water temperature in the return to the tower, pump selection and head loss of the valves.

MILP 2: It uses as input data the geometry of the heat exchangers (tube length, tube diameter, tube layout, tube pitch ratio, number of baffles, shell diameter, and number of tube passes) obtained in the previous step. The solution of this MILP problem determines the pipe diameters, pump selection, head loss of the valves and water temperature in the return to the tower.

Convergence criterion: If the variation of the objective function along an iteration is less than a tolerance value, it is considered that the problem has converged, otherwise, the algorithm returns to MILP 1.

Figure 7 presents a scheme of the iterative procedure.

Figure 7 - Scheme of the iterative procedure



Fonte: A autora, 2022

3.3. Results

In this problem, an optimization of a cooling water network is performed, including heat exchangers, piping and pump, with the temperature of the water return to the tower and at the outlet of the exchangers, as variables.

The investigated example presents similar characteristics of the network in Example 2 of the previous chapter. In this new problem, the cooling water temperature at the outlet of the exchangers and the return to the tower are not previously defined. The temperature range of the water return was 39 to 41^{o}C and the bounds on the water temperature at the outlet of the heat exchangers were 37 to 47^{o}C.

The data of the problem were the same of those presented in the previous chapter, with exception of the standard values of the discrete design variables of the coolers presented in Table 30. Additionally, the cooling water in heat exchanger he2 flows in the tube-side, according to the proposed formulation of the current optimization problem.

In order to verify the performance of the proposed procedure, the optimization of the same network was also carried out with the water temperature at the outlet of the exchangers and return to the tower fixed at 40 ^{o}C, using the proposed optimization procedure of the previous chapter.

Table 30 - Standard values of the discrete design variables of the coolers (KAKAÇ et al., 2012)

Variable	Values
Outer tube diameter, $\widehat{pdte}_{sd}$ (m)	0.019, 0.025, 0.032, 0.038, 0.051
Tube length, $\widehat{pL}_{sL}$ (m)	1.220,1.829,2.439, 3.049, 3.659, 4.877, 6.098
Number of baffles, $\widehat{pNb}_{sNb}$	17, 19, 20
Number of tube passes, $\widehat{pNpt}_{sNpt}$	1, 2, 4, 6
Tube pitch ratio, $\widehat{prp}_{srp}$	1.25, 1.33, 1.50
Shell diameter, $\widehat{pDs}_{sDs}$ (m)	0.48895, 0.6858, 0.889
Tube layout, $\widehat{play}_{slay}$	1 = square, 2 = triangular

Table 31 presents the number of equations and variables of the NLP problem associated with the solution of the system of equations for the determination of the outlet cooling water temperature for each heat exchanger service. Table 32 presents the number of equations and variables of MILP1 and Table 33 presents the number of equations and variables of MILP 2. The NLP problem for the determination of the water temperature at the outlet of each heat exchanger was solved to one heat exchanger service at a time. The attempt to solve the problem for all services simultaneously in a single problem was not successful because the available computer memory was not enough.

Table 34 presents the number of heat exchangers obtained after each constraint of the first Set Trimming step. Table 35 presents the number of heat exchangers obtained after each constraint of the second Set Trimming step.

Table 36 show the values of the objective function using the approach presented in the previous chapter and the total annualized cost obtained for each loop. These values show how the model converges.

Table 31 – Number of equations and variables of NLP (for each heat exchanger service)

Equations	3720
Variables	4006

Fonte: A autora, 2022

Table 32 – Number of equations and variables of MILP 1

Equations	1202722
Variables	401978
Discrete Variables	399892

Fonte: A autora, 2022

Table 33 – Number of equations and variables of MILP 2

Equations	1158224
Variables	387120
Discrete Variables	386082

Fonte: A autora, 2022

Table 34 – Number of heat exchangers obtained after each constraint of the first set trimming

Constraints	**Number of heat exchangers**
Initial	11340
tube length / shell diameter ratio	5400
baffle spacing	4320
water temperatures	3280
P temperature ratio	2648
heat tranfer area	2648
hot stream flow velocities	1062
hot stream Reynolds number	1062

Fonte: A autora, 2022

Table 35 – Number of heat exchangers obtained after each constraint of the second set trimming

Constraints	Number of heat exchangers
From previous set trimming	1062
Reynolds number in the tubes	1062
Flow velocity in the tubes	989

Fonte: A autora, 2022

Table 36 – Total annualized cost obtained for each loop

Loop	Cost ($/year)
Result of the problem of Chapter 2	86352.764
Loop 1	83267.556
Loop 2	81934.010
Loop 3	81934.010

Fonte: A autora, 2022

According to Table 34, it can be observed that the number of heat exchangers obtained at the end of the first Set trimming step represents only 9.4% of the initial number of possible design alternatives, a significant reduction in the problem domain. It is verified that the constraint related to heat transfer area did not reduce the number of heat exchangers obtained by the previous constraint (P temperature ratio). This can be explained by the fact that both constraints are based on the definition of the minimum water temperature value. The constraint related to the Reynolds number also did not show gains in relation to the velocity constraint. This could be because both constraints are directly related.

Table 35 indicates that the second Set Trimming step did not present a significant reduction in the number of candidates, the number of heat exchangers at the end of this step represents 93% of the total at the beginning of this step.

Considering the two Set Trimmings of the solution procedure, the number of heat exchangers at the end of the second Set Trimming step constitutes 8.7% of the initial number of options, which shows that Set Trimming was an effective way to reduce the problem domain.

Tables 37 to 40 show the heat exchanger design results, for the heat exchangers he1, he2, he3 and he4 respectively, for the model used in the previous problem and for the iterative method proposed here.

Tables 41 to 44 show the thermo-fluid dynamic results for the heat exchangers he1, he2, he3 and he4 respectively, for the model used in the previous problem and for the iterative method.

Table 45 presents the values for pump head and valve head losses, for both methods. Table 46 presents the diameters and head loss of the pipes.

Tables 47 and 48 show the water outlet temperature of each heat exchanger and the water temperature in the return to the tower, respectively, obtained by the iterative method.

Finally, Table 49 presents the annualized costs obtained by the simultaneous design and the iterative method.

Table 37 - Heat exchanger design results (he1)

	Results of Problem Chapter 2	**Iterative method**
Area (m^2)	123.465	106.869
Tube diameter (m)	0.016	0.016
Tube length (m)	3.049	3.049
Number of baffles	19	20
Number of tube passes	2	2
Tube pitch ratio	1.25	1.25
Shell diameter (m)	0.686	0.686
Tube layout	2	1
Total number of tubes	677	586
Baffle spacing (m)	0.152	0.145

Fonte: A autora, 2022

Table 38- Heat exchanger design results (he2)

	Results of Problem Chapter 2	**Iterative method**
Area (m^2)	248.822	197.491
Tube diameter (m)	0.016	0.016
Tube length (m)	3.659	4.877
Number of baffles	19	20
Number of tube passes	4	2
Tube pitch ratio	1.25	1.25
Shell diameter (m)	0.889	0.686
Tube layout	2	2
Total number of tubes	1137	677
Baffle spacing (m)	0.183	0.232

Fonte: A autora, 2022

Table 39 - Heat exchanger design results (he3)

	Results of Problem Chapter 2	Iterative method
Area (m^2)	197.491	186.744
Tube diameter (m)	0.016	0.022
Tube length (m)	4.877	3.659
Number of baffles	20	19
Number of tube passes	2	4
Tube pitch ratio	1.25	1.25
Shell diameter (m)	0.686	0.889
Tube layout	2	2
Total number of tubes	677	640
Baffle spacing (m)	0.232	0.183

Fonte: A autora, 2022

Table 40 - Heat exchanger design results (he4)

	Results of Problem Chapter 2	Iterative method
Area (m^2)	75.281	65.344
Tube diameter (m)	0.016	0.022
Tube length (m)	3.659	4.877
Number of baffles	17	20
Number of tube passes	2	2
Tube pitch ratio	1.25	1.25
Shell diameter (m)	0.489	0.489
Tube layout	2	1
Total number of tubes	344	168
Baffle spacing (m)	0.203	0.232

Fonte: A autora, 2022

Table 41 - Thermo-fluid dynamic results (he1)

	Results of Problem Chapter 2	Iterative method
Shell-side flow velocity (m/s)	0.972	1.020
Tube-side flow velocity (m/s)	1.151	1.889
Shell-side coefficient (W/(m^2K))	5112.722	4554.625
Tube-side coefficient (W/(m^2K))	5466.148	6211.044
Overall coefficient (W/(m^2K))	964.401	966.798
Shell-side pressure drop (Pa)	152531.523	120129.664
Tube-side pressure drop (Pa)	9507.376	18647.815

Fonte: A autora, 2022

Table 42 - Thermo-fluid dynamic results (he2)

	Results of Problem Chapter 2	Iterative method
Shell-side flow velocity (m/s)	1.140	1.164
Tube-side flow velocity (m/s)	1.027	1.232
Shell-side coefficient (W/(m^{2}K))	2681.511	2712.462
Tube-side coefficient (W/(m^{2}K))	4990.902	5016.304
Overall coefficient (W/(m^{2}K))	748.886	751.973
Shell-side pressure drop (Pa)	152590.609	128360.859
Tube-side pressure drop (Pa)	17862.512	17342.789

Fonte: A autora, 2022

Table 43 - Thermo-fluid dynamic results (he3)

	Results of Problem Chapter 2	Iterative method
Shell-side flow velocity (m/s)	1.198	1.174
Tube-side flow velocity (m/s)	1.189	1.825
Shell-side coefficient (W/(m^{2}K))	1314.038	1141.304
Tube-side coefficient (W/(m^{2}K))	5611.315	6368.050
Overall coefficient (W/(m^{2}K))	589.410	570.857
Shell-side pressure drop (Pa)	194116.592	163957.252
Tube-side pressure drop (Pa)	14798.491	38319.815

Fonte: A autora, 2022

Table 44 - Thermo-fluid dynamic results (he4)

	Results of Problem Chapter 2	Iterative method
Shell-side flow velocity (m/s)	0.652	0.571
Tube-side flow velocity (m/s)	1.084	1.610
Shell-side coefficient (W/(m^{2}K))	4106.230	2907.571
Tube-side coefficient (W/(m^{2}K))	5210.413	5042.899
Overall coefficient (W/(m^{2}K))	912.645	856.959
Shell-side pressure drop (Pa)	47533.401	21248.927
Tube-side pressure drop (Pa)	9850.440	14253.700

Fonte: A autora, 2022

ble 45 - Pump head and valve head losses

Design	Pump head (m)	Head loss he1(m)	Head loss he2(m)	Head loss he3(m)	Head loss he4(m)
Results of Problem Chapter 2	6	0.458	0.017	0.004	0.075
Iterative method	6	0.674	0.563	0.004	0.169

Fonte: A autora, 2022

Table 46 - Diameters and head loss of the pipes

Pipe section	Results of Problem Chapter 2		Iterative method	
	Diameter (in)	**Head loss (m)**	**Diameter (in)**	**Head loss (m)**
pi1	18	0.963	16	0.683
pi2	8	0.507	8	0.522
pi3	16	0.177	16	0.099
pi4	10	0.123	8	0.520
pi5	10	0.123	8	0.520
pi6	8	0.507	8	0.522
pi7	14	0.202	16	0.084
pi8	8	0.194	8	0.196
pi9	10	0.119	14	0.044
pi10	12	0.050	14	0.044
pi11	8	0.108	6	0.414
pi12	8	0.108	8	0.109
pi13	8	0.194	6	0.745
pi14	14	0.202	16	0.084
pi15	16	0.177	16	0.099
pi16	20	0.579	18	0.392
pi17	20	0.012	16	0.014

Fonte: A autora, 2022

Table 47 – Water outlet temperature

Heat exchanger	he1	he2	he3	he4
Water outlet temperature (^{o}C)	39.8	38.3	38.4	39.9

Fonte: A autora, 2022

Table 48 – Water temperature in the return to the tower

Water temperature in return to the tower (^{o}C)	39.0

Fonte: A autora, 2022

Table 49 - Annualized costs

Cost ($/year)	Results of Problem Chapter 2	Iterative method
Pump cost	1455.215	1455.214
Heat exchanger cost	48485.605	43117.266
Pipe cost	15672.868	14339.323
Operation cost	20739.075	23022.207
Total cost	86352.764	81934.010

Fonte: A autora, 2022

Tables 47 indicates that cooling water outlet temperatures lower than 40 ^{o}C (fixed value used in the problem in Chapter 2) were selected at the optimal solution for the heat exchangers. This logically entails a higher water flow rate, which ultimately generates a higher operational cost. This can be explained by the fact that the optimization has sought water temperatures in the heat exchangers outlet so that the increase in operational costs was offset by the decrease in the costs of the heat exchangers. By choosing lower temperatures, the optimization sought to reduce the size of the heat exchangers, but without this having a strong impact on the system pressure drop, as the same pump was selected (Table 45).

The decrease in water temperature leads to an increase in flow in the pipes, so it would be expected that the optimization would choose larger diameter pipes in the iterative method, but as verified by the cost value of the pipes, the optimization selected smaller diameters for some pipes (in sections: pi1, pi4, pi5, pi11, pi16 and pi17) than those chosen by the model in Chapter 2. The sections pi1 and pi16 are the longest in the system, when choosing smaller diameters for these sections, there is an effective reduction of the cost. The choice of smaller piping diameters, as well as the choice of smaller exchangers results in greater head loss, but even so, the optimization managed to respect the limit given by the head of the same pump. Although the resulting operating cost was higher in the iterative method, the decrease in costs in choosing exchangers and pipe diameters outweighed the increase in operating costs, resulting in a lower total cost in the iterative method. Observing the values of the total annualized cost presented in Table 49, it can be seen that the iterative method showed a cost improvement of 5.2% in relation to the simpler model of Chapter 2.

4. **OPTIMIZATION OF A MECHANICAL INDUCED DRAFT COUNTER FLOW COOLING WATER TOWER**

In this problem, the optimization of a mechanic induced draft counter flow cooling water tower is performed through the use of Set Trimming and Smart Enumeration. As defined by Carvalho et al. (2019), the Smart Enumeration is based on a search of the alternatives organized according to an increasing order of a given lower bound on the objective function, therefore, when the current lower bound on the objective function is larger or equal to the objective function of the incumbent solution, the procedure stops. Because of the prior ordering, no solution can be found that has lower cost. Therefore, the proposed approach always attains the global optimum.

In order to reduce the computational effort, before the Smart Enumeration procedure, a Set Trimming step was performed to reduce the number of solution candidates. A possible alternative approach to solve this problem, without using enumeration techniques, would be through mixed-integer nonlinear programming (MINLP), which would be affected by convergence difficulties. The utilization of metaheuristic methods would be more robust, but it cannot guarantee global optimality.

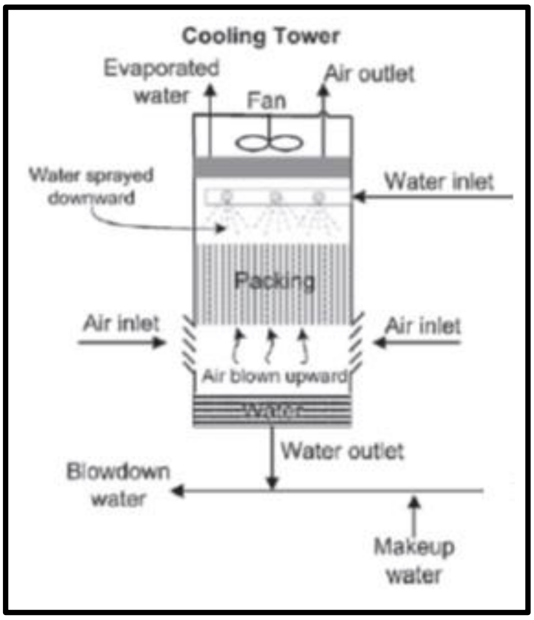
4.1. Problem description

The problem focuses on the design of a mechanic induced draft counter flow cooling water tower. The discrete variables of the problem that characterize each solution candidate are: height of the filling (Δ_{pack}), cross sectional area of the tower (Atw), number of cells (N_{cell}), type of filling and the total dry air mass flow rate (m_{airT}). The flow of humid air, the fraction of evaporated water and the water temperature at the outlet of the tower are continuous variables that can be calculated for each set of values of the discrete variables.

The goal of the optimization is to minimize the total annualized cost, encompassing capital and operating costs. The problem solution must be able to cool the cooling water flow rate ($\hat{q}_w$) to a maximum outlet temperature ($\widehat{Tbot}^{w}_{max}$).

Figure 8 shows a schematic of a countercurrent cooling water tower with induced air draft.

Figure 8 – Counter flow water cooling tower



Reference: PONCE-ORTEGA et al., 2010

4.2. Solution procedure

The proposed solution procedure involves two steps, as follows (a more detailed description of each step is presented later):

1) Application of Set Trimming to reduce the number of cooling tower candidates.
2) Identification of the optimal solution of the cooling tower in the remaining search space using Smart Enumeration

4.2.1. <u>Step 1: Reduction the number of cooling tower candidates</u>

Set Trimming is used in this step to reduce the number of cooling tower candidate solutions. The unique constraints explored to reduce the number of candidates are the bounds on the mass flux of cooling water in the tower, according to the engineering practice:

$$1.8 \leq \frac{\hat{\rho}_w \hat{q}_w}{Atw\ N_{cell}} \leq 2.8 \qquad (165)$$

4.2.2. <u>Step 2: Identification of the optimal solution of the cooling water tower in the remaining search space</u>

4.2.2.1. Cooling tower model

The cooling water tower is divided into cells. The flow rate of each cell can be defined according to the equation below.

$$q_{cell} = \frac{\widehat{qw}}{N_{cell}} \qquad (166)$$

The mass air flow rate in each cell can be defined by the equation below:

$$m_{air} = \frac{m_{airT}}{N_{cell}} \qquad (167)$$

where m_{airT} is the total mass air flow rate of the cooling tower.

During the Smart Enumeration, solution candidates must be checked to verify its feasibility ($Tbot^w < \widehat{Tbot}^w_{max}$). This task involves the solution of the mathematical model depicted below based on the Merkel model (SINGHAM, 1983).

According to the Merkel equation (3):

$$I_P = I_M \tag{168}$$

where I_M is the Merkel integral and I_P is the cooling tower characteristic.

The Merkel integral can be evaluated by a numerical integration as shown by:

$$I_M = \widehat{Cp}_w \frac{\widehat{Ttop}^w - Tbot^w}{4} \sum_{m=1}^{4} \frac{1}{\widehat{H}_m^{sat} - H_m} \tag{169}$$

where $\widehat{H}_m^{sat}$ and H_m are the enthalpy of air in the saturation and bulk conditions, respectively, $\widehat{Ttop}^w$ is the temperature of the water entering the cell of the cooling tower, and $Tbot^w$ is the temperature of the water leaving the cell of the cooling water tower.

The enthalpy of the air at the quadrature points $m = 1,\ldots,4$ can be evaluated through an energy balance in the tower:

$$H_m = \widehat{H}^{atm} + \widehat{Cp}_w \frac{\hat{\rho}_w q_{cell}}{m_{air}} \left(T_{w,m} - Tbot^w\right) \tag{170}$$

where $\widehat{H}^{atm}$ is the air enthalpy at the ambient conditions, m_{air} is the mass flow rate of the air in the cell of the cooling tower, and $T_{w,m}$ is the cooling water temperature along the quadrature points ($\hat{\alpha}_1 = 0.1, \hat{\alpha}_2 = 0.4, \hat{\alpha}_3 = 0.6, \hat{\alpha}_4 = 0.9$), which is presented below.

$$T_{w,m} = Tbot^w + \left(\widehat{Ttop}^w - Tbot^w\right)\hat{\alpha}_m \qquad m = 1, \ldots, 4 \tag{171}$$

The characteristic of the cooling tower depends on the ratio between the water and air flow rates:

$$I_P = \Delta_{pack}\, \hat{c}_{un} \left(\frac{\hat{\rho}_w q_{cell}}{m_{air}}\right)^{-\hat{n}} \tag{172}$$

where $\boldsymbol{\Delta_{pack}}$ is the height of the filling, and $\hat{n}$ and $\hat{c}_{un}$ are model parameters that depend on the filling type.

The energy balance in the cell of the tower can be defined by:

$$m_{air}\left(H_{out} - \widehat{H}_{in}\right) = \widehat{Cp}_w(\widehat{Ttop}^w - \ Tbot^w)\hat{\rho}_w q_{cell}$$

173)

The temperature of satured air, in the outlet of the cell of the tower, can be obtained using the expression adjusted by Serna-Gonzáles et al. (2010):

$$H_{out} = \left[\hat{a} + \hat{b}\,T_{out}^{air} + \hat{c}exp\left(\hat{d}\,T_{out}^{air}\right)\right]10^3$$

174)

where $\hat{a}$ = -6.38887667, $\hat{b}$ = 0.86581791, $\hat{c}$ = 15.7153617, and $\hat{d}$ = 0.05439778.

The Antoine equation applied below represents the correlation between the vapor pressure of the water in the cell of the tower outlet (P_{out}^{sat}) and the temperature of satured air in the outlet of the cell of tower (T_{out}^{air}) (SMITH et al., 2018).

$$P_{out}^{sat} = 133\left(10^{8.07131 - \frac{1730.63}{T_{out}^{air} + 233.426}}\right)$$

175)

where T_{out}^{air} is in Celsius degrees and P_{out}^{sat} is in Pa.

Assuming that the air is saturated at the tower outlet, the corresponding value of the humidity can be obtained by placing the vapor pressure in the Raoult's Law equation:

$$W_{out} = \frac{P_{out}^{sat}\,\widehat{M}_w}{\left(\hat{P} - \ P_{out}^{sat}\right)\widehat{M}_{air}}$$

176)

where $\hat{P}$ is the local atmospheric pressure, $\widehat{M}_w$ and $\widehat{M}_{air}$ are the molar mass of the water and air, respectively, and W_{out} is the air humidity at the outlet of the cell of the tower, defined in kg of water vapor per kg of dry air.

The density of the humid air at the outlet of the cooling tower cell (ρ_{out}) can be calculated using the ideal gas law (SERNA-GONZALES et al., 2010):

$$\rho_{out} = \frac{\hat{P}}{287.08\ \ (T_{out}^{air} + 273)}\left(1 - \frac{W_{out}}{W_{out} + 0.62198}\right)(1 + W_{out})$$

177)

The average wet air density ρ_{hum} is obtained from the harmonic mean of the densities of the air in the filling (SERNA-GONZALES et al., 2010):

$$\rho_{hum} = 2/\left(\frac{1}{\hat{\rho}_{in}} + \frac{1}{\rho_{out}}\right)$$

178)

The average flow of humid air is calculated using:

$$m_{hum} = m_{air} + m_{air}\left(\frac{W_{out} + \widehat{W}_{in}}{2}\right)$$

179)

The mass flowrate of humid air at outlet of the cell is given by:

$$m_{out} = m_{air} + m_{air}W_{out}$$

180)

Assuming that the area delimited by the fan ring is 50% of the cross-sectional area of the cooling tower cell, the velocity of the humid air, at the outlet of the cell, is calculated by:

$$vair_{out} = 2\,\frac{(m_{out}/\,\rho_{out})}{Atw}$$

(

181)

The relation between the outlet mass and volumetric flow rates is:

$$m_{out} = \ \rho_{out}qair_{out}$$

182)

The mechanical energy balance in the tower considers the pressure loss in the cell, by the filling and other components, as well as the gain in kinetic energy, which must be counterbalanced by the head pressure of the fan.

$$\Delta P_t = \Delta P \;+\; \rho_{out}\,\frac{{vair_{out}}^2}{2} \tag{183}$$

The head loss in the cooling tower filling (for each cell) is calculated using Eq. (184) (KLOPPERS; KROGER, 2003).

$$\Delta P_{pack} = Kfi\;\Delta_{pack}\;\frac{m_{hum}^2}{2\rho_{hum}Atw^2} \tag{184}$$

where Kfi is the pressure drop coefficient of the filling.

The pressure drop coefficient of the filling can be calculated using the following equation (KLOPPERS; KROGER, 2003):

$$Kfi = \left[\hat{d}_1\left(\frac{\hat{\rho}_w q_{cell}}{Atw}\right)^{\hat{d}_2}\left(\frac{m_{hum}}{Atw}\right)^{\hat{d}_3} + \hat{d}_{4_f}\left(\frac{\hat{\rho}_w q_{cell}}{Atw}\right)^{\hat{d}_5}\left(\frac{m_{hum}}{Atw}\right)^{\hat{d}_6}\right]\Delta_{pack} \tag{185}$$

in which $\hat{d}_1, \hat{d}_2, \hat{d}_3, \hat{d}_4, \hat{d}_5\ and\ \hat{d}_6$ are empirical coefficients for each filling.

The miscellaneous pressure losses encompass drift eliminators, air inlet, water distribution piping, column supports are calculated by (MILLS, 1999):

$$\Delta P_{misc} = \widehat{Kmis}\;\rho_{hum}\;\frac{{v_{hum}}^2}{2} \tag{186}$$

where:

$$v_{hum} = \frac{(m_{hum}/\,\rho_{hum})}{Atw} \tag{187}$$

Adding the sources of head loss, the total head loss is obtained, expressed by:

$$\Delta P = \Delta P_{misc} + \Delta P_{pack} \tag{188}$$

The fraction of evaporated water in the cooling tower cell can be calculated by the difference between the air humidity at the outlet and inlet of the tower:

$$mvap = m_{air}\left(W_{out} - \widehat{W}_{in}\right)$$

189)

The tower blowdown of the cell can be calculated by:

$$mbld = \frac{mvap}{\hat{n}_{cycles} - 1}$$

190)

where $\hat{n}_{cycles}$ is the number of concentration cycles (ratio between the concentration of solids in the circulating water and the concentration of solids in the make-up water) of the cooling tower.

The loss of water from the tower cell (mass flow), in the form of splashes (drift), can be evaluated by:

$$mdrift = 0.0005\hat{\rho}_w q_{cell}$$

191)

The water make up is defined by the sum of all tower water losses.

$$mmk = mdrift + mvap + mbld$$

192)

4.2.2.2. Economic model

In this item, the economic equations of the cooling tower model will be presented.

The power consumed by the fan (for each cell) of the induced air tower is defined by the equation below (SERNA-GONZÁLEZ, 2010).

$$Pot_{fan} = \frac{qair_{out}\Delta P_t}{\hat{\tau}_{fan}}$$
193)

where $\hat{\tau}_{fan}$ is the efficiency of the fan.

The annualized investment cost associated with the cooling tower is calculated by (KINTNER-MEYER; EMERY, 1995):

$$C_{cap} = N_{cell}\hat{r}\left(\hat{C}_{tower} + \hat{C}_{pack}Atw\Delta_{pack} + \hat{C}_{air}m_{air}\right)$$
194)

where $\hat{C}_{tower}$ is the investment cost of the cooling tower, $\hat{C}_{pack}$ *and* $\hat{C}_{air}$ are the costs of the cooling tower associated with the filling and air flow, respectively. The cost associated with the filling depends on the type of filling and $\hat{r}$ is the annualizing factor.

The fan operation cost is defined by the following equation as a function of the fan power and the number of hours operated per year.

$$C_{opf} = N_{cell}\widehat{N_{OP}}\,\widehat{pc}\,\frac{Pot_{fan}}{1000}$$
195)

The cost of consumption of make up water is calculated based on the water losses and the price of water.

$$C_{mk} = N_{cell}3600\widehat{N_{OP}}\hat{C}_w mmk$$
196)

where $\hat{C}_w$ is the cost of water.

4.2.2.3. Objective function

The objective function is to minimize the total annualized cost, including cooling tower costs, and operational cost associated with the fan and make up water consumption.

$$\min Ct = C_{cap} + C_{mk} + C_{opf} \quad (197)$$

4.2.3. Smart Enumeration

Each cooling tower candidate has a different combination of design variables. The Set Trimming step reduces the number of candidates, which will form the remaining search space to be explored using the Smart enumeration procedure.

The Smart Enumeration procedure starts by ordering the remaining candidates according to an increasing value of a lower bound of the objective function. This lower bound must be built and its computational cost has to be the lowest possible, without compromising the gap between its value and the rigorous solution too much. Then, the candidates are evaluated one by one, that is, the full rigorous model is solved for each candidate following the increasing lower bound order. When a candidate is feasible, the corresponding objective function is evaluated and, if this value is lower than the current incumbent objective function, the incumbent is updated (i.e. it is an upper bound on the optimal solution). The procedure continues until the current candidate's lower bound becomes larger than the objective function of the incumbent. This guarantees that the global optimum was found (NAHES, 2022).

The model presented above is a non-linear problem, which will be solved within the enumeration procedure depicted below. The resolution of the model corresponds to the simulation of the cooling tower to check if the outlet temperature is feasible in relation to the design specification ($Tbot^{w} < \widehat{Tbot}_{max}^{w}$). If the solution candidate is feasible, then the objective function is evaluated to verify if the incumbent can be updated.

Enumeration steps:

1) Select the cooling towers from the Set Trimming step
2) Calcule a lower bound of the objective function of all the candidates
3) Put candidates in ascending order of the lower bound of the objective function
4) Pick the candidate with the lowest value of objective function lower bound

5) Test the feasibility of the selected candidate through the resolution of the tower model
6) If the candidate is feasible, then calculate the objective function and go to Step 7, otherwise, discard this candidate, and go to Step 8
7) If the objective function of the current candidate is smaller than the incumbent objective function, it becomes the new incumbent.
8) Pick the next candidate in the lower bound list of the objective function
9) If the objective function of the incumbent is smaller than or equal to the lower bound of the selected candidate, then stop, the incumbent is the solution to the problem, otherwise, go to Step 5

The lower bound of the objective function encompasses a lower bound of the annualized capital costs and the make-up water consumption costs:

$$Clb = C_{caplb} + C_{mklb}$$

198)

The lower bound on the water consumption includes only the replacement water consumption corresponding to the drift from each cell of the cooling tower. That is, the equation for calculating the mass flow rate of the make up water (for each cell) is:

$$mmklb = 0.0005\hat{\rho}_w q_w$$

199)

The lower bound on the cost of make up water is then:

$$C_{mklb} = 3600\widehat{N_{OP}}\hat{C}_w mmklb$$

200)

The lower bound on the annualized investment is calculated by:

$$C_{caplb} = N_t \hat{r}\left(\hat{C}_{tower} + \hat{C}_{pack}\, Atw\, \Delta_{pack}\right)$$

201)

4.3. Results

One example of cooling water design was solved, with cooling water flow rate of 0.2 m^3/s, associated with an outlet temperature of the water leaving the tower below 31 ºC. The inlet temperature of the water in the tower was 41 ºC.

Table 50 presents the cost related parameters, obtained from Serna-Gonzalez et al. (2010). Table 51 shows the number of concentration cycles, fan efficiency and miscellaneous pressure loss constant (SERNA-GONZALEZ et al., 2010). Table 52 shows the air related parameters. Table 53 shows the parameters related to the environment.

Table 54 presents the available options of the design variables (the tower area and height options were obtained from the Alpina website (ALPINA EQUIPAMENTOS, 2021)). Table 55 presents the empirical coefficients for each filling option (SERNA-GONZALEZ et al., 2010). Table 56 presents the water related parameters.

Table 50 – Cost related parameters (SERNA-GONZÁLEZ et al., 2010)

Parameter	Value
Investment cost of the cooling tower, $\hat{C}_{tower}$ (USD)	31850
Costs of the cooling tower associated with the filling, $\hat{C}_{pack}$ (USD)	2006.6
Costs of the cooling tower associated with mass air flow, $\hat{C}_{air}$ (USD)	1097.5
Water cost, $\hat{C}_w$ (USD/kg)	$5.2 \cdot 10^{-4}$
Energy cost, $\widehat{pc}$ (USD/kWh)	0.1308
Interest rate, $\hat{\imath}$	0.05

Fonte: A autora, 2022

Table 51 - Problem prameters

Parameter	Value
Number of concentration cycles, $\hat{n}_{cycles}$	4
Fan efficiency, $\hat{\tau}_{fan}$	0.80
Miscellaneous pressure loss constant, $\widehat{Kmis}$	6.5

Fonte: A autora, 2022

Table 52 – Air related parameters

Parameter	Value
Air enthalpy under ambient conditions, $\widehat{H}_{in}$ (J/kg)	64667
Inlet air temperature in the tower, $\hat{T}_{in}^{air}$ (ºC)	25
Intlet air humidity, $\widehat{W}_{in}$ (kg water/kg dry air)	0.016
Inlet humid air density, $\hat{\rho}_{in}$ (kg/m^3)	1.122

Fonte: A autora, 2022

Table 53 - Parameters related to the environment

Parameter	Value
Wet bulb temperature, $\widehat{TWB}_{in}$ (°C)	22
Atmospheric pressure, $\hat{P}$ (Pa)	101325

Fonte: A autora, 2022

Table 54 - Available options of the design variables (ALPINA EQUIPAMENTOS, 2021)

Input data	Cooling tower
Filling height (m)	2.280, 2.550, 2.580, 2.635, 3.110, 2.590, 2.610, 2.620
Cross sectional area (m^2)	8.49, 8.34, 10.75, 19.61, 25.81, 32.00
Number of cells	1, 2, 3, 4
Filling types	Splash, Film
Dry air flowrate (kg/s)	90, 95, 100, 105, 110, 115, 120, 125, 130, 135

Fonte: A autora, 2022

Table 55 - Empirical coefficients for each filling options (SERNA- GONZÁLEZ et al., 2010)

Coefficients	Splash fill	Film fill
$\hat{d}_1$	3.179688	3.897830
$\hat{d}_2$	1.083916	0.777271
$\hat{d}_3$	-1.965418	-2.114727
$\hat{d}_4$	0.639088	15.327472
$\hat{d}_5$	0.684936	0.215975
$\hat{d}_6$	0.642767	0.079696
$\hat{c}_{un}$	0.309	0.691
$\hat{n}$	0.5	0.69

Fonte: A autora, 2022

Table 56 – Water related parameters (SERNA-GONZÁLEZ et al., 2010)

Parameter	Value
Water specific density, $\hat{\rho}_w$ (kg/m^3)	995
Water specific heat capacity, $\widehat{Cp}_w$ (J/kg.K)	4187

Fonte: A autora, 2022

Table 57 presents the number of equations and variables of the cooling tower model. The number of equations and variables is relatively small, because in the Smart Enumeration procedure the model is solved only for one tower at a time. Table 58 presents the number of cooling towers alternatives before and after the Set Trimming step. It is verified that the number of towers selected by Set Trimming represents only 16.7% of the initial dimension of the search space.

Table 57 – Number of equations and variables of each NLP of cooling tower

Equations	3 4
Variables	3 4

Fonte: A autora, 2022

Table 58 – Number of cooling towers obtained before and after the Set Trimming

Constraints	Number of cooling towers
Before Set Trimming	3840
After Set Trimming	640

Fonte: A autora, 2022

The design results are presented in the Tables 59-63. Table 59 shows the optimal values of the design variables. Table 60 shows the water flowrates. Table 61 shows the hydraulic results of each cell. Table 62 shows the air flow variables. Table 63 presents the annualized costs. The optimal value of the cooling water outlet temperature is 25.08 ºC.

Table 59 – Optimal design variables

Variable	Value
Cross sectional area of the cell (m^2)	25.81
Cell height of each cell (m)	2.280
Type of filling	Splash
Total dry air flowrate (kg/s)	135
Number of cells	3

Fonte: A autora, 2022

Table 60 - Water flowrates for the all cells of the cooling tower

Water flow rate	Value
Evaporated (kg/s)	2.051
Blowdown (kg/s)	0.684
Drift (kg/s)	0.100
Make up (kg/s)	2.834

Fonte: A autora, 2022

Table 61 – Hydraulic results of each cell

Variable	Value
Filling pressure drop (Pa)	34.38
Miscellaneous pressure drop (Pa)	9.33
Gain in kinetic energy (Pa)	5.91
Fan pressure variation (Pa)	49.62
Fan power (W)	2628

Fonte: A autora, 2022

Table 62 - Air flow stream results

Variable	Value
Total humid outlet air flowrate (kg/s)	139.2
Outlet temperature (^oC)	39.55
Outlet air humidity (kg water/kg dry air)	0.031
Outlet humid air density (kg/m^3)	1.108
Outlet air enthalpy (J/kg)	162937

Fonte: A autora, 2022

Table 63 - Annualized costs

Cost	Value
Tower investment ($/year)	77438.36
Make up water ($/year)	47183.03
Fan Operation ($/year)	9034.88
Total cost ($/year)	133656.26

Fonte: A autora, 2022

5. OPTIMIZATION OF A COOLING WATER SYSTEM CONSISTING OF A COOLING TOWER, HEAT EXCHANGERS, PIPE SECTIONS AND PUMP

The fourth problem of the thesis aims to optimize the cooling water tower and the cooling water network simultaneously, using the models previously described. This proposal will attain the higher costs reduction.

5.1. Problem formulation

The optimization problem corresponds to the minimization of the total annualized cost of a cooling water system, encompassing the heat exchangers, the pipe sections of the network for cooling water distribution, the pump and a mechanic induced draft counterflow cooling water tower.

The problem design variables are the pipe diameters of the pipe sections, the pump size, the head losses of the valves associated with each heat exchanger, the heat exchanger geometry (tube length, tube diameter, tube layout, tube pitch ratio, number of baffles, shell diameter, and number of tube passes), the mass flow rate of the cooling water in each heat exchanger and in each pipe section, the temperatures of the water streams (outlet of the heat exchangers and top and bottom cooling tower temperatures), the tower filling height, the cross sectional area of the each tower cell, the number of cells, the type of the filling and the air flow rate.

The objective function is the total annualized cost of the system, encompassing heat exchangers (investment cost), pipe sections (investment cost), pump (investment and operating costs), and cooling tower (investment and operating costs).

5.2. Optimization approach

The proposed solution procedure is based on the recursive solution of two optimization problems previously presented: (1) Design optimization problem of the cooling tower through application of Set Trimming and Smart Enumeration - Procedure presented in Chapter 4 with a additional constraint that establishes that the bottom temperature must be within a given interval (here called P1); and (2) Design optimization of a cooling water system - Procedure presented in Chapter 3 (here called P2).

The procedure involves the following steps:

1) Split of the search space related to the top cooling tower temperature (i.e. the return stream of water to the tower) and the bottom cooling tower temperature (i.e. the stream of water leaving the tower and feeding heat exchangers) in a set of intervals.
2) Let S be a set of pairs of intervals of top and bottom cooling tower temperatures considering all possible combination of the discretized values.
3) Select a pair of temperature intervals, *TI,* from the set S.
4) Let $T_0 = (T_{top}, T_{bottom})$ be an initial pair of values of the top and bottom cooling tower temperatures within the interval (e.g. the middle point of the interval)
5) Set a iteration counter $t = 1$
6) Set $T_t = T_{t-1}$
7) Solve the problem P1 using T_{top} subjected to the bottom cooling tower temperature bound associated with *TI*.
8) Update T_t with the optimal value of the bottom cooling tower temperature obtained in the previous step.
9) Solve the problem P2 using T_{bottom} subjected to the top cooling tower temperature bound associated with *TI*.
10) Update T_t with the optimal value of the top cooling tower temperature obtained in the previous step.
11) If the difference between the top and bottom cooling tower temperatures of T_t and T_{t-1} is higher than the tolerance, go to Step 6.
12) Obtain the value of the final objective function within the interval *TI* through the sum of the objective functions of P1 and P2.
13) Eliminate *TI* from the Set S.
14) If $S \neq \varnothing$, then go to Step 3
15) Identify the lowest cost solution among the combinations of cooling tower bottom and top temperatures.

5.3. Results

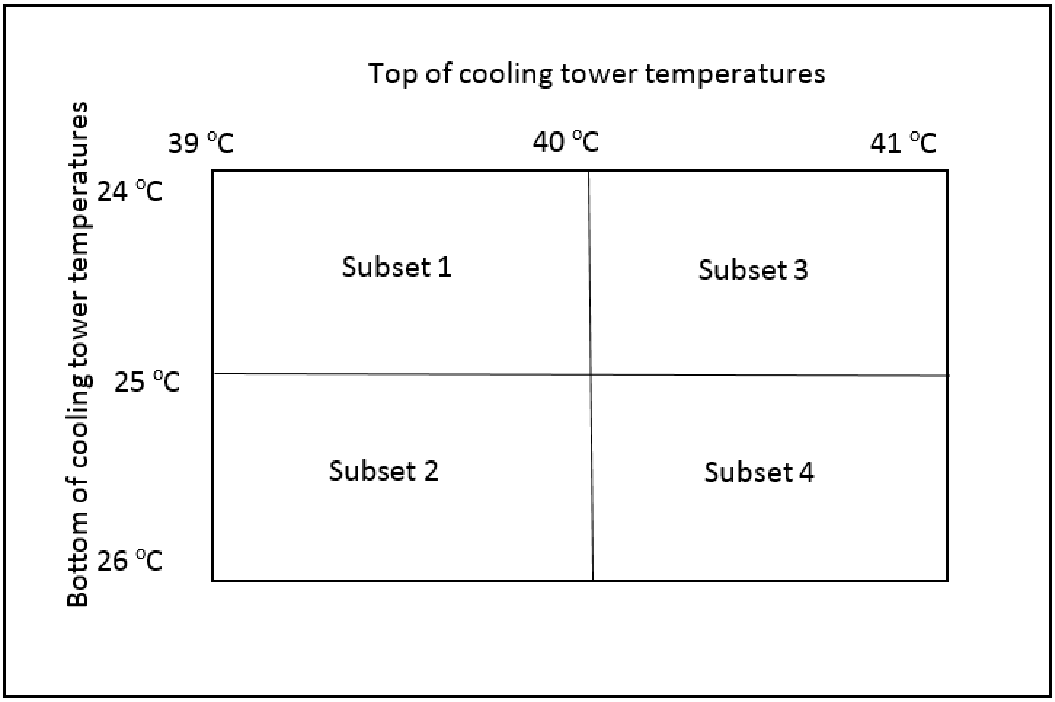
The optimization of a cooling water system consisting of the cooling water tower, pump, pipes and four heat exchangers was carried out.

The design example data for the cooling tower correspond to the following tables: Table 50 presents the cost related parameters, obtained from Serna-Gonzalez et al. (2010); Table 51 shows the number of concentration cycles, fan efficiency and miscellaneous pressure loss constant (SERNA-GONZALEZ et al., 2010); Table 52 shows the air related parameters; Table 53 shows the parameters related to the environment; Table 54 presents the available options of the design variables (the tower area and height options were obtained from a Brazilian vendor of cooling towers (ALPINA EQUIPAMENTOS, 2021)); Table 55 presents the empirical coefficients for each filling option (SERNA-GONZALEZ et al., 2010);and Table 56 presents water related parameters.

The data of the heat exchangers network are the same of those presented in Chapter 4, with exception of the standard values of the discrete design variables of the coolers, presented in Table 30. Additionally, the cooling water in heat exchanger he2 flows in the tube-side and the hot stream inlet temperature, of this same heat exchanger, was 80 ^{o}C and the outlet temperature was 50 ^{o}C.

The search space and its corresponding subsets are shown in Figure 9.

Figure 9 – Search space and its corresponding subsets



Fonte: A autora, 2022

Table 64 presents the total annualized cost values obtained for the cooling water system, for each subset of the search space. It is possible to observe in this table that the lowest-cost solution is located within Subset 1. The optimal solution of this subset is also associated with lower costs in both main components of the system, i.e. the cooling tower and the cooling water system.

Table 64 - Total annualized cost of the cooling water system

Subsets	**Total annualized cost Cooling tower**	**Total annualized cost Cooling water system**	**Total annualized cost of the entire system**
Subset 1	84605.98	68417.68	153023.65
Subset 2	89974.97	70857.22	160832.18
Subset 3	87137.41	68570.43	155707.85
Subset 4	90140.77	69375.04	159515.81

Fonte: A autora, 2022

The optimal values of the cooling tower identified in the Subset 1 are present in Tables 65-69.

Table 65 – Optimal design variables

Variable	Value
Cross sectional area of the cell (m^2)	32
Cell height of each cell (m)	2.280
Type of filling	Splash
Total dry air flowrate (kg/s)	130
Number of cells	2

Fonte: A autora, 2022

Table 66 - Water flowrates for the all cells of the cooling tower

Water flow rate	Value
Evaporated (kg/s)	0.382
Blowdown (kg/s)	0.127
Drift (kg/s)	0.08
Make up (kg/s)	0.589

Fonte: A autora, 2022

Table 67 – Hydraulic results of each cell

Variable	Value
Filling pressure drop (Pa)	39.28
Miscellaneous pressure drop (Pa)	12.43
Gain in kinetic energy (Pa)	7.71
Fan pressure variation (Pa)	59.42
Fan power (W)	4427

Fonte: A autora, 2022

Table 68 - Air flow stream results

Variable	Value
Total humid outlet air flowrate (kg/s)	132.4
Outlet temperature (^oC)	37.17
Outlet air humidity (kg water/kg dry air)	0.019
Outlet humid air density (kg/m^3)	1.116
Outlet air enthalpy (J/kg)	144469

Fonte: A autora, 2022

Table 69 - Annualized costs

Cost	Value
Tower investment ($/year)	64645.84
Make up water ($/year)	9814.04
Fan Operation ($/year)	10146.09
Total cost ($/year)	84605.98

Fonte: A autora, 2022

The results of the optimal cooling tower network are present in Tables 70-82.

Table 70 - Heat exchanger design results (he1)

	Results

Area (m^2)	111.2
Tube diameter (m)	0.025
Tube length (m)	3.659
Number of baffles	20
Number of tube passes	4
Tube pitch ratio	1.25
Shell diameter (m)	0.686
Tube layout	2
Total number of tubes	381
Baffle spacing (m)	0.174

Fonte: A autora, 2022

Table 71 - Heat exchanger design results (he2)

	Results
Area (m^2)	123.5
Tube diameter (m)	0.019
Tube length (m)	3.049
Number of baffles	19
Number of tube passes	4
Tube pitch ratio	1.25
Shell diameter (m)	0.686
Tube layout	2
Total number of tubes	677
Baffle spacing (m)	0.152

Fonte: A autora, 2022

Table 72 - Heat exchanger design results (he3)

	Results
Area (m^2)	186.7
Tube diameter (m)	0.025
Tube length (m)	3.659
Number of baffles	19
Number of tube passes	6
Tube pitch ratio	1.25
Shell diameter (m)	0.889
Tube layout	2
Total number of tubes	640
Baffle spacing (m)	0.183

Fonte: A autora, 2022

Table 73 - Heat exchanger design results (he4)

	Results
Area (m^2)	76.9
Tube diameter (m)	0.032
Tube length (m)	3.659
Number of baffles	19
Number of tube passes	6
Tube pitch ratio	1.25
Shell diameter (m)	0.686
Tube layout	1
Total number of tubes	211
Baffle spacing (m)	0.183

Fonte: A autora, 2022

Table 74 - Thermo-fluid dynamic results (he1)

	Results
Shell-side flow velocity (m/s)	0.85
Tube-side flow velocity (m/s)	1.310
Shell-side coefficient (W/(m^2K))	4173.9
Tube-side coefficient (W/(m^2K))	5665.7
Overall coefficient (W/(m^2K))	963.8
Shell-side pressure drop (Pa)	89341
Tube-side pressure drop (Pa)	20345

Fonte: A autora, 2022

Table 75 - Thermo-fluid dynamic results (he2)

	Results
Shell-side flow velocity (m/s)	1.173
Tube-side flow velocity (m/s)	1.086
Shell-side coefficient (W/(m^2K))	3419.1
Tube-side coefficient (W/(m^2K))	5216.7
Overall coefficient (W/(m^2K))	803.6
Shell-side pressure drop (Pa)	262124
Tube-side pressure drop (Pa)	17088

Fonte: A autora, 2022

Table 76 - Thermo-fluid dynamic results (he3)

	Results
Shell-side flow velocity (m/s)	1.174
Tube-side flow velocity (m/s)	1.207
Shell-side coefficient (W/(m^2K))	1141.3
Tube-side coefficient (W/(m^2K))	5305.2
Overall coefficient (W/(m^2K))	559.3
Shell-side pressure drop (Pa)	163957
Tube-side pressure drop (Pa)	26204

Fonte: A autora, 2022

Table 77 - Thermo-fluid dynamic results (he4)

	Results
Shell-side flow velocity (m/s)	0.517
Tube-side flow velocity (m/s)	1.026
Shell-side coefficient (W/(m^{2}K))	2489.5
Tube-side coefficient (W/(m^{2}K))	4428.5
Overall coefficient (W/(m^{2}K))	809.8
Shell-side pressure drop (Pa)	18176
Tube-side pressure drop (Pa)	15634

Fonte: A autora, 2022

Table 78 - Pump head and valve head losses

Design	Pump head (m)	Head loss he1(m)	Head loss he2(m)	Head loss he3(m)	Head loss he4(m)
Results	7	0.115	0.435	0.123	0.024

Fonte: A autora, 2022

Table 79 – Diameters and head loss of the pipes

Pipe section	Results	
	Diameter (in)	Head loss (m)
pi1	16	0.727
pi2	6	0.820
pi3	12	0.228
pi4	6	0.599
pi5	6	0.599
pi6	6	0.820
pi7	12	0.136
pi8	6	0.315
pi9	8	0.152
pi10	8	0.152
pi11	5	0.429
pi12	5	0.429
pi13	6	0.315
pi14	12	0.136
pi15	12	0.228
pi16	18	0.418
pi17	16	0.015

Fonte: A autora, 2022

Table 80 – Water outlet temperature

Heat exchanger	he1	he2	he3	he4
Water outlet temperature (^{o}C)	39.8	39.7	39.8	39.9

Fonte: A autora, 2022

Table 81 – Water temperature in the return to the tower

Water temperature in return to the tower (^oC)	39.8

Fonte: A autora, 2022

Table 82 - Annualized costs

Cost ($/year)	Results
Pump cost	1134.91
Heat exchanger cost	39665.39
Pipe cost	12373.02
Operation cost	15244.36
Total cost	68417.68

Fonte: A autora, 2022

The System with minimum total annualized cost selected by the optimization corresponds to Subset 1.

Observing the values of the total annualized cost of the cooling tower for the Subsets 1 and 2, shown in Table 64 it can be seen that there was an increase of 6.3%. Comparing the values for Subsets 3 and 4, there is an increase of 3.4%.

Table 83 presents the annualized costs obtained for the items of the cooling water tower selected in Subset 2. It can be observed that the tower investment and fan operation costs are lower than those found for Subset 1 (Table 69) and while the make up water cost is superior.

This is because the cooling water tower selected for Subset 2 has the same area and height values as for Subset 1, however the total dry air flowrate used is 115 kg/s, lower than the value used for Subset 1. In Subset 2 the make up water flowrate is 1.17 kg/s, higher than the value used in Subset 1.

In Subset 2, the cooling water temperature difference is smaller than in Subset 1, so the resulting water flowrate in Subset 2 is higher than in Subset 1, which results in higher evaporation and higher blowdown flowrate, respectively 0.4 kg/s e 0.136 kg/s.

Table 83 - Annualized costs of the cooling tower of Subset 2

Cost	Value
Tower investment ($/year)	62513.88
Make up water ($/year)	19513.90
Fan Operation ($/year)	7947.18
Total cost ($/year)	89974.96

Fonte: A autora, 2022

Observing the values of total annualized cost of the cooling tower for Subset 1 and Subset 3, presented in Table 69, it can be seen that there was an increase of 3%. Comparing the values found for Subsets 1 and 4, there was an increase of 6%,

Table 84 presents the annualized costs obtained for the items of the cooling water tower selected in Subset 3. It can be observed that the tower investment and fan operation costs are lower than those found for Subset 1 and while the make up water cost is superior. Because the higher water temperature in Subset 3 results in greater evaporation, which is demonstrated by the outlet air humidity of 0.021 kg water/kg dry air, higher than the value obtained in Subset 1. The outlet air temperature, in the Subset 3 was 37.99 ^{o}C, also higher than that obtained for Subset 1. The subset 3 presents a make up water flowrate of 0.95 kg/s, higher than the value used in Subset 1.

Table 84 - Annualized costs of the cooling tower for Subset 3

Cost	Value
Tower investment ($/year)	63224.53
Make up water ($/year)	15825.62
Fan Operation ($/year)	8087.26
Total cost ($/year)	87137.41

Fonte: A autora, 2022

Table 85 presents the values of the annualized costs obtained for the items of the cooling water tower of Subset 4. It is verified that the tower investment and the fan operation cost present lower values than those obtained in Subset 1. This is because the selected tower has the same area and height as Subset 1, but the total dry air flowrate obtained is 115 kg/s. The make up water cost obtained for Subset 4 is higher than for Subset 1, this is because the evaporate and blowdown flow rates of Subset 4 are respectively 0.84 and 0.28 kg/s, higher than the values found in Subset 1. Because the higher water temperature in Subset 4 results in greater evaporation, which is demonstrated by the outlet air humidity of 0.023 kg water/kg dry air, higher than the value obtained in Subset 1. The outlet air temperature, in the Subset 4, was 38.68 ^{o}C, also higher than that obtained for Subset 1.

Table 85 - Annualized costs of the cooling tower for Subset 4

Cost	Value
Tower investment ($/year)	62513.87
Make up water ($/year)	19969.78
Fan Operation ($/year)	7657.12
Total cost ($/year)	90140.77

Fonte: A autora, 2022

Observing the values of the total annualized cost of the cooling water system for Subsets 1 and 2, shown in Table 69, it can be seen that there was an increase of 3.5%. The 1.2% increase can be verified by comparing the values found for Subsets 3 and 4.

Table 86 presents the annualized costs obtained for the items of the cooling water system selected in Subset 2. It can be observed that the heat exchanger cost remained the same as in Subset 1, but all other items presented higher costs. Although the pump head selected for Subset 2 is the same as for Subset 1, the cooling water flow rate is higher in subset 2 (since the difference between the entering and leaving water temperatures of the heat exchangers is smaller), which leads to a more expensive pump, operating and piping costs due to the consequent higher pressure drop.

Table 86 **-** Annualized costs of the cooling water system for Subset 2

Cost ($/year)	**Results**
Pump cost	1193.45
Heat exchanger cost	39665.39
Pipe cost	13436.97
Operation cost	16561.41
Total cost	70857.22

Fonte: A autora, 2022

Comparing the values of the total annualized cost of the cooling water system for Subset 1 and Subset 3, presented in Table 64, it can be seen that there was a small increase (0.2%). While the value found for Subset 4 represents an increase of 1.2% in relation to Subset 3. These variations in costs are not significant.

It appears that the cost variations for the cooling tower were more significant than for the cooling water network. The costs of the cooling water tower were more sensitive to temperatures in the analyzed range than the cooling water network.

CONCLUSIONS AND SUGGESTIONS

This chapter presents conclusions and suggestions for improvement around the study that was presented and discussed in this thesis.

Conclusions

This thesis presents the resolution of four problems about cooling water systems. These problems address different scope levels of the design task involving cooler water systems, ranging from the design of the cooling tower only to the design of the entire system encompassing the cooling tower, pump, pipe sections and heat exchangers, associated with the interconnections of all models in a recursive algorithm. This set of solution alternatives allows the designer to choose the problem option according to his/her needs.

The conclusions about each problem are discussed in the next paragraphs.

- Optimization of Cooling Water Systems Including the Design of Heat Exchangers

The first problem corresponds to the optimization of a cooling water network, which includ heat exchangers, pipes and pumps, for a given pair of values of the inlet and outlet temperatures.

As presented in the literature review, the previous attempts to solve the design problem of cooling water networks are dominated by nonlinear mathematical programming solutions, typically MINLP problems, which leads to convergence problems and the need for good initial estimates. Although a previous work solved the design problem using a linear formulation (Souza et al., 2014), it was limited to the pipe network, without including the heat exchangers. Aiming at to eliminate this limitations, a new formulation for the design problem based on a mixed-integer linear programming (MILP) was developed. The linearity of the formulation allows attaining the global optimum, without the need of a good initial estimate or a specialized global solver. The results obtained showed that the simultaneous design presented a lower total annualized cost than the two-step designs A (the coolers are designed seeking to minimize the heat transfer surface according to maximum pressure drop constraints, and then the pipe network is optimized considering the total annualized cost of the pipe sections and pumps) and B (The coolers are first optimized using the total annualized cost associated with the corresponding capital and operational costs, and then the pipe network is designed through the minimization of the total annualized cost of the pipe sections and pumps), which presented solutions in steps similar to the procedures performed by the industry.

- Optimization of Cooling Water Systems Together with the Distribution of the Flow Rates

The optimization of a cooling water network was performed, including heat exchangers, piping and pump, with the temperature of the water return to the tower and at the outlet of the exchangers as variables. The obstacles of the convergence of the resultant MINLP formulation and of the dimension of the linear formulation were overcome through the development of an optimization procedure, which employs Set trimming technique, solution of a system of nonlinear algebraic equations of the heat exchanger model, according to the LMTD method and Identification of the optimal solution through a decomposition procedure involving two MILPs problems. The results obtained showed that the iterative method showed a cost improvement of 5.2% in relation to the simpler model with a fixed flow rate distribution.

- Optimization of a Mechanical Induced Draft Counter Flow Cooling Water Tower

As shown in the literature review of the design optimization of cooling towers, there is the dominance of two types of methods: metaheuristic methods and mathematical programming. However, Mathematical Programming presents nonconvergence problems for nonlinear models, such as those models employed for the optimization of cooling towers and global optimality cannot be guaranteed by Metaheuristic methods. So, this thesis presents a solution procedure for the cooling tower design problem that avoids the cited drawbacks: Set Trimming + Smart Enumeration (Costa and Bagajewicz, 2019).

- Optimization of a Cooling Water System Consisting of a Cooling Tower, Heat Exchangers, Pipe Sections and Pump

The literature review shows that with the exception of Ponce-Ortega et al. (2010), the previous papers in the literature employed simplified models for the heat exchangers in the analysis of cooling water systems. Ponce-Ortega et al. (2010) included a heat exchanger model based on the Kern correlations, but it ignores the pressure drop in the pipe sections. Avoiding the limitations presented in the literature, this thesis proposed a general formulation, including all the elements: cooling tower, pump, pipe sections and heat exchangers, represented by the Kern model. The most general formulation corresponds to a MINLP, but instead to use a single mathematical programming approach, the proposed procedure is based on the split of the search space, followed by a recursive convergence of the design of the cooler water network, using a conventional MILP algorithm, and the design of the cooling tower, using Set Trimming + Smart Enumeration. Despite global optimality cannot be guaranteed, the robustness of the proposed approach allows an easier utilization. The results showed that a cooling water system with the lowest annualized total cost was selected and that the cost variations for the cooling tower were more significant than for the cooling water network.

Suggestions

The investigated problem could be extended to consider other alternatives for the cooling water system:

- Optimization of a cooling water system with multiple pumps and cells in the cooling water tower.
- Utilization of the same approach of this work for the optimization of a cross-current cooling water tower. Therefore, the two cooling water tower options (cross-current and counter-current) could be inserted into into the optimization of the cooling water system, including heat exchangers, pumps and piping.

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ANNEX I – MILP formulations

In this item, part of the work of Gonçalves et al. (2017) is presented, in order to explain how the model that was used for the development of this work.

I. DEVELOPMENT OF THE MILP FORMULATIONS

The new MILP formulations are built starting from the MINLP model through three main steps: the organization of the data table of the discrete variables, the model reformulation, and the conversion to a linear model.

I.1. **Organization of the data table of the discrete variables**

The original relations between the discrete variables and the corresponding binaries are given by:

$$dte = \sum_{sd=1}^{sdmax} \widehat{pdte}_{sd}\, yd_{sd} \tag{202}$$

$$dti = \sum_{sd=1}^{sdmax} \widehat{pdti}_{sd}\, yd_{sd} \tag{203}$$

$$Ds = \sum_{sDs=1}^{sDsmax} \widehat{pDs}_{sDs}\, yDs_{sDs} \tag{204)}$$

$$lay = \sum_{slay=1}^{slaymax} \widehat{play}_{slay}\, ylay_{slay} \tag{205)}$$

$$Npt = \sum_{sNpt=1}^{sNptmax} \widehat{pNpt}_{sNpt}\, yNpt_{sNpt} \tag{206)}$$

$$rp = \sum_{srp=1}^{srpmax} \widehat{prp}_{srp} yrp_{srp} \tag{207)}$$

$$L = \sum_{sL=1}^{sLmax} \widehat{pL}_{sL}\, yL_{sL} \tag{208)}$$

$$Nb = \sum_{sNb=1}^{sNbmax} \widehat{pNb}_{sNb}\, yNb_{sNb} \tag{209)}$$

with the following equations needed to guarantee only one choice among many:

$$\sum_{sd=1}^{sdmax} yd_{sd} = 1 \tag{210}$$

$$\sum_{sDs=1}^{sDsmax} yDs_{sDs} = 1 \tag{211}$$

$$\sum_{slay=1}^{slaymax} ylay_{slay} = 1 \tag{212}$$

$$\sum_{sNpt=1}^{sNptmax} yNpt_{sNpt} = 1 \tag{213}$$

$$\sum_{srp=1}^{srpmax} yrp_{srp} = 1 \tag{214}$$

$$\sum_{sL=1}^{sLmax} yL_{sL} = 1 \tag{215}$$

$$\sum_{sNb=1}^{sNbmax} yNb_{sNb} = 1 \tag{216}$$

According to the aggregation strategy employed in the development of the new MILP formulation, the parameters that represent the discrete values can be grouped in one or more tables. Therefore, several discrete values of the design variables are identified by the same index (a multi-index related to the corresponding original indices). The multi-index *srow* represents the discrete values of all design variables. The corresponding set of parameters are defined from the original ones, as follows:

$$\widehat{Pdte}_{srow} = \widehat{pdte}_{sd}$$

217)

$$\widehat{Pdti}_{srow} = \widehat{pdti}_{sd}$$

218)

$$\widehat{PDs}_{srow} = \widehat{pDs}_{sDs}$$

219)

$$\widehat{Play}_{srow} = \widehat{play}_{slay}$$

220)

$$\widehat{PNpt}_{srow} = \widehat{pNpt}_{sNpt}$$

221)

$$\widehat{Prp}_{srow} = \widehat{prp}_{srp}$$

222)

$$\widehat{PL}_{srow} = \widehat{pL}_{sL}$$

223)

$$\widehat{PNb}_{srow} = \widehat{pNb}_{sNb}$$

224)

Consequently, different discrete variables become associated with the same set of binaries. All discrete variables are described by the set of binaries $yrow_{srow}$, thus yielding:

$$dte = \sum_{srow} \widehat{Pdte}_{srow}\, yrow_{srow}$$

225)

$$dti = \sum_{srow} \widehat{Pdti}_{srow}\, yrow_{srow} \quad 226)$$

$$Ds = \sum_{srow} \widehat{PDs}_{srow}\, yrow_{srow} \quad 227)$$

$$lay = \sum_{srow} \widehat{P}lay_{srow}\, yrow_{srow} \quad 228)$$

$$Npt = \sum_{srow} \widehat{PNpt}_{srow}\, yrow_{srow} \quad 229)$$

$$rp = \sum_{srow} \widehat{Prp}_{srow} yrow_{srow} \quad 230)$$

$$L = \sum_{srow} \widehat{PL}_{srow}\, yrow_{srow} \quad 231)$$

$$Nb = \sum_{srow} \widehat{PNb}_{srow}\, yrow_{srow} \quad 232)$$

$$\sum_{srow} yrow_{srow} = 1 \quad 233)$$

I.2. **Model reformulation**

In this step, the model equations are modified through the substitution of the discrete variables by their binary representation. This reformulation step also involves a procedure for the organization of the resultant expressions containing binary variables, as described in the following paragraphs.

The substitution of a set of discrete variables p, q, … , z by its binary representation in the heat exchanger model yields terms of the form $p^{n1}q^{n2} \cdots z^{nm}$ that are substituted as follows:

$$p^{n1}q^{n2} \cdots z^{nm} = [\textstyle\sum_i \widehat{pd}_i\, yp_i]^{n1}\, [\sum_j \widehat{qd}_j\, yq_j]^{n2}\, [\sum_k \widehat{zd}_k\, yz_k]^{nm} \quad (234)$$

Because all the binary variables are equal to 1 only once in the corresponding set, this equation is equivalent to:

$$p^{n1}q^{n2}\cdots z^{nm} = \sum_{i,j,..k} \widehat{pd}_i^{\,n1}\widehat{qd}_j^{\,n2} \ldots\ldots \widehat{qd}_k^{\,nm}\; yp_i\; yq_j \ldots yz_k \tag{235}$$

After the application of this procedure, the reformulated model becomes composed of several expressions containing multiple summations of products of binary variables.

I.3. **Conversion to a linear model**

The product of binaries obtained from the discrete variable substitution can be reorganized in equivalent linear expressions.

Let the product of binaries be substituted by a nonnegative variable $w_{i,j,\ldots,k}$:

$$p^{n1}q^{n2}\cdots z^{nm} = \sum_{i,j,..k} \widehat{pd}_i^{\,n1}\widehat{qd}_j^{\,n2} \ldots\ldots \widehat{qd}_k^{\,nm}\; w_{i,j,\ldots k} \tag{236}$$

where:

$$w_{i,j,\ldots,k} = yp_i\, yq_j \ldots yz_k \tag{237}$$

However, the nonlinearity existent in this equation can be eliminated through the substitution of this expression by the equivalent set of linear inequality constraints:

$$w_{i,j,\ldots,k} \leq yp_i \tag{238}$$

$$w_{i,j,\ldots,k} \leq yq_j \tag{239}$$

$$w_{i,j,\dots,k} \leq yz_k \tag{240}$$

$$w_{i,j,\dots,k} \geq yp_i + yq_j + \cdots + yz_k - (m-1) \tag{241}$$

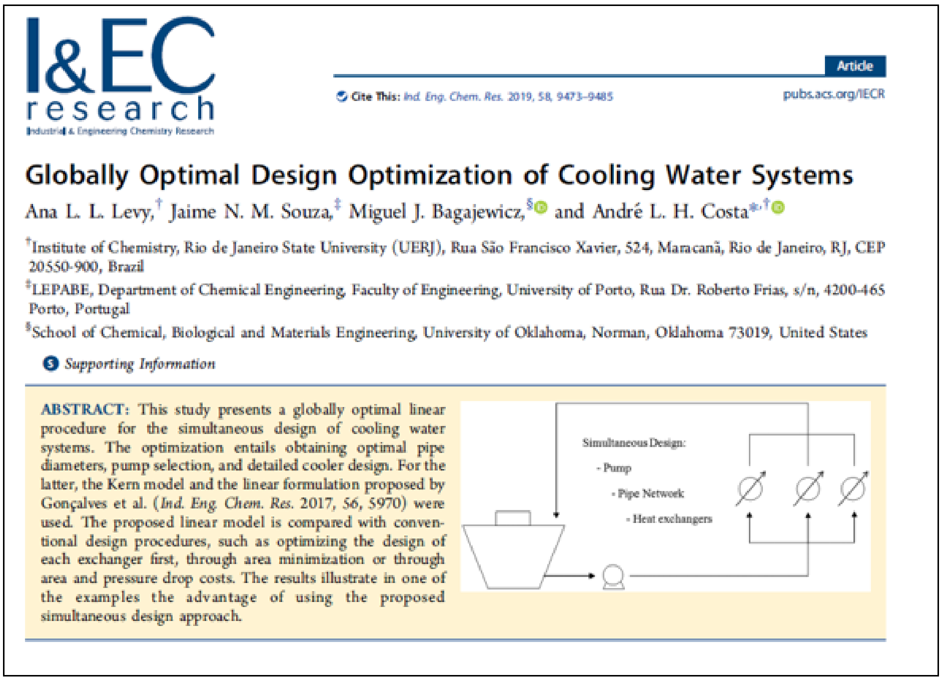
where m is the number of binary variables in the product.

ANNEX II – Published **a**rticles

This annex presents the articles published in an international journal resultant of the work developed in this doctoral thesis (shown in Figure 10).

LEVY, ANA L. L.; SOUZA, J. N. M.; BAGAJEWICZ, M. J.; COSTA, A. L. H. *Globally Optimal Design Optimization of Cooling Water System*, Industrial & Engineering Chemistry Research, v. 58, p. 9473-9485, 2019.

Figure 10 - Article published in an international journal



I&EC research
Industrial & Engineering Chemistry Research

Article

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Globally Optimal Design Optimization of Cooling Water Systems

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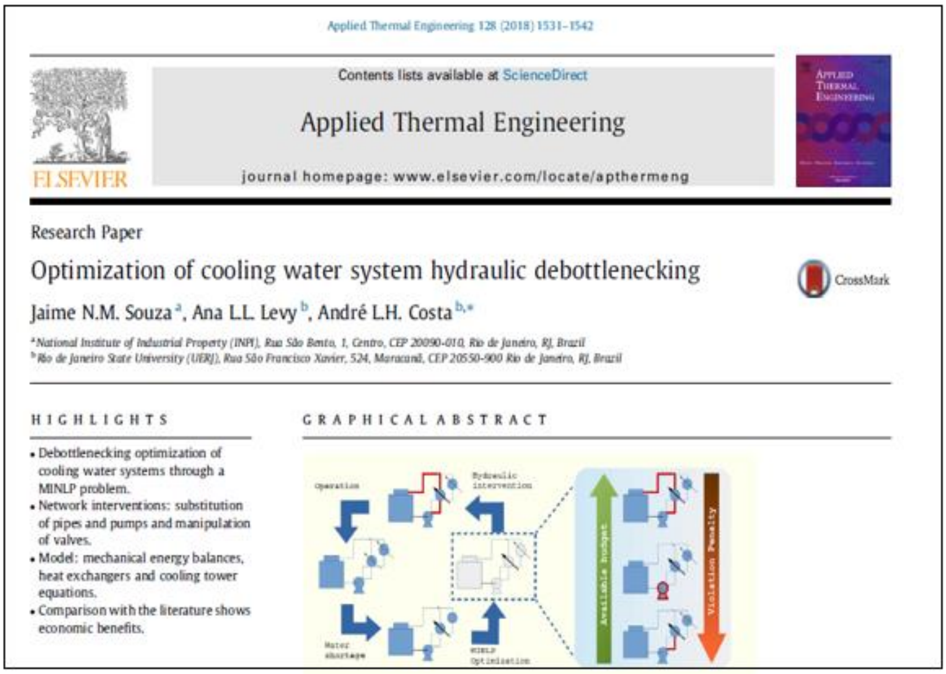
Supporting Information

ABSTRACT: This study presents a globally optimal linear procedure for the simultaneous design of cooling water systems. The optimization entails obtaining optimal pipe diameters, pump selection, and detailed cooler design. For the latter, the Kern model and the linear formulation proposed by Gonçalves et al. (Ind. Eng. Chem. Res. 2017, 56, 5970) were used. The proposed linear model is compared with conventional design procedures, such as optimizing the design of each exchanger first, through area minimization or through area and pressure drop costs. The results illustrate in one of the examples the advantage of using the proposed simultaneous design approach.

Additionally, during the development of this thesis, another paper about cooling water systems was also published (shown in Figure 11).

SOUZA, J. N. M.; LEVY, A. L. L.; COSTA, A. H. *Optimization of Cooling Water System Hydraulic Debottlenecking*, Applied Thermal Engineering, v. 128, p. 1531–1542, **2018**.

Figure 11- Article published in an international journal



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Research Paper

Optimization of cooling water system hydraulic debottlenecking

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HIGHLIGHTS

- Debottlenecking optimization of cooling water systems through a MINLP problem.
- Network interventions: substitution of pipes and pumps and manipulation of valves.
- Model: mechanical energy balances, heat exchangers and cooling tower equations.
- Comparison with the literature shows economic benefits.

GRAPHICAL ABSTRACT